Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

MONASH
University

Universal Quality Metrics for Graph Drawings: Which Graphs Excite Us Most?

Gavin J. Mooney **Tim Hegemann**

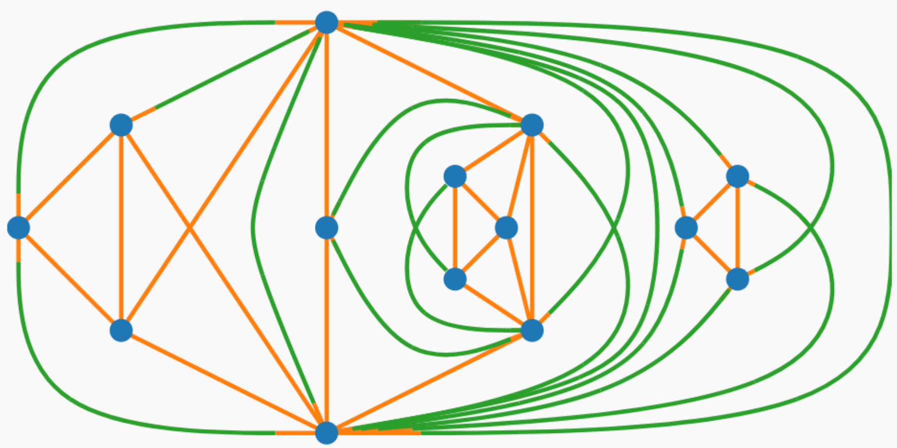
Alexander Wolff Michael Wybrow Helen C. Purchase

The 33rd International Symposium on Graph Drawing and Network Visualization

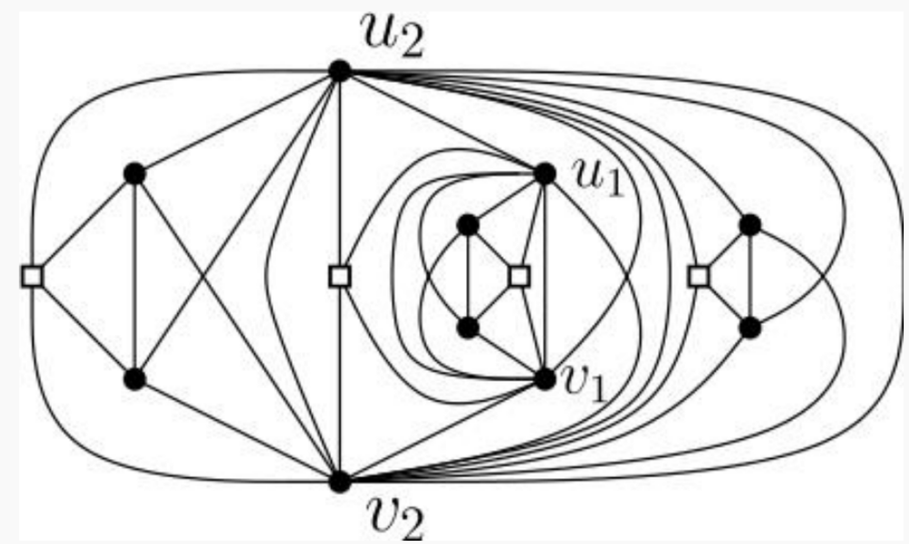
At Last Year's GD...

[Deynet, Hegemann, Kempf, Wolff. GD'24]
graph drawing from [Bekos et al. J. DAM 2022]

Previous 12/18 Next New Upload



Graph produced by Graph Harvester



Original from PDF

Fig. 8. Illustration for the hierarchical contraction technique. Thick edges are drawn thicker (and red).

Graph6 String	HoG Link
>>graph6<<M`rMEFo{M@x_ooKK?	Not found on HoG. Add/Edit via this link

Extracting Drawings

Opportunity: What does the metrics landscape of drawings from GD look like?

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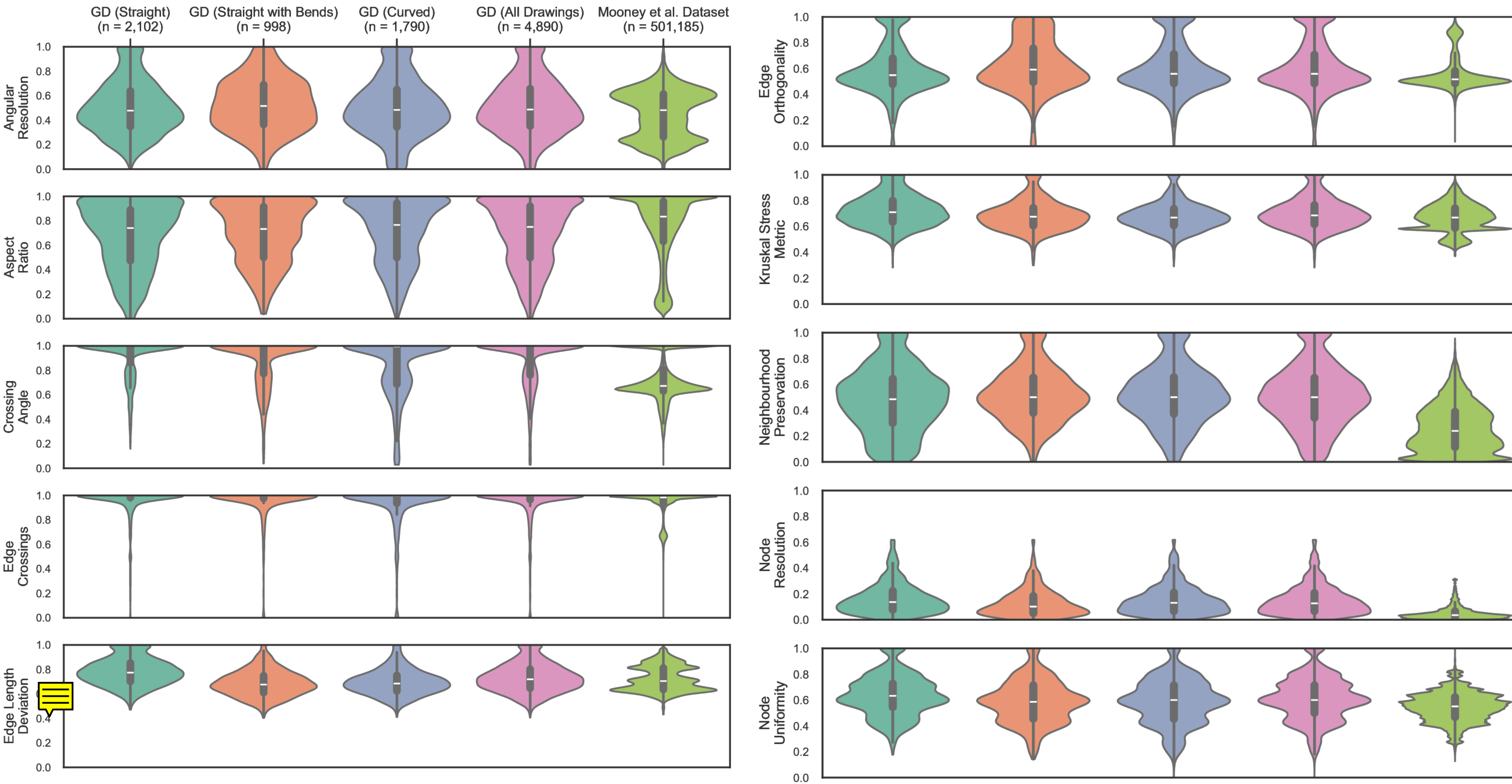

Challenges

- **Extracting drawings**
  - detecting figures
  - bitmap images
  - varied drawing styles
  - and many more...
- **Evaluating drawings with curves**
  - non-standard representation/format
  - existing metrics assume straight-line edges

Challenges: Extracting Graphs

The Graph Harvester Rulebook

Nodes

- squares or circles
- dark fill or stroke
- clearly separated

Challenges: Extracting Graphs

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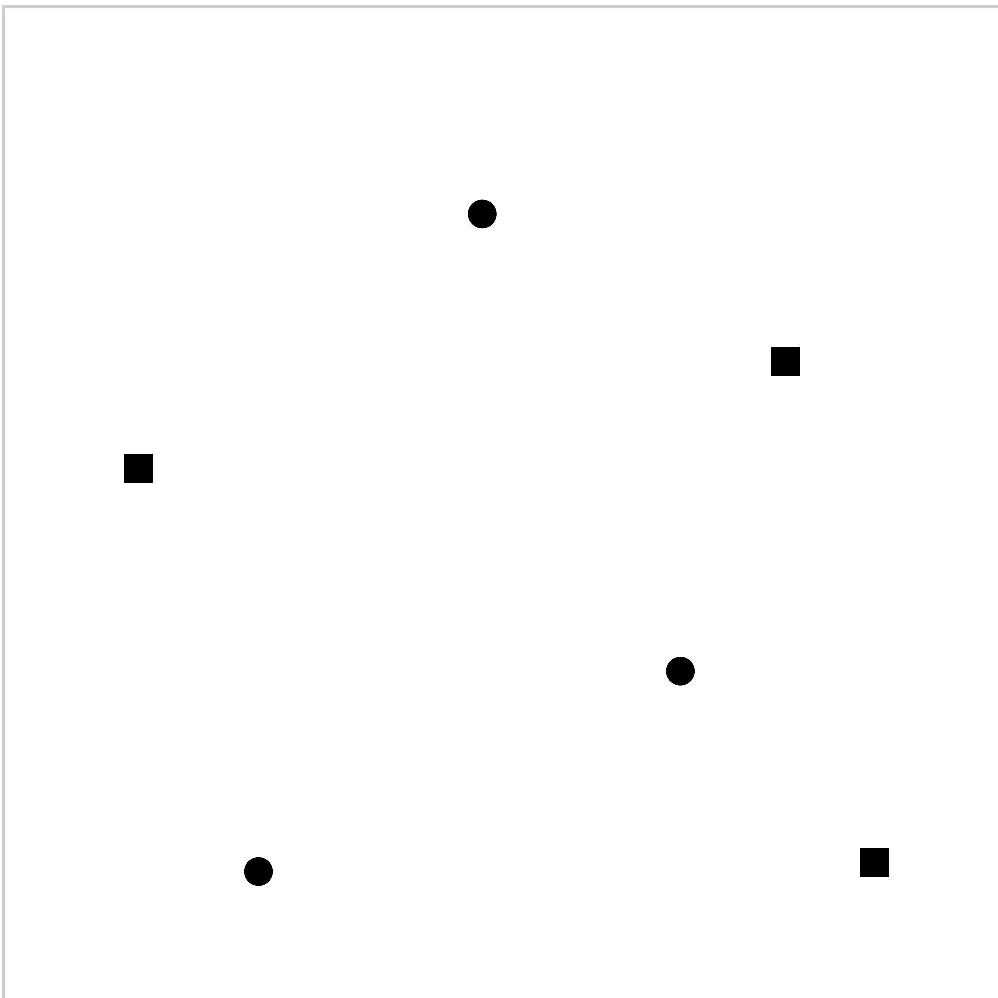

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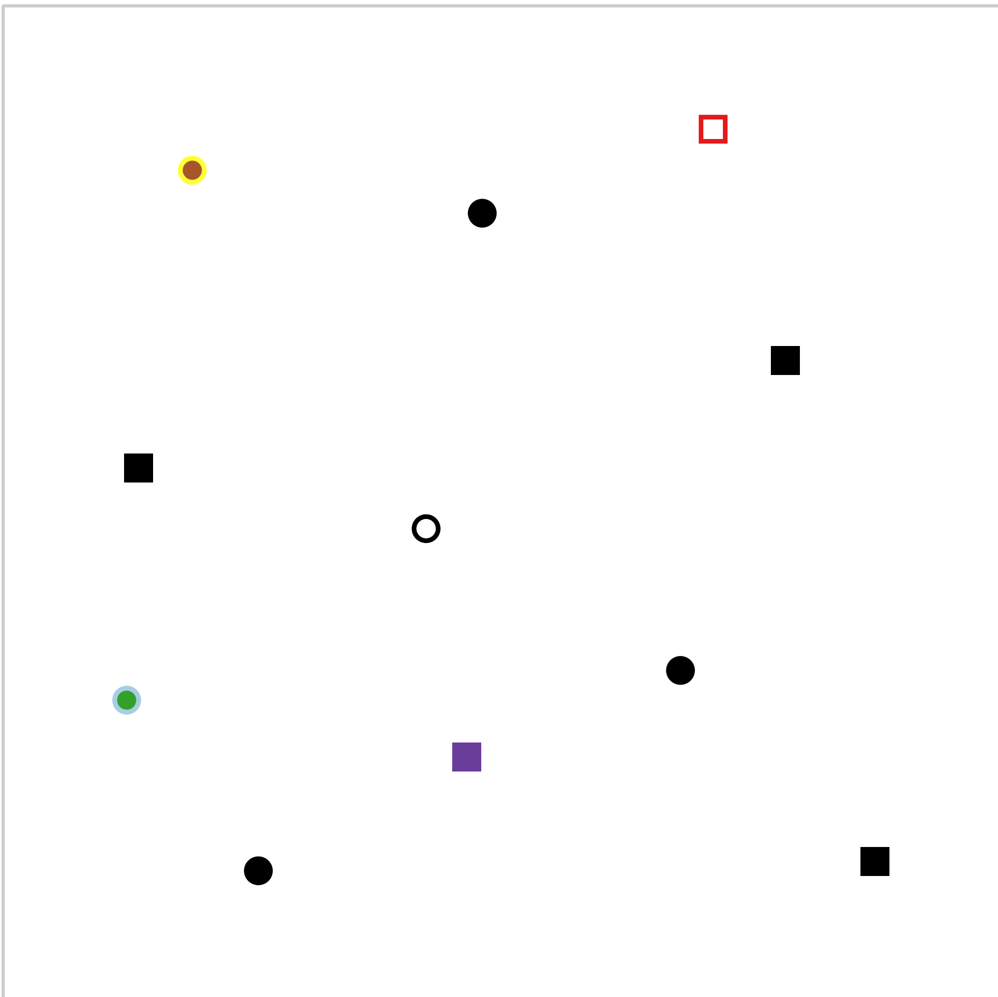

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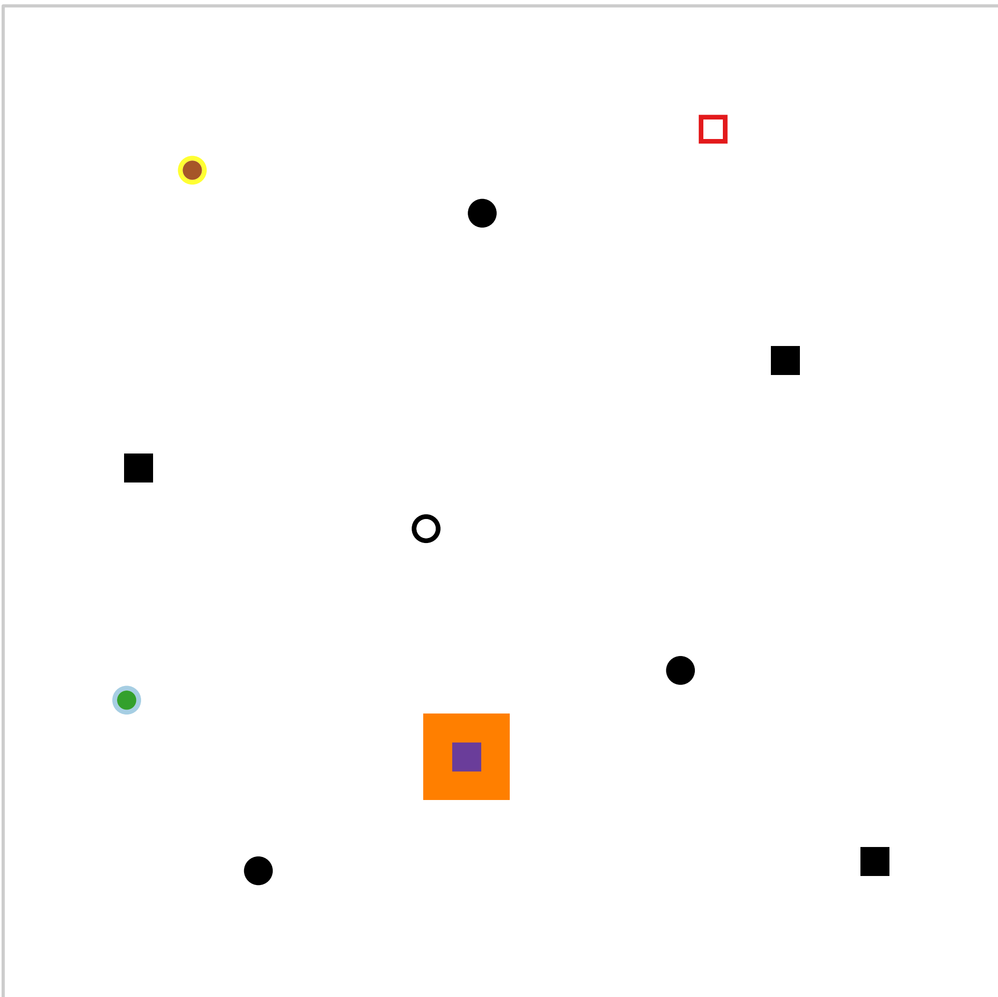

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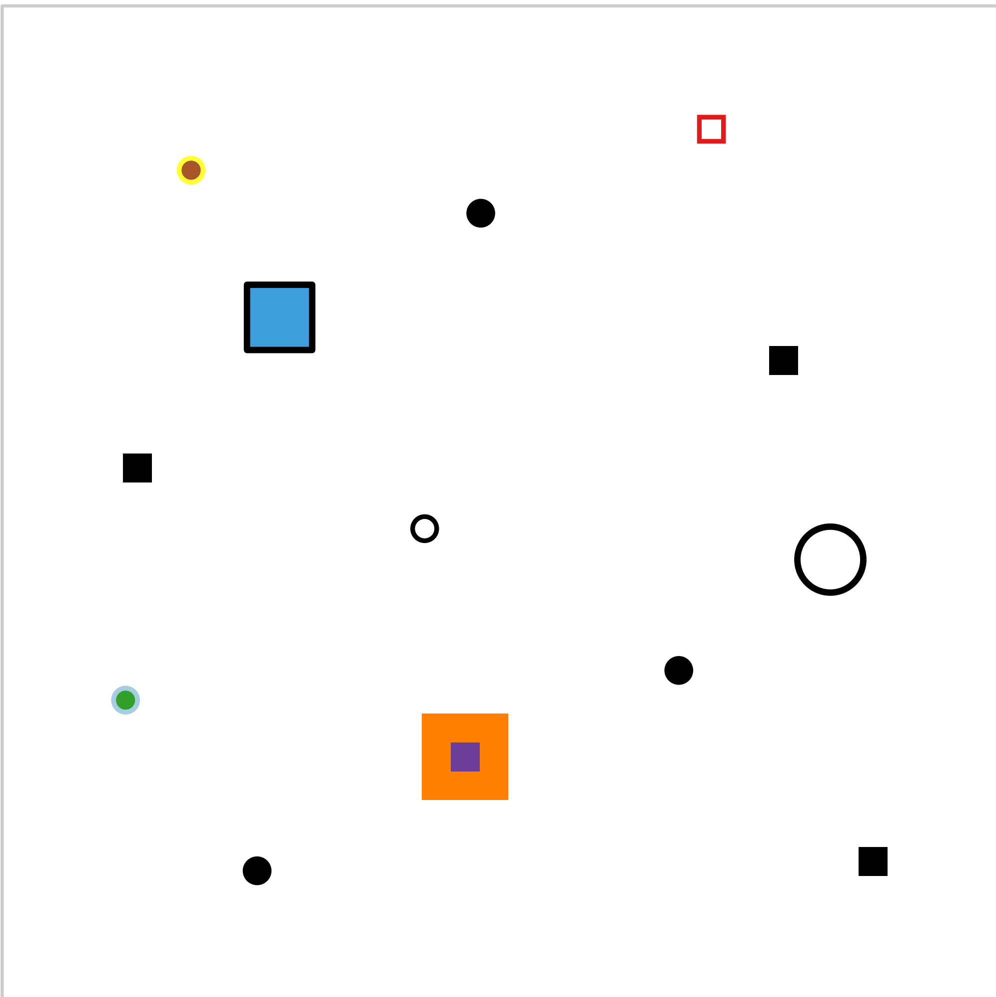

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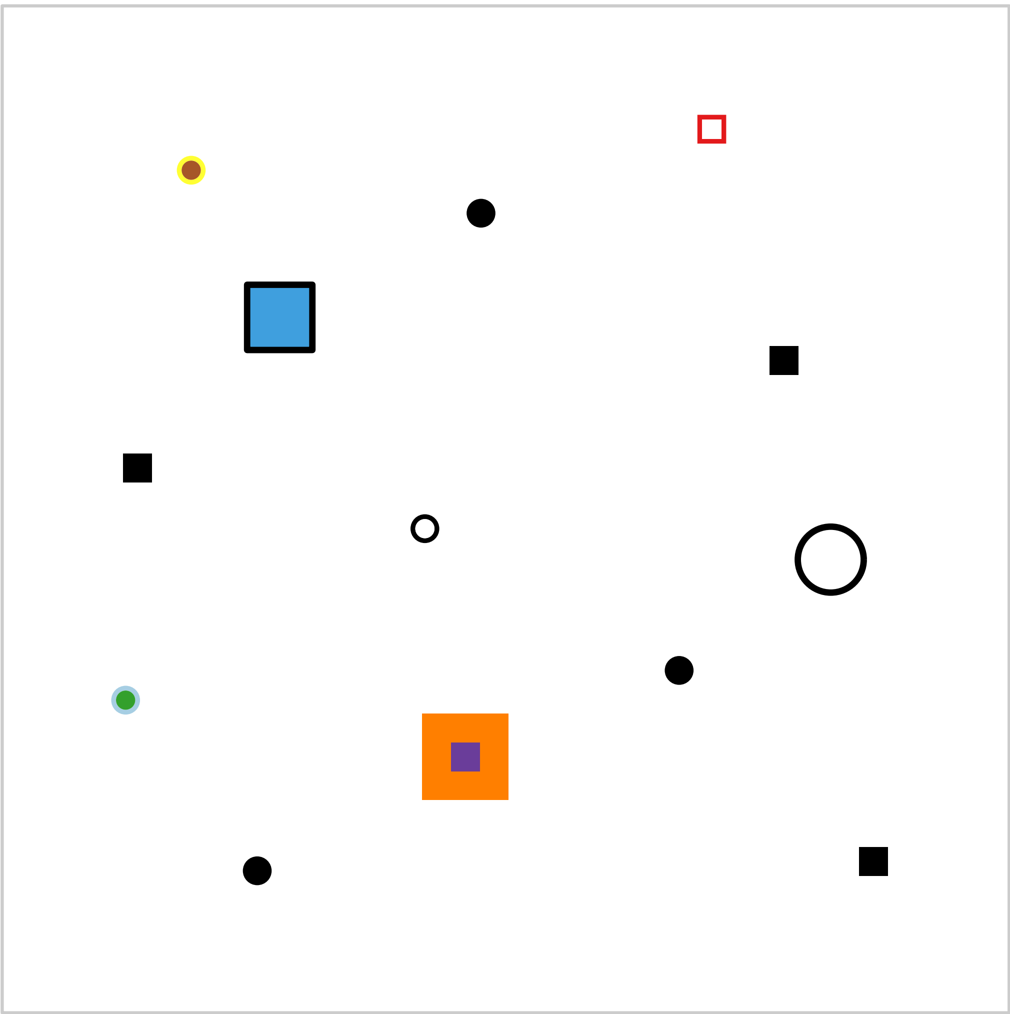

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Edges

- sequence of line segments or Bézier curves
- dark stroke
- one node at each end

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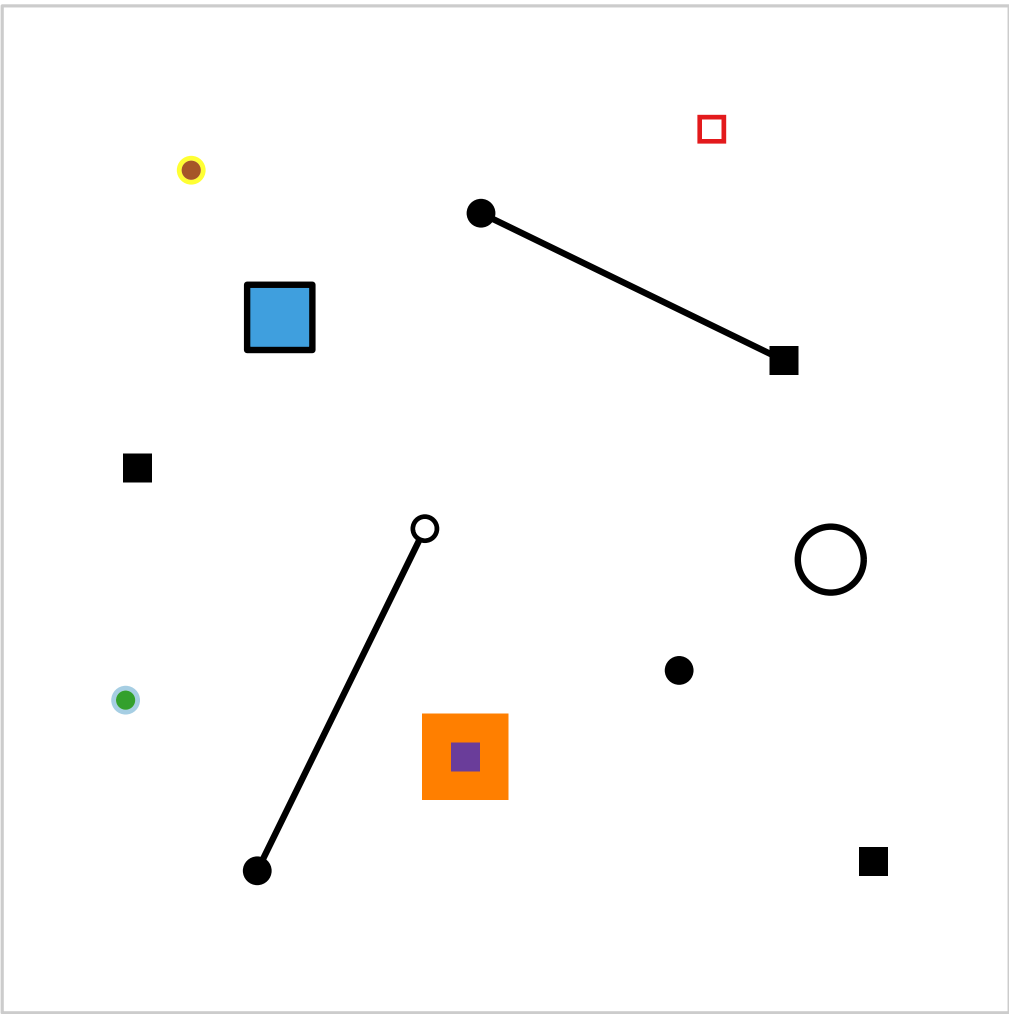

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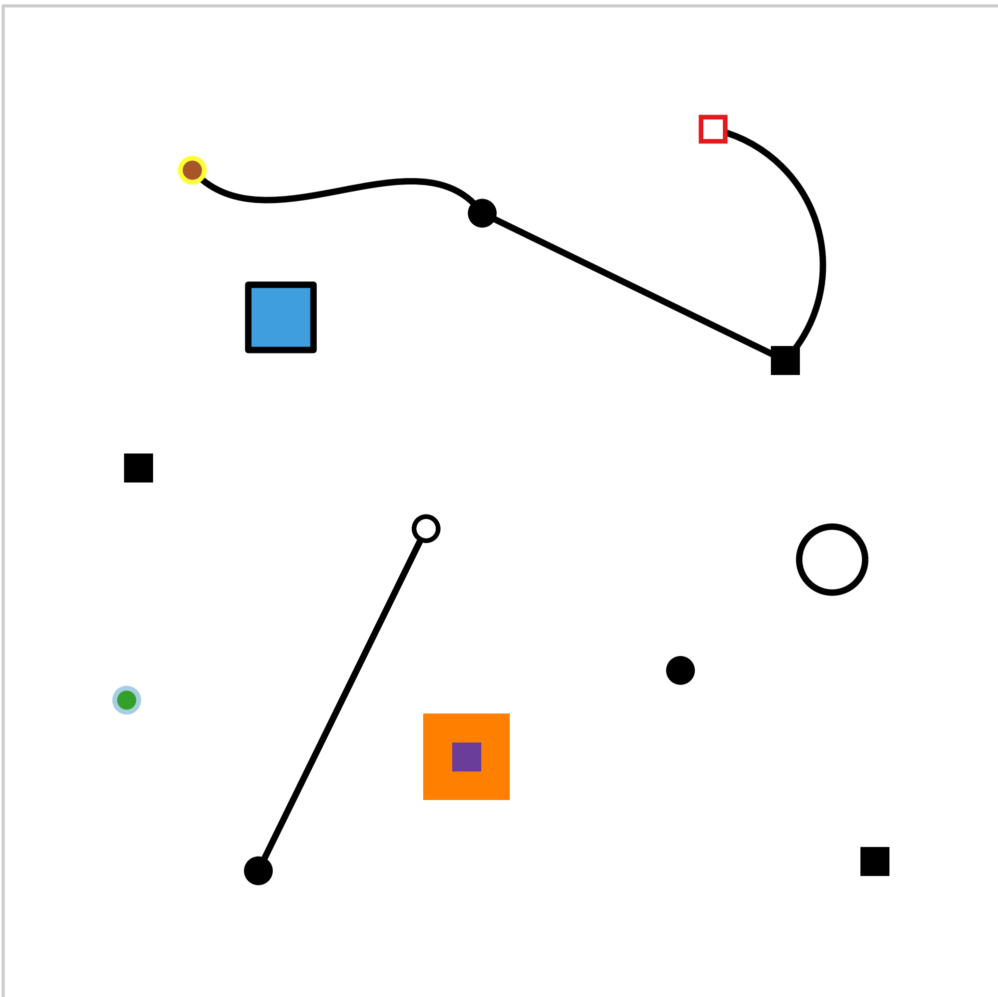

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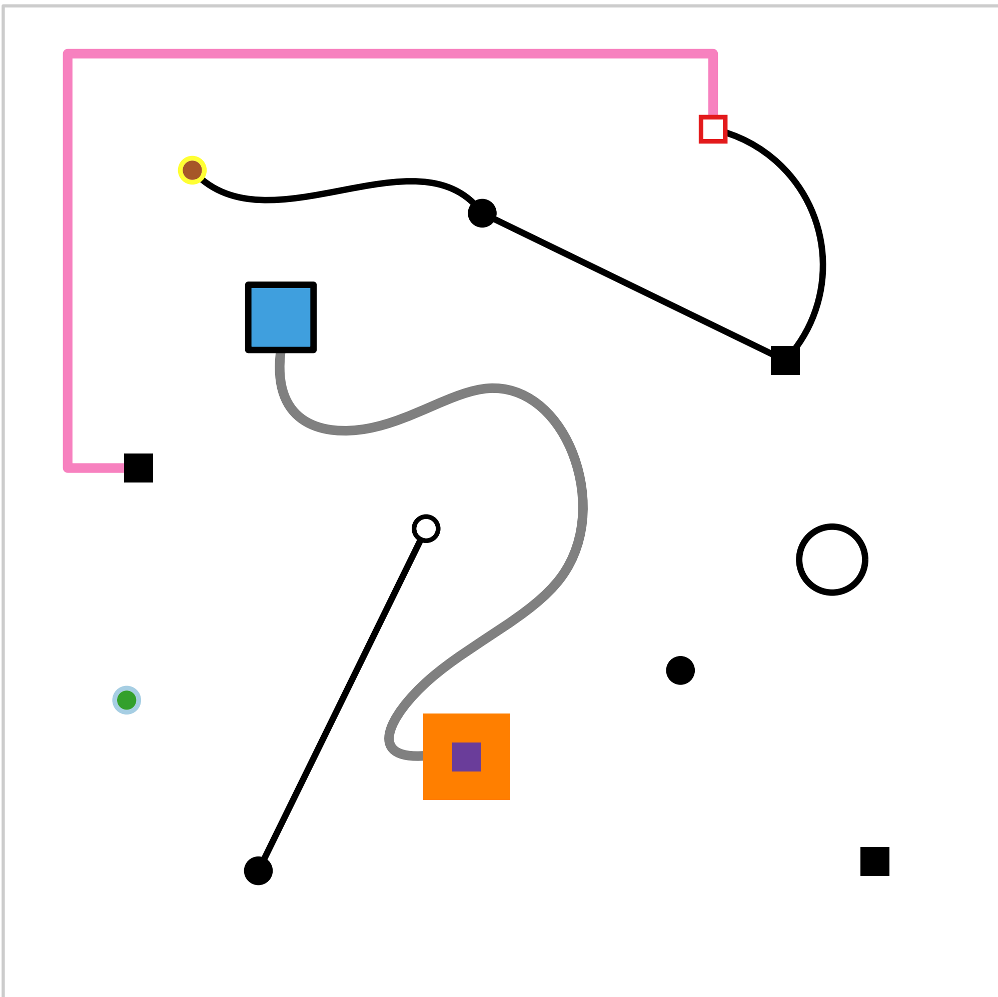

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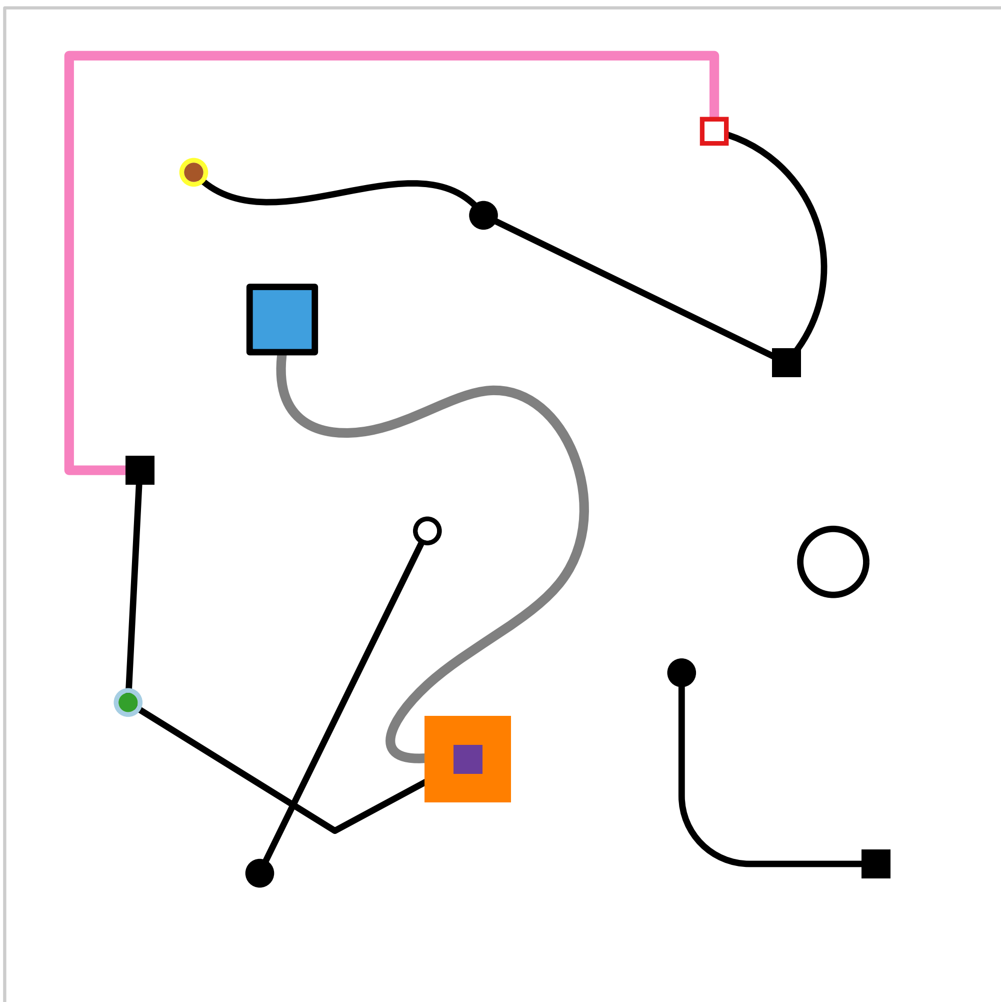

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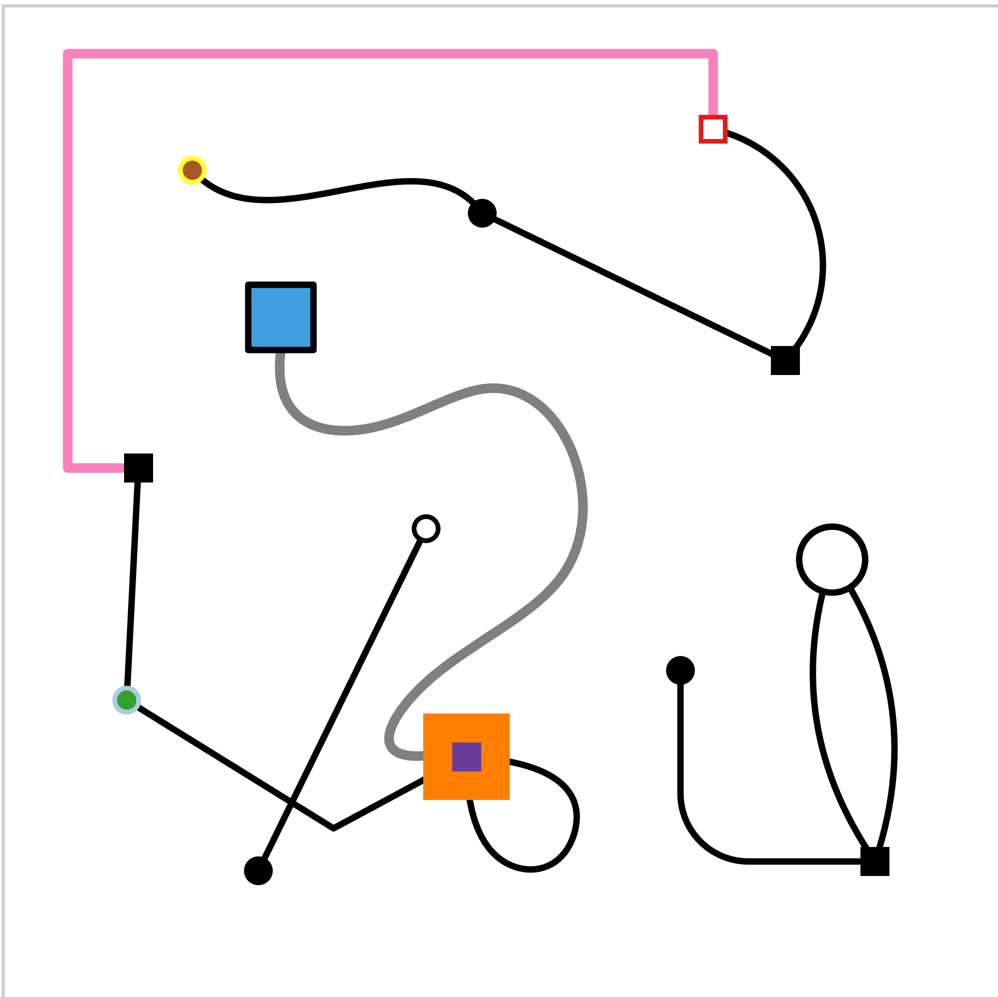

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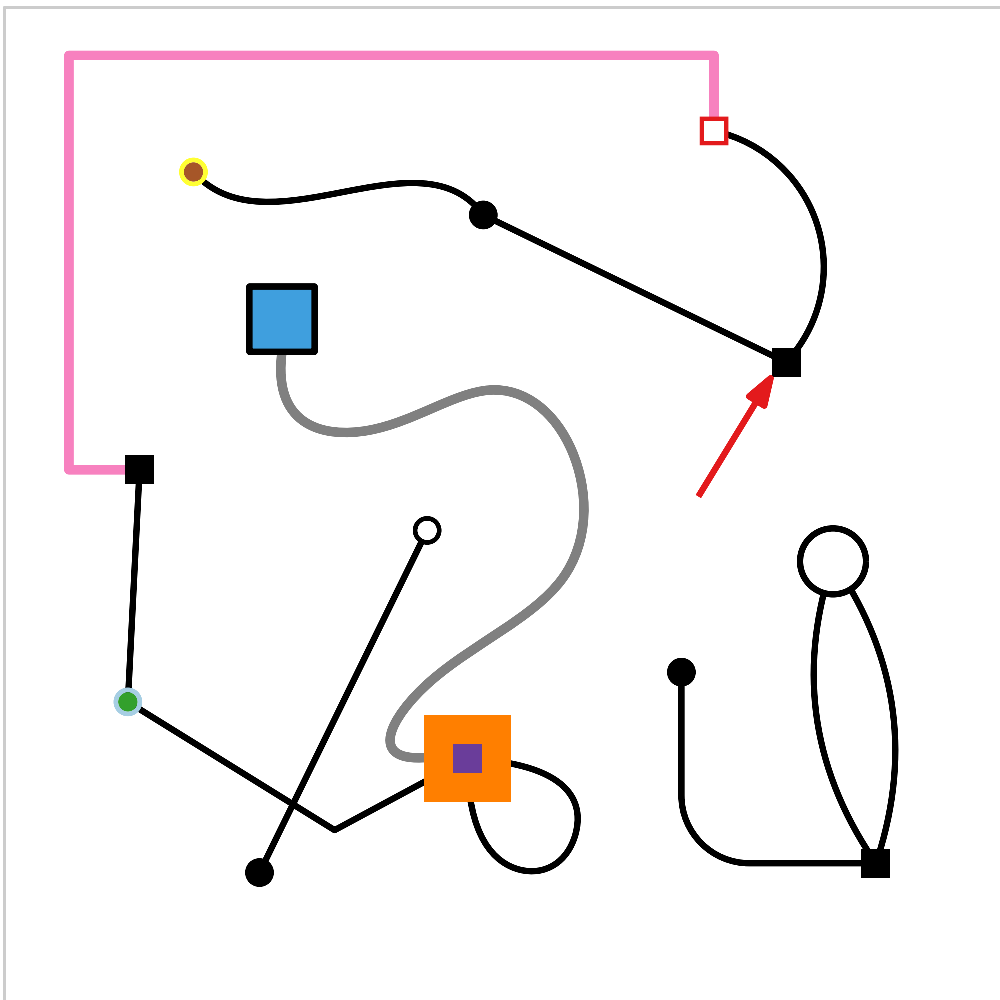

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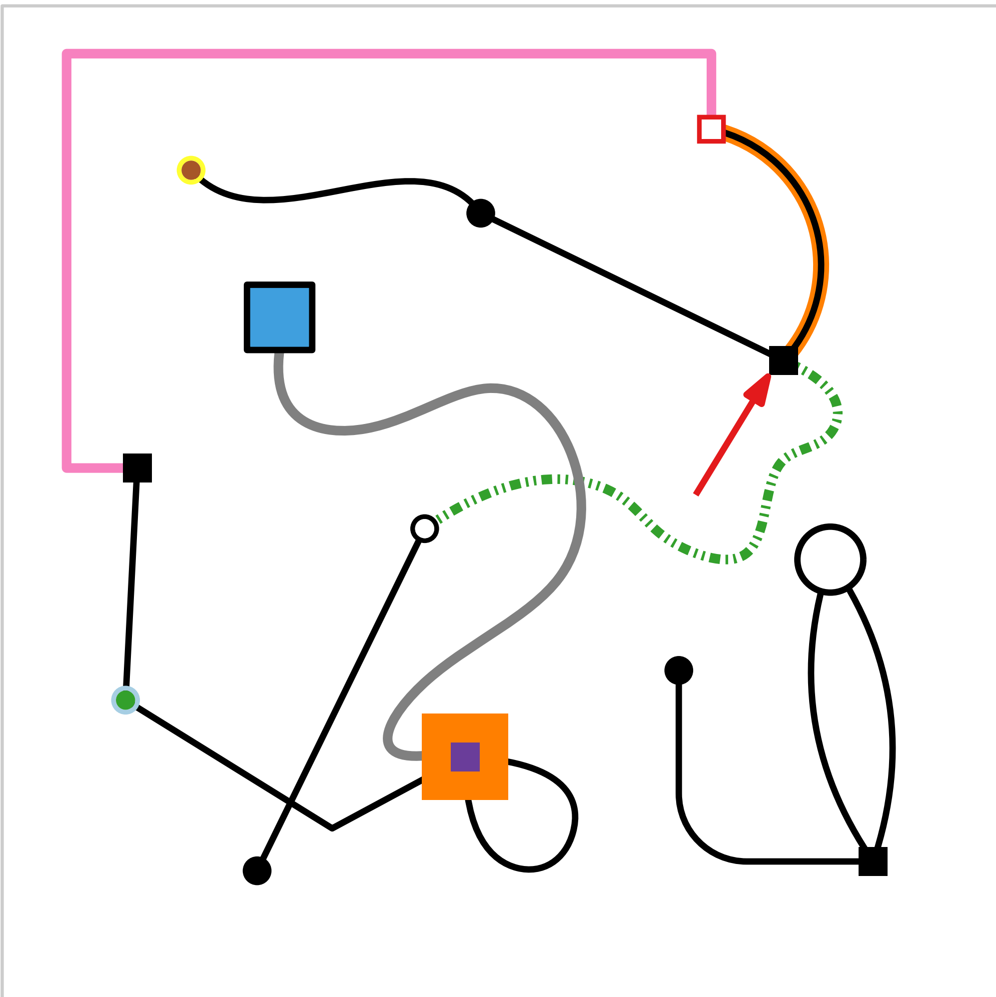

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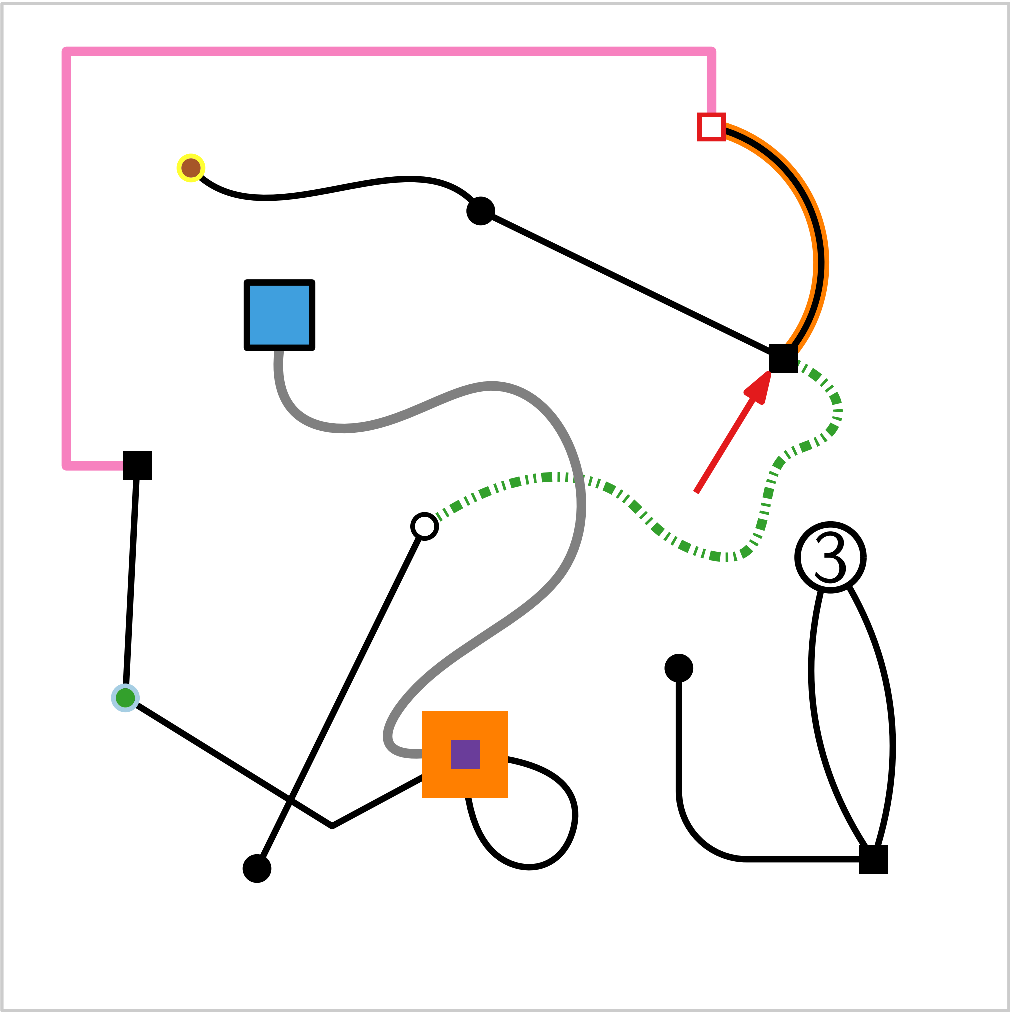

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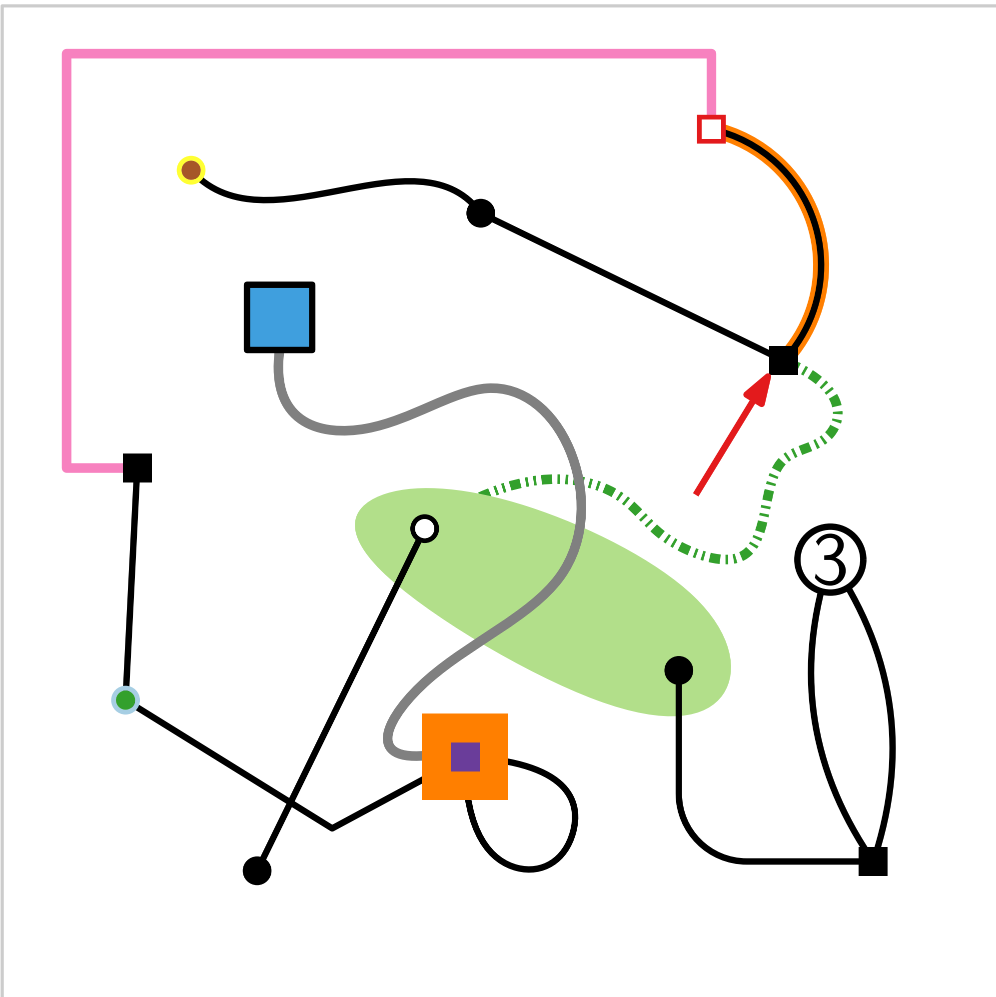

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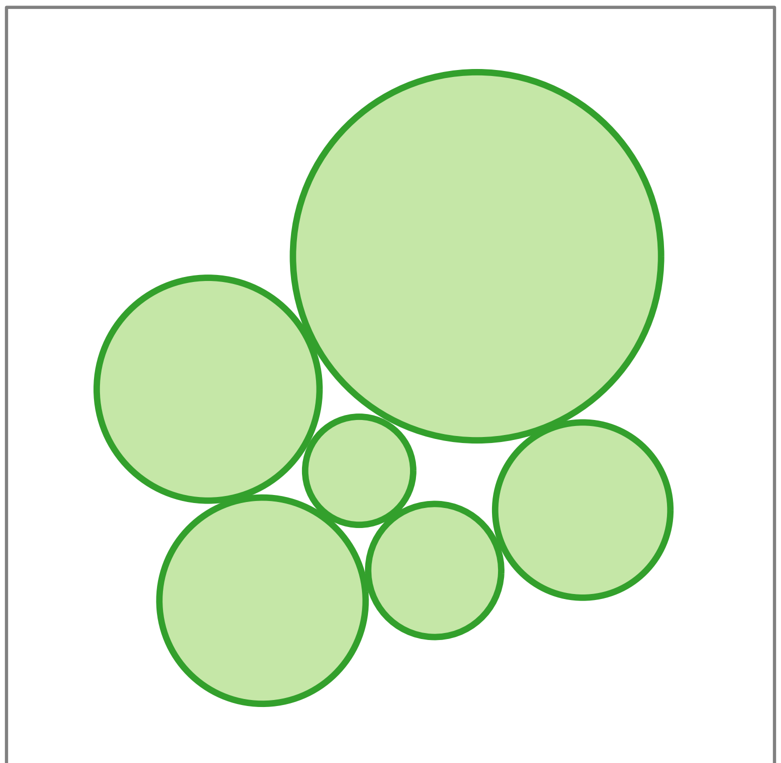

Not a node-link-diagram

Challenges: Extracting Drawings

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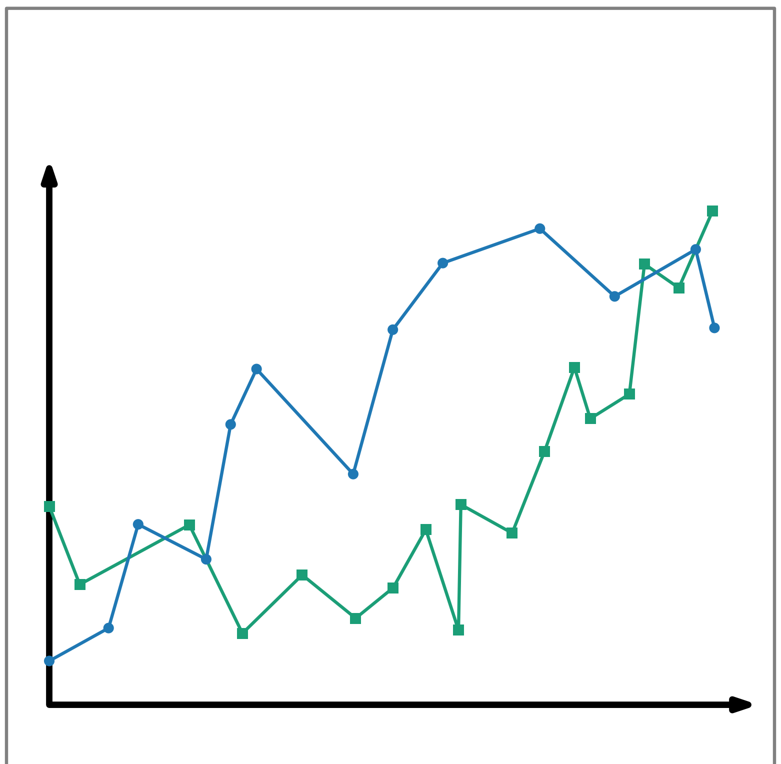
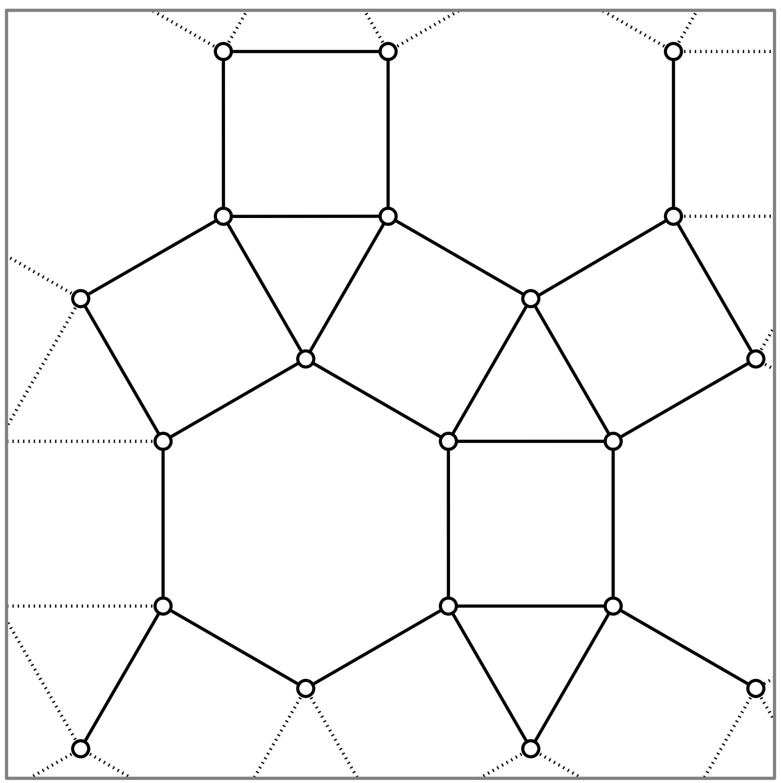


Not a graph drawing

but could be mistaken
for a node-link-diagram

Challenges: Extracting Drawings

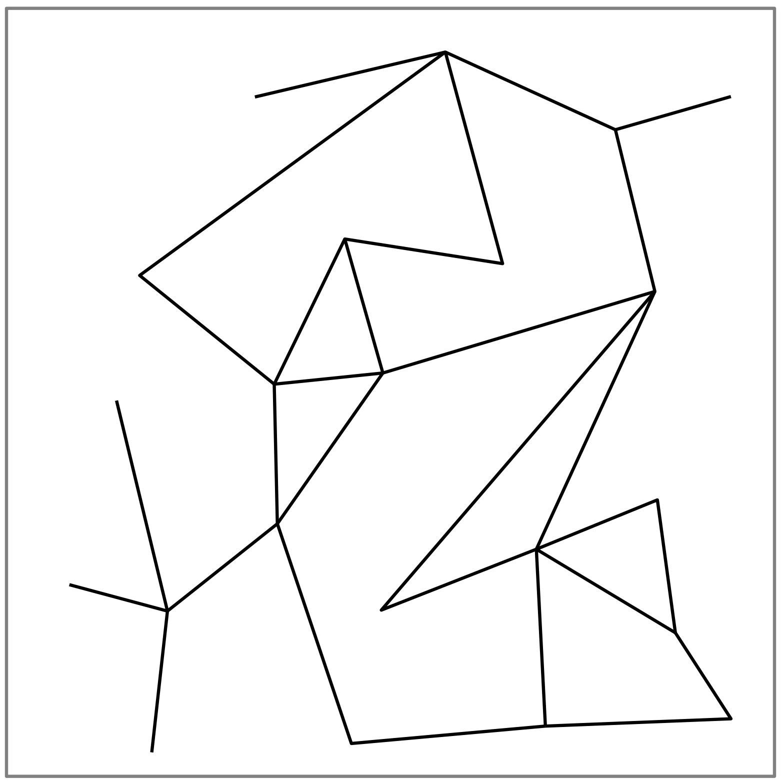
What we don't extract



Incomplete drawing

Challenges: Extracting Drawings

What we don't extract



No vertex markers

Challenges: Extracting Drawings

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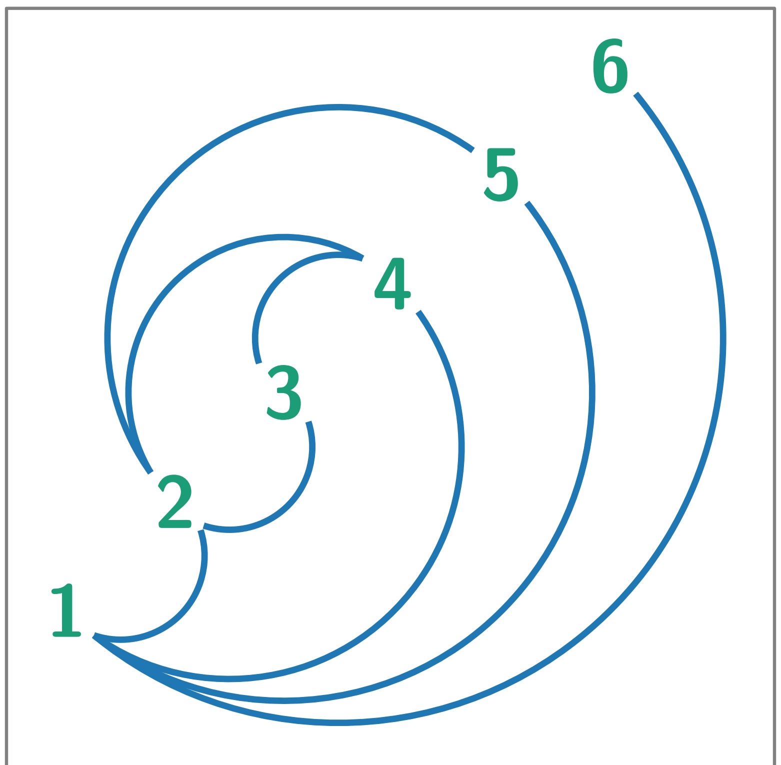

Text-only nodes

Challenges: Extracting Drawings

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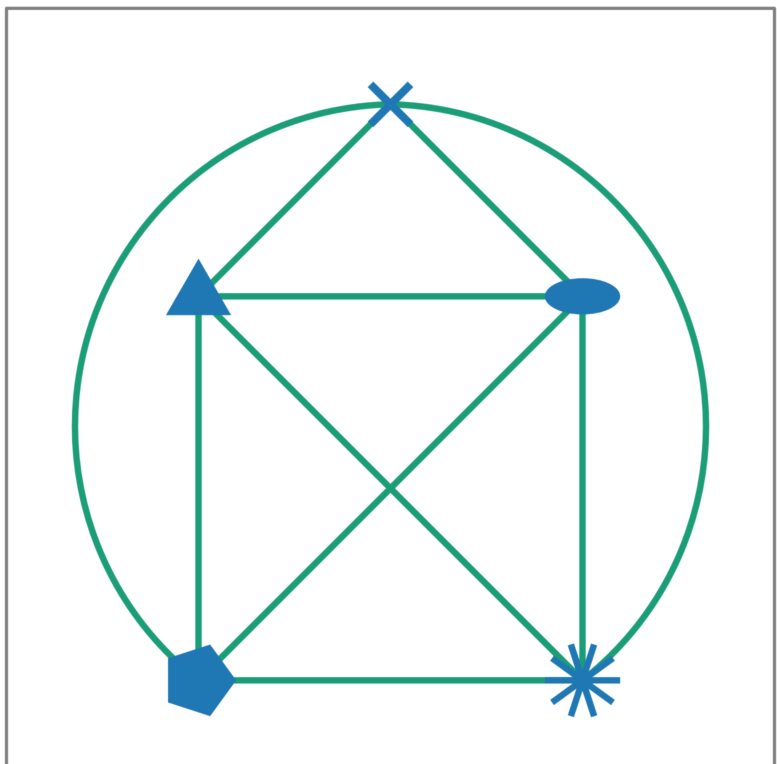

Unusual Node Shapes

Challenges: Extracting Drawings

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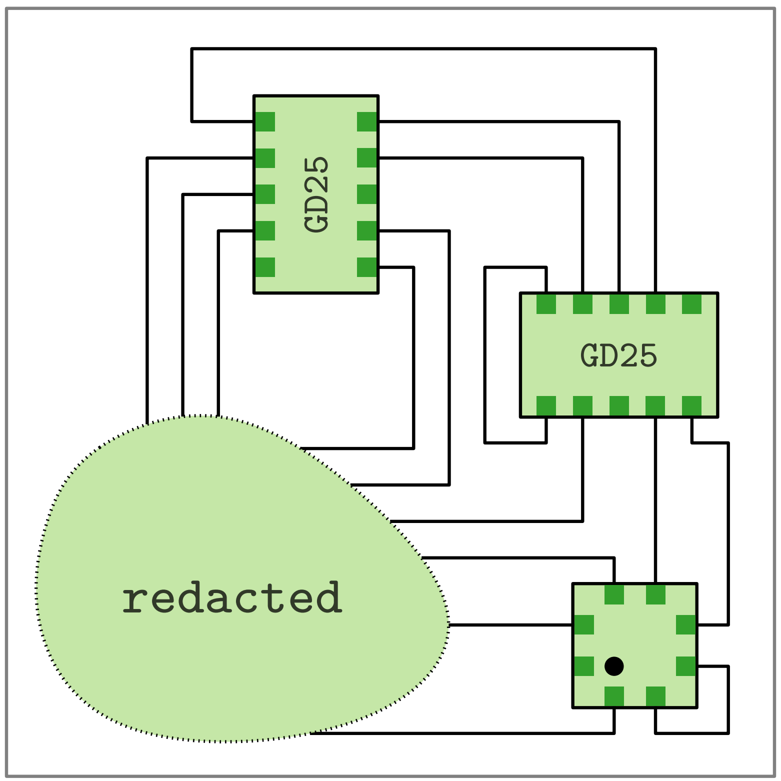

Ports or compose nodes

Challenges: Extracting Drawings

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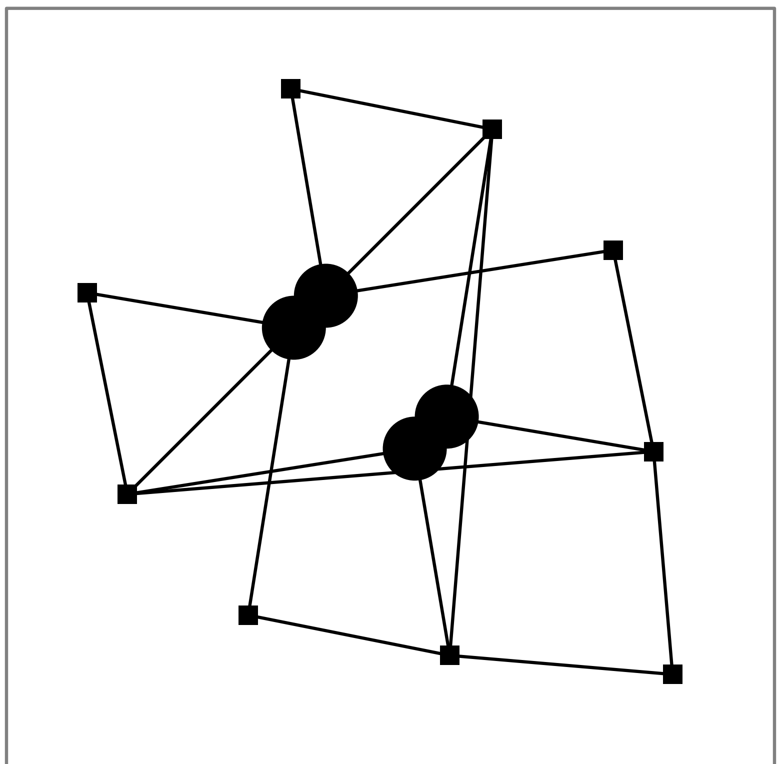

Partially overlapping and/or ambiguously placed nodes

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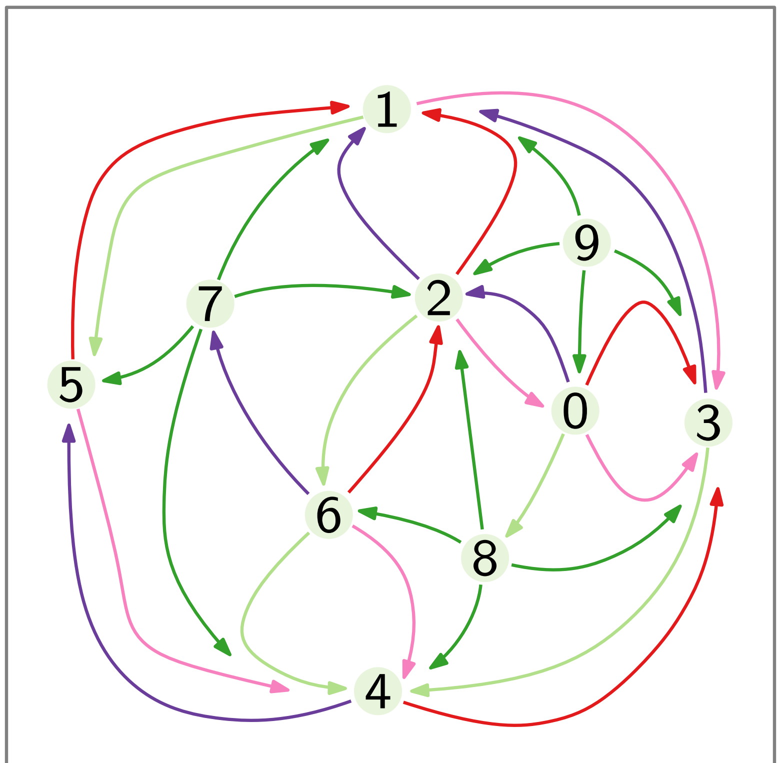

Edges not reaching their target

Challenges: Extracting Drawings

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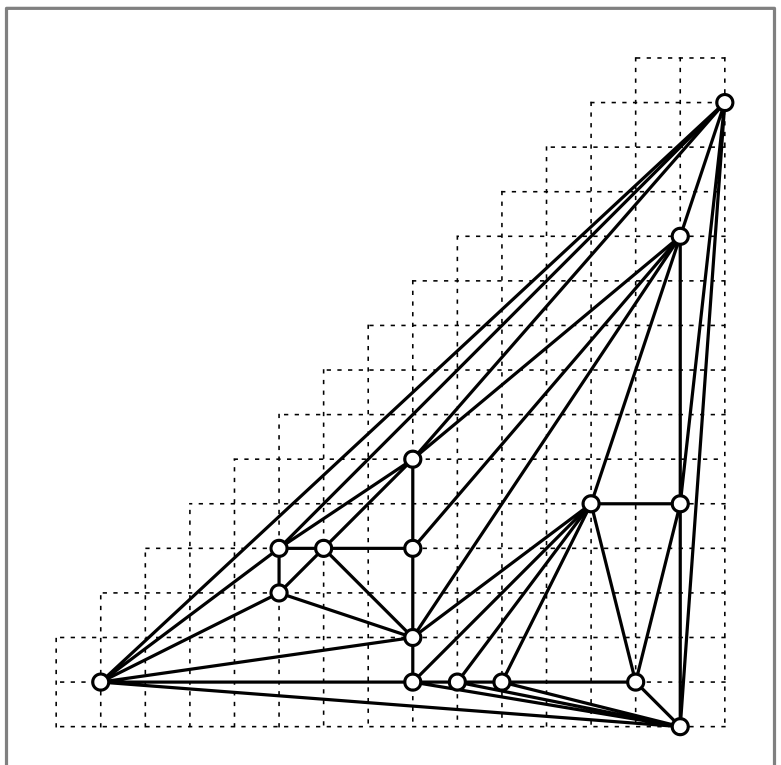

Grid or level lines

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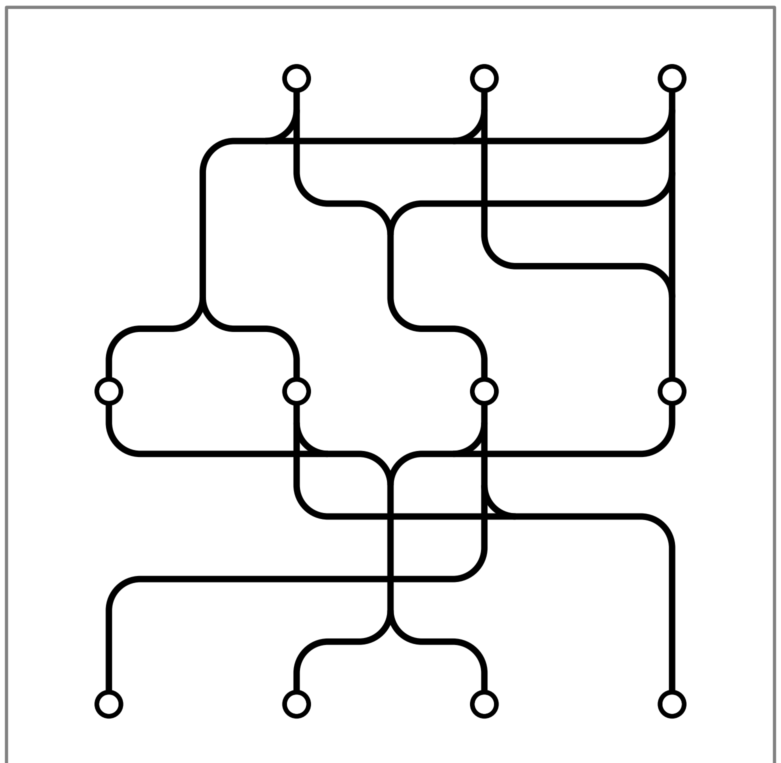


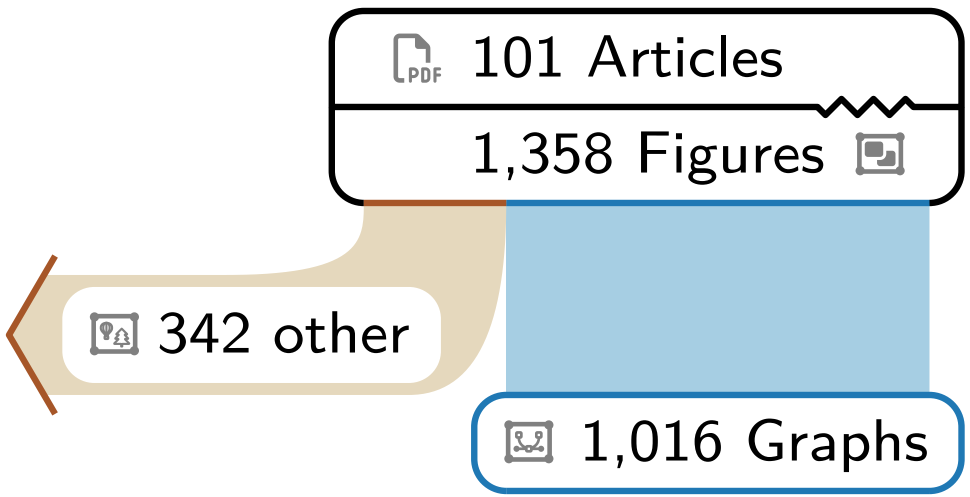
Confluent edges and hyper edges

What we Found

101 Articles

1,358 Figures

What we Found



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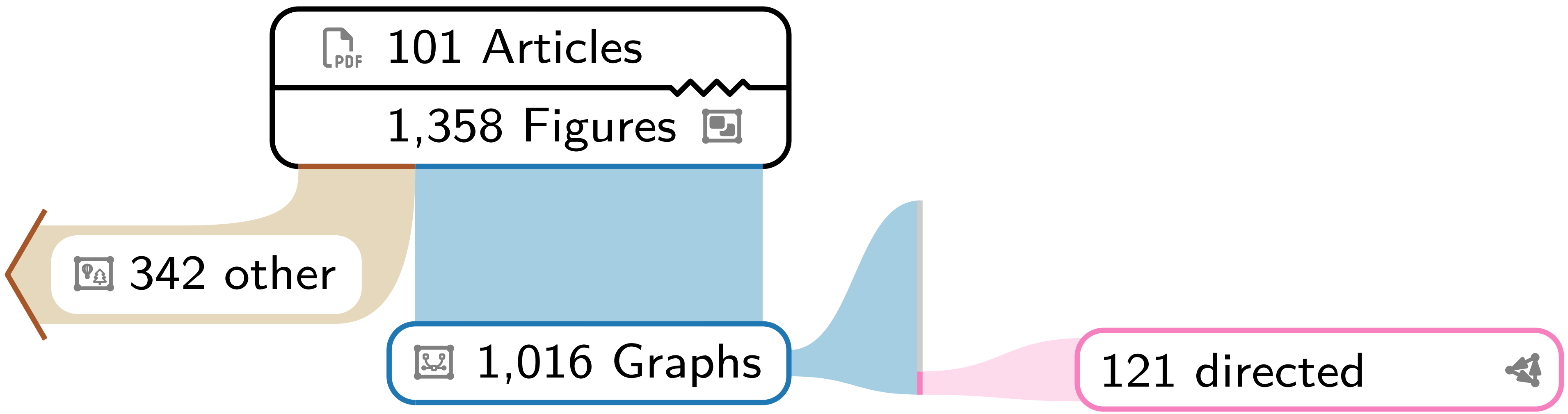

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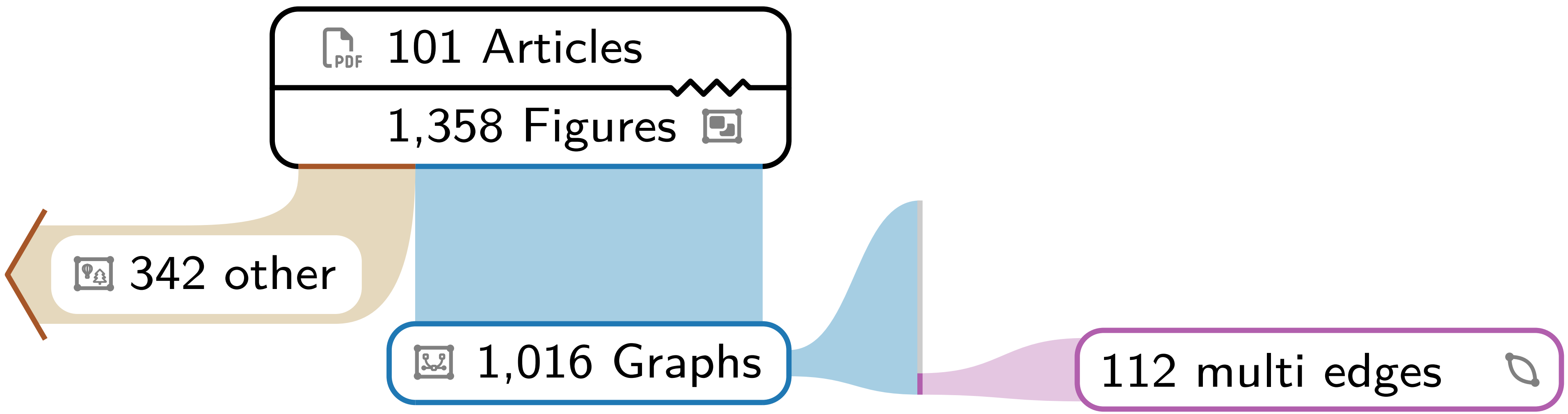

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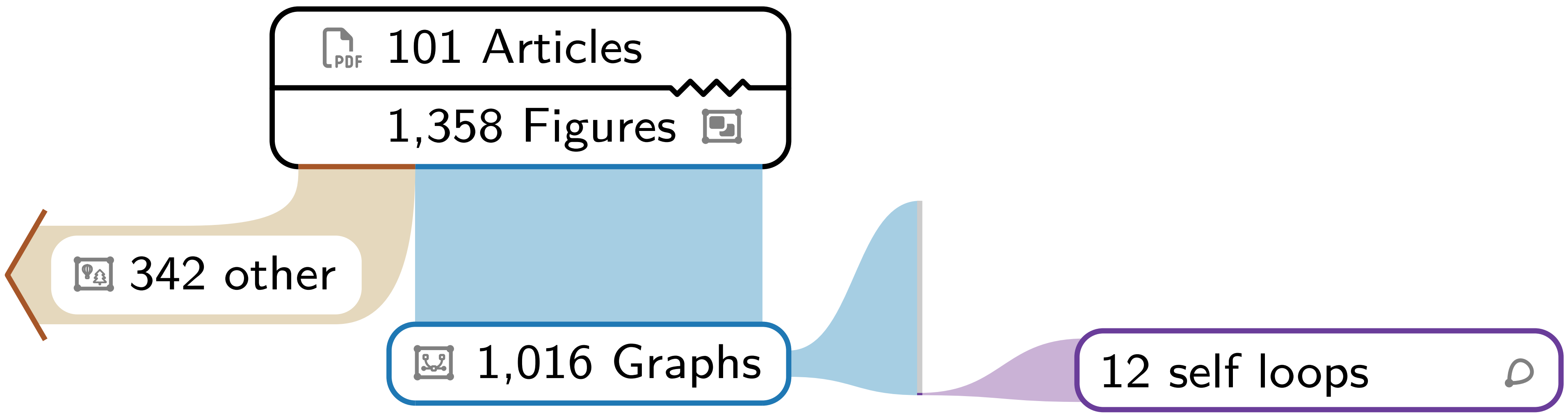

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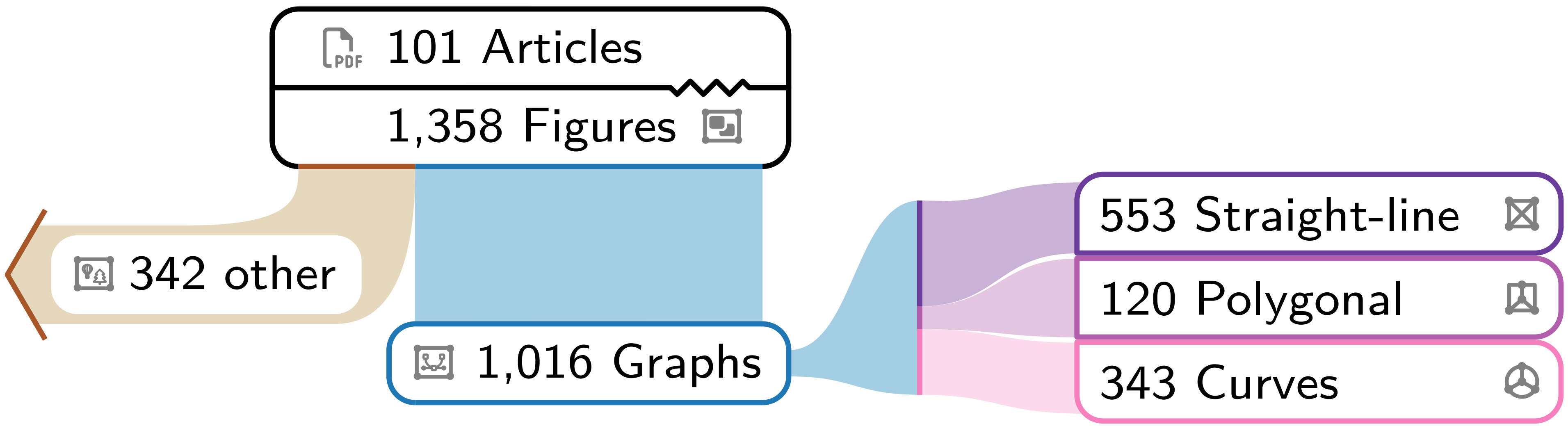

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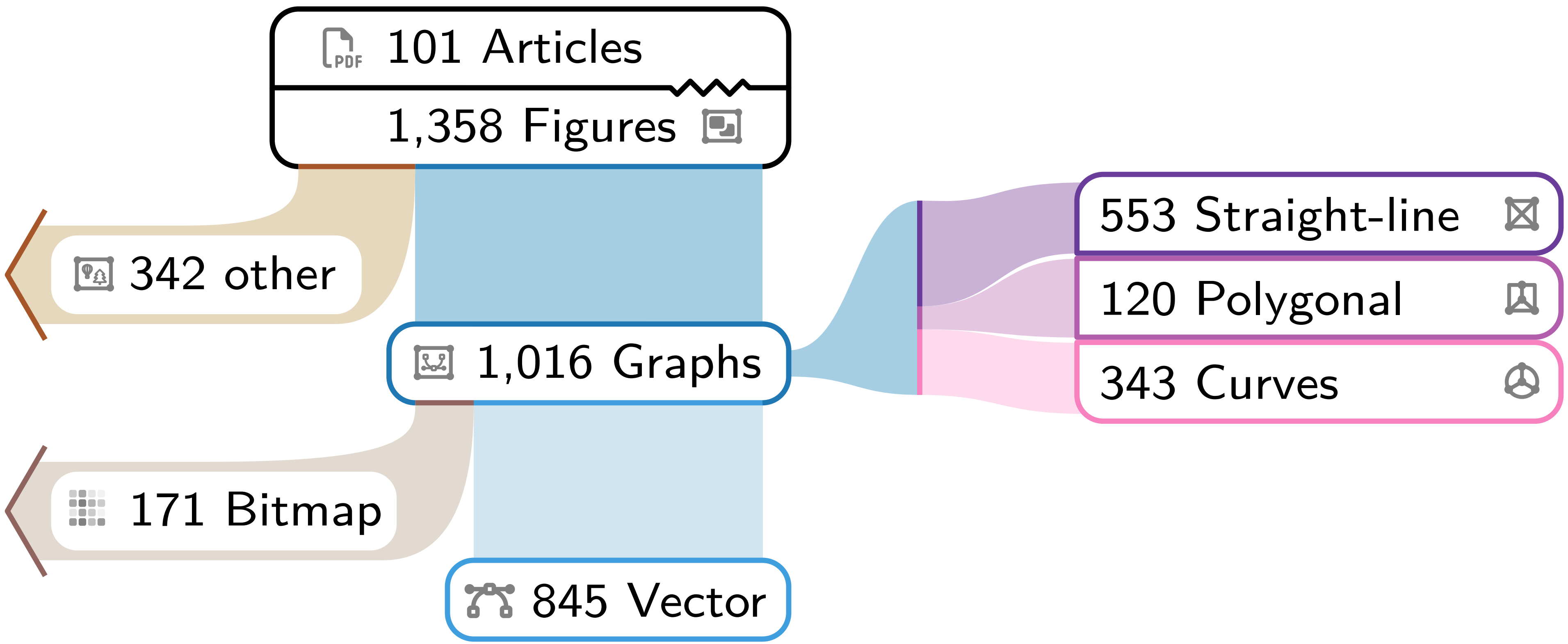

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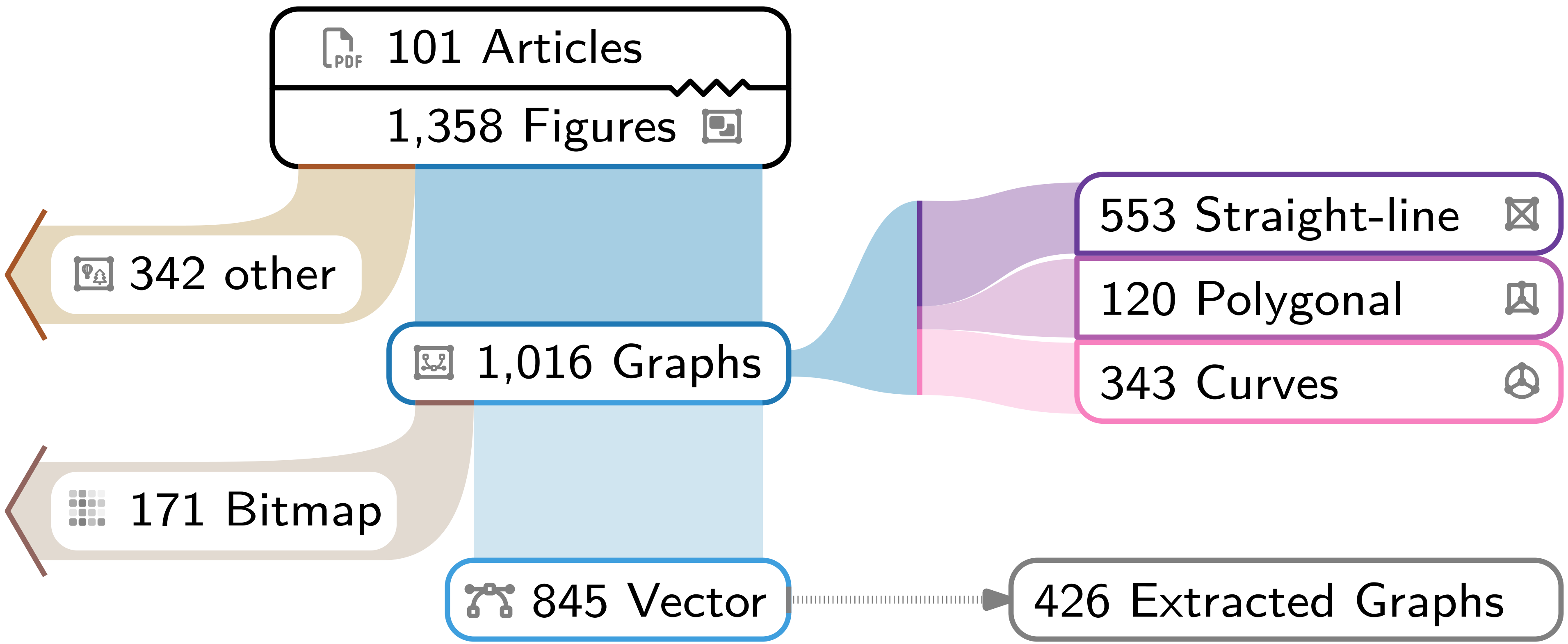

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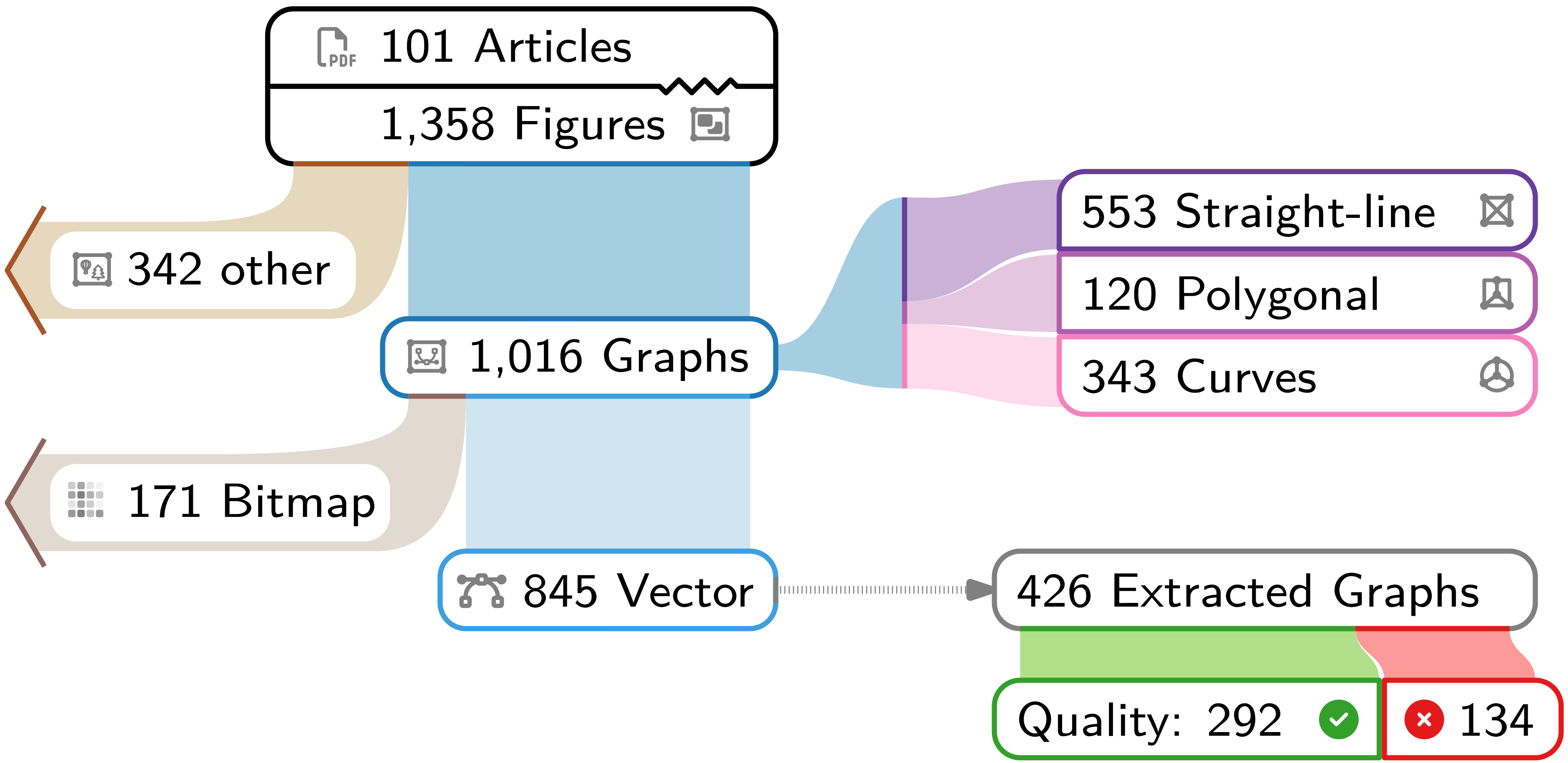

Challenges: Evaluating Drawings

Lack of standard representation for curves

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- Some extensions exist (GraphML, DOT) but are not widely deployed/implemented

Challenges: Evaluating Drawings

Lack of standard representation for curves

- Some extensions exist (GraphML, DOT) but are not widely deployed/implemented
- Curves could be approximated with polylines
  - Difficult to extract
  - Unclear how many segments to use

Challenges: Evaluating Drawings

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- Our solution:
  GEG (GEG Encodes Graphs)

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  - Nodes defined by an ID with a position (x,y)
  - Edges defined by node IDs (source→target) and optional *path*

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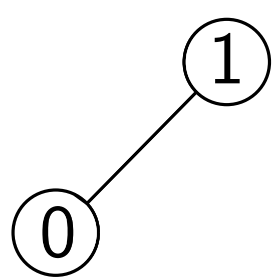
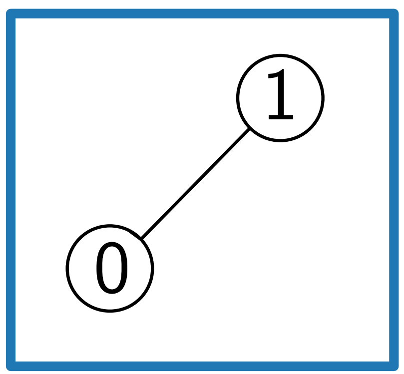



```
{"graph": {"directed": false},
  "nodes": [
    {"id" : "0",
     "position" : [0,0],
     "shape" : "ellipse"},
    {"id" : "1",
     "position" : [1,1],
     "shape" : "ellipse"},
  ],
  "edges": [
    {"id" : "2",
      "source" : "0",
      "target" : "1",
      "polyline" : false,
      "path" : "M0,0 L1,1"},
  ]
}
```

Challenges: Evaluating Drawings

Lack of standard representation for curves

- Edge geometries represented by SVG paths

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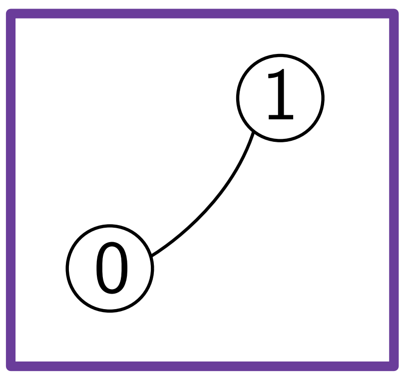


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"path" : "M0,0 Q0.5,0 1,1"},
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Challenges: Evaluating Drawings

Metrics typically assume straight-line edges

- **Aspect Ratio** (Asp): Bounding box should be ‘square’
- **Angular Resolution** (AR): Maximise the minimum angle between adjacent edges
- **Crossing Angle** (CA): Edges cross at right angles
- **Edge Crossings** (EC): Minimise the number of edge crossings
- **Edge Length Deviation** (ELD): Make edge lengths uniform
- **Edge Orthogonality** (EO): Keep edges vertical or horizontal
- **Neighborhood Preservation** (NP): K-nearest neighbours in the graph positioned close together in the drawing
- **Node Resolution** (NR): Avoid overlapping (or very close) nodes
- **Node Uniformity** (NU): Distribute nodes evenly in the plane
- **Stress** (KSM): Geometric distances matches graph-theoretic distance

Challenges: Evaluating Drawings

[Mooney, Purchase, Wybrow, Kobourov]
Pacific VIS 2024

Some are defined on node positions only, and require no changes

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Some only require changes in their implementations

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Edge Length Deviation

Edge Length Deviation (ELD) of a drawing D is the average relative deviation of edge lengths from an ideal value:

$$\mathrm{ELD}(D) = 1 \Bigg/ \left(1 + \frac{1}{|E(D)|} \sum_{e \in E(D)} \left| \frac{L(e) - L_{\text{ideal}}(D)}{L_{\text{ideal}}(D)} \right| \right)$$

where $L(e)$ is the geometric length of edge e, either as the arc length of a curved path or the summed Euclidean length of straight-line segments, and $L_{\text{ideal}}(D) = (\sum_{e \in E(D)} L(e))/|E(D)|$ is the average edge length of D.

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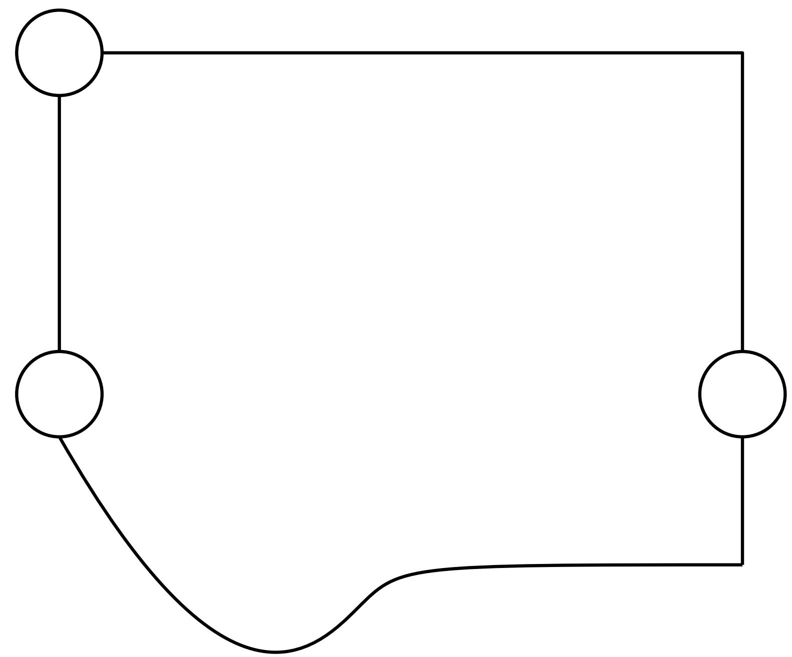

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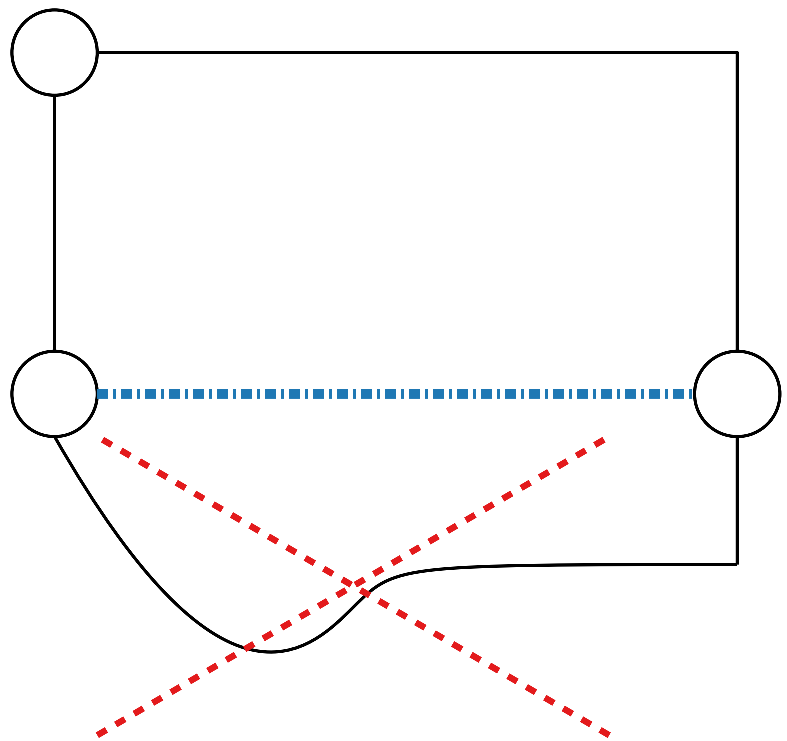

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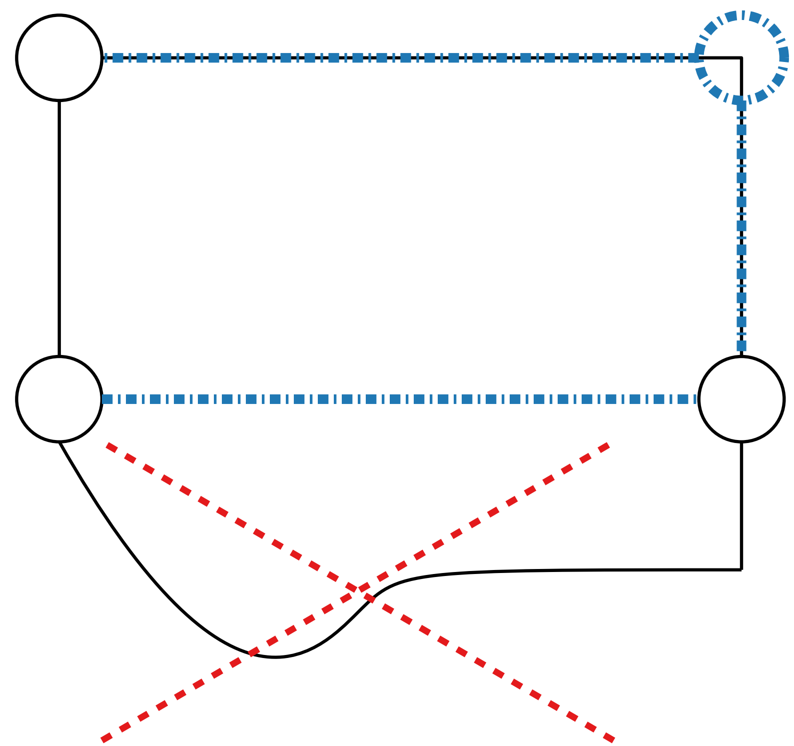

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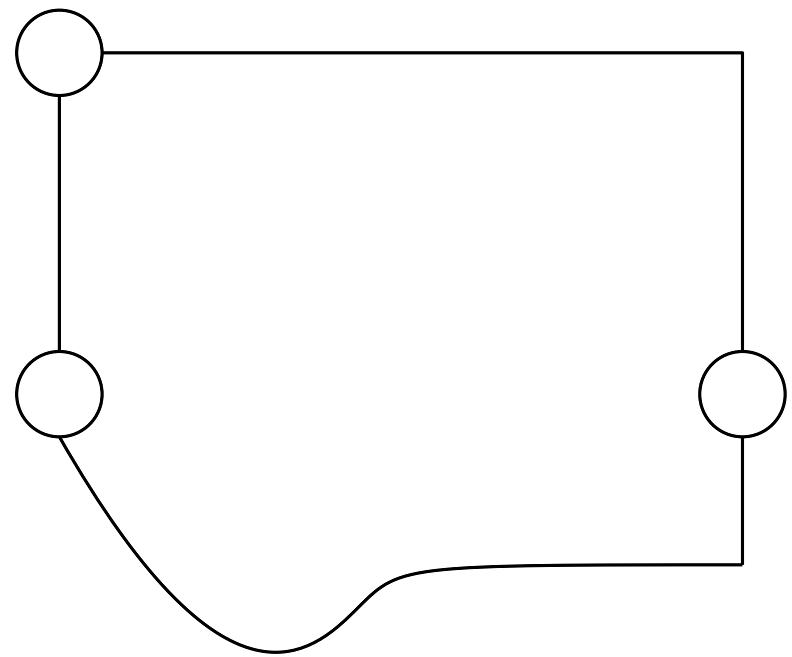


Edge Orthogonality

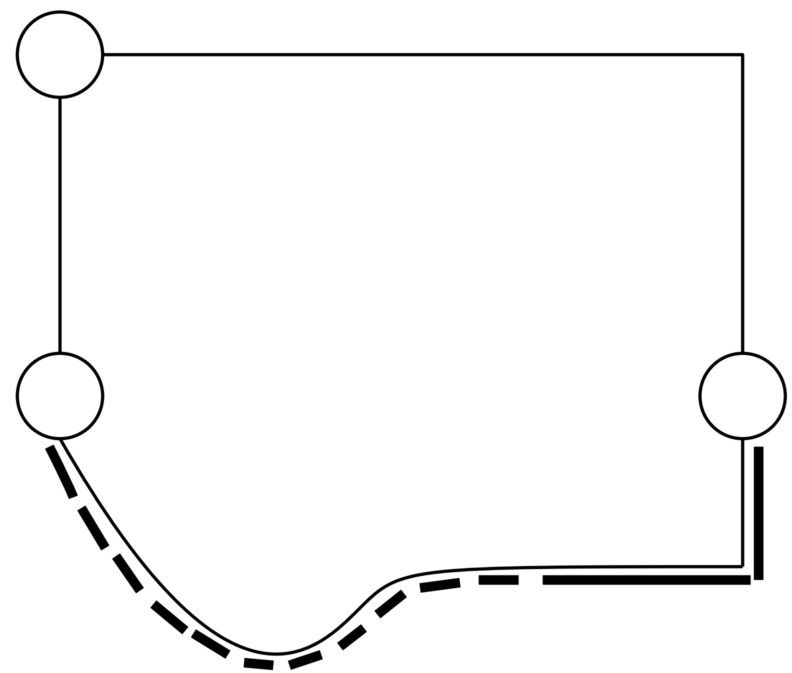
Edge Orthogonality (EO) of a drawing D is the average angular deviation of edges from the horizontal or vertical axes:

$$\mathrm{EO}(D) = 1 - \frac{1}{|E(D)|} \sum_{e \in E(D)} \delta_e, \text{ where}$$

$$\delta_e = \sum_{j=1}^{k_e} \frac{\min(\theta_{e,j}, |90^\circ - \theta_{e,j}|, 180^\circ - \theta_{e,j})}{45^\circ} \cdot \frac{\ell_{e,j}}{\tilde{L}(e)}$$

is the length-weighted orthogonality deviation of edge e, each edge e is approximated by a polygonal line $P(e)$ with k_e straight-line segments, $\theta_{e,j}$ is the absolute angle between segment j of $P(e)$ and the horizontal axis, $\ell_{e,j}$ is the length of segment j of $P(e)$, and $\tilde{L}(e) = \sum_{j=1}^{k_e} \ell_{e,j}$ is the total length of $P(e)$.

Edge Orthogonality



GD Proceedings: The Papers

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A: 71.4% (740/1,037)*

*Not all drawings were successfully extracted

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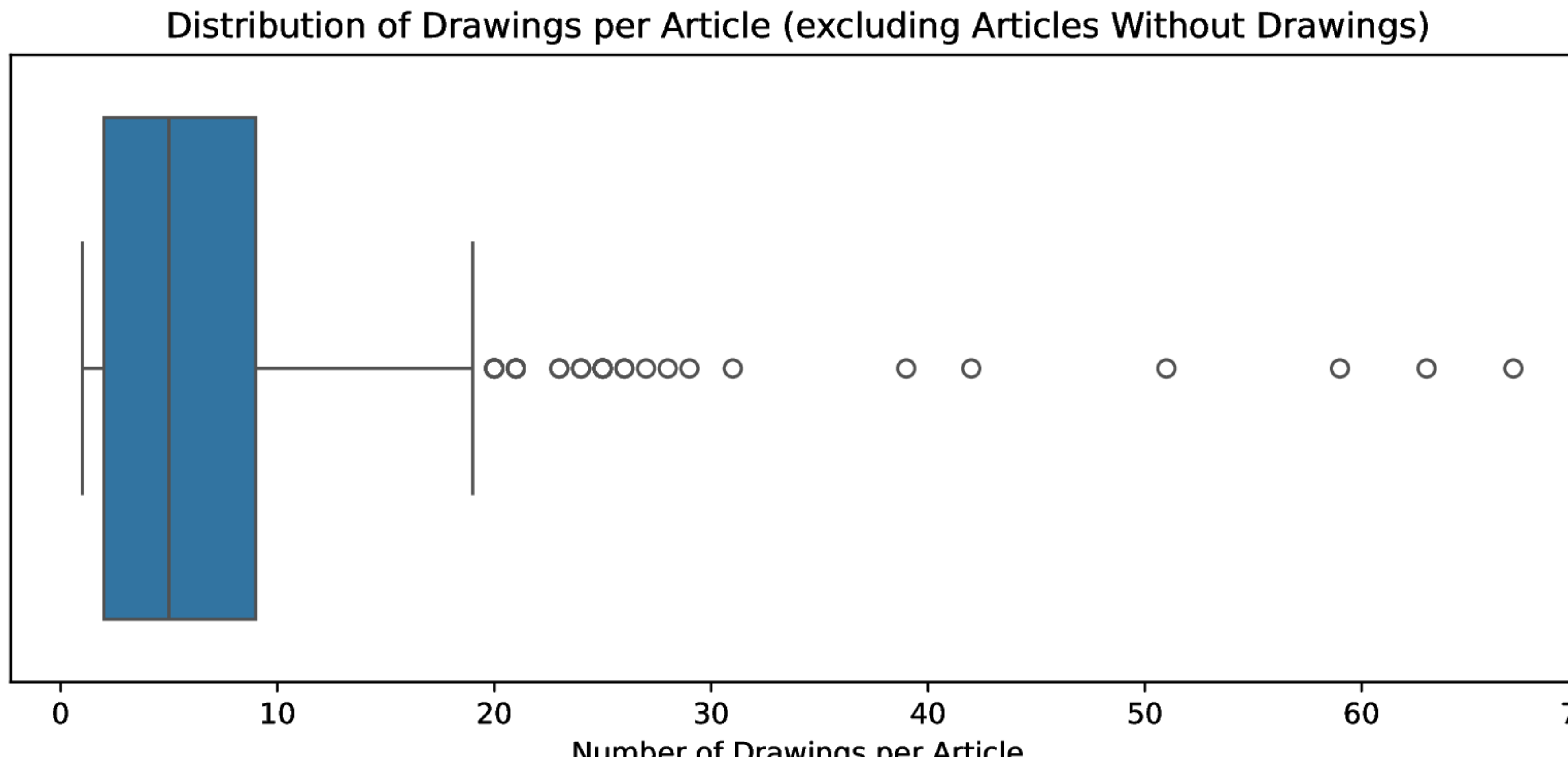
Median 5 drawings per paper

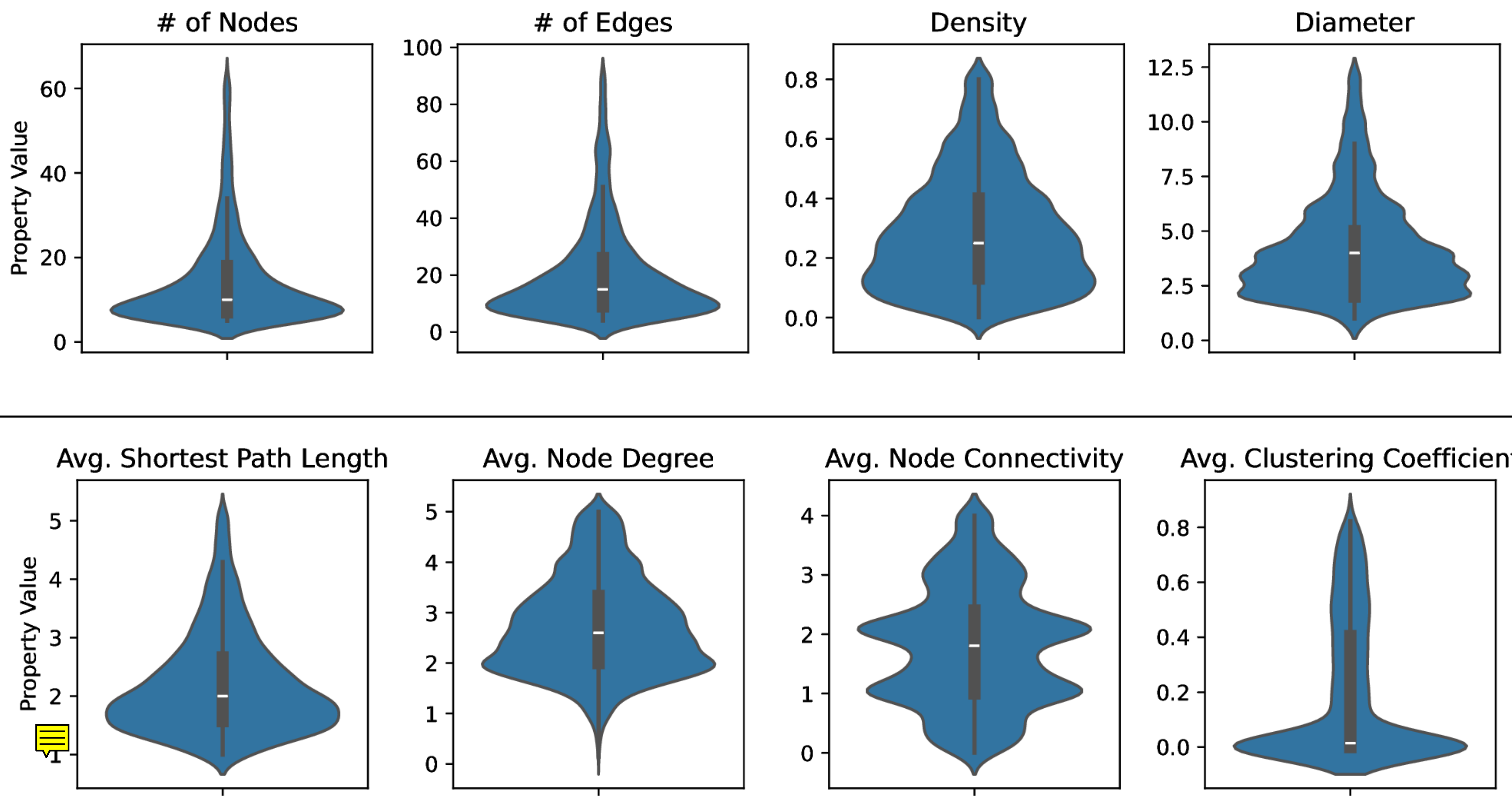


GD Proceedings: The Graphs

We extracted 4,890 graph drawings

- 77.1% connected
- 17.6% multi-graphs
- 86.1% planar
  - but only 57.4% drawn without crossings

GD Proceedings: The Graphs



GD Proceedings: The Drawings

We categorise the extracted drawings into three sets based on edge geometry:

- **Straight** (43.0%), **Polygonal** (20.4%), **Curved** (36.6%)

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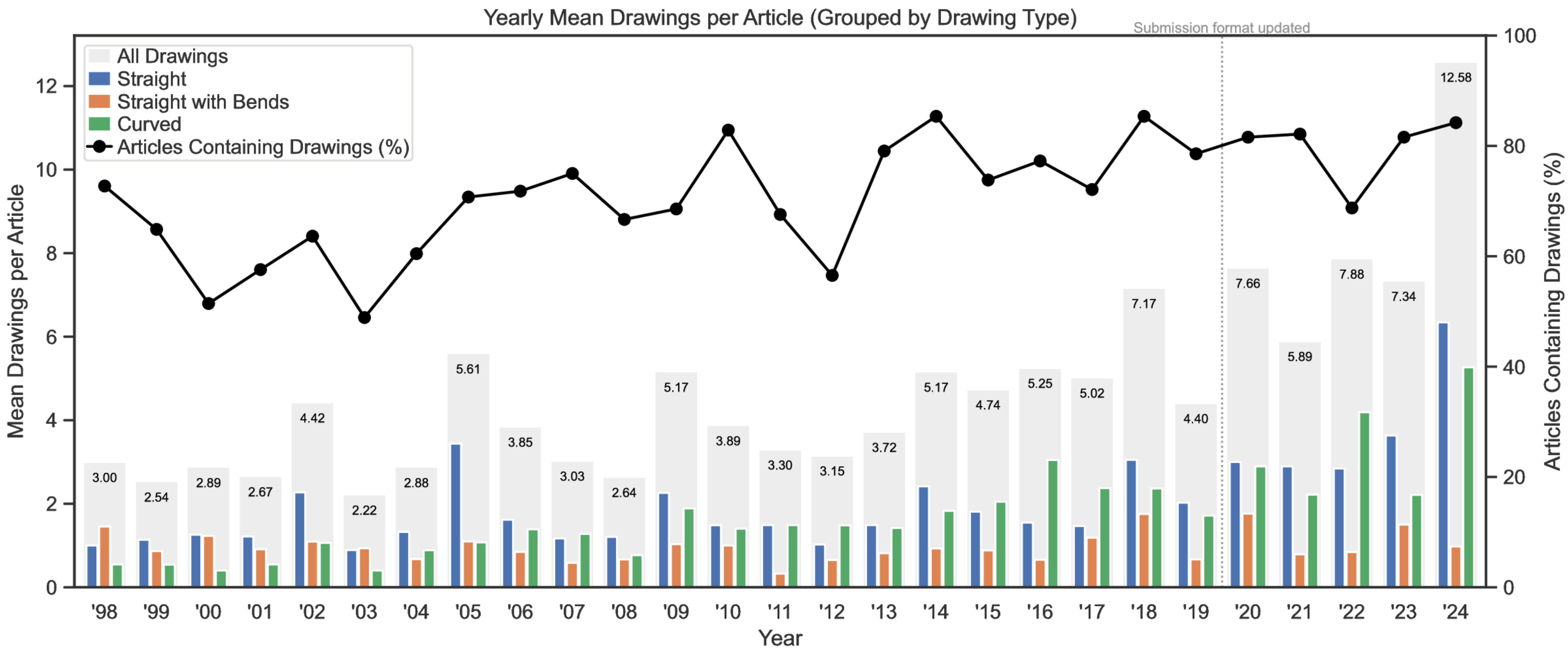
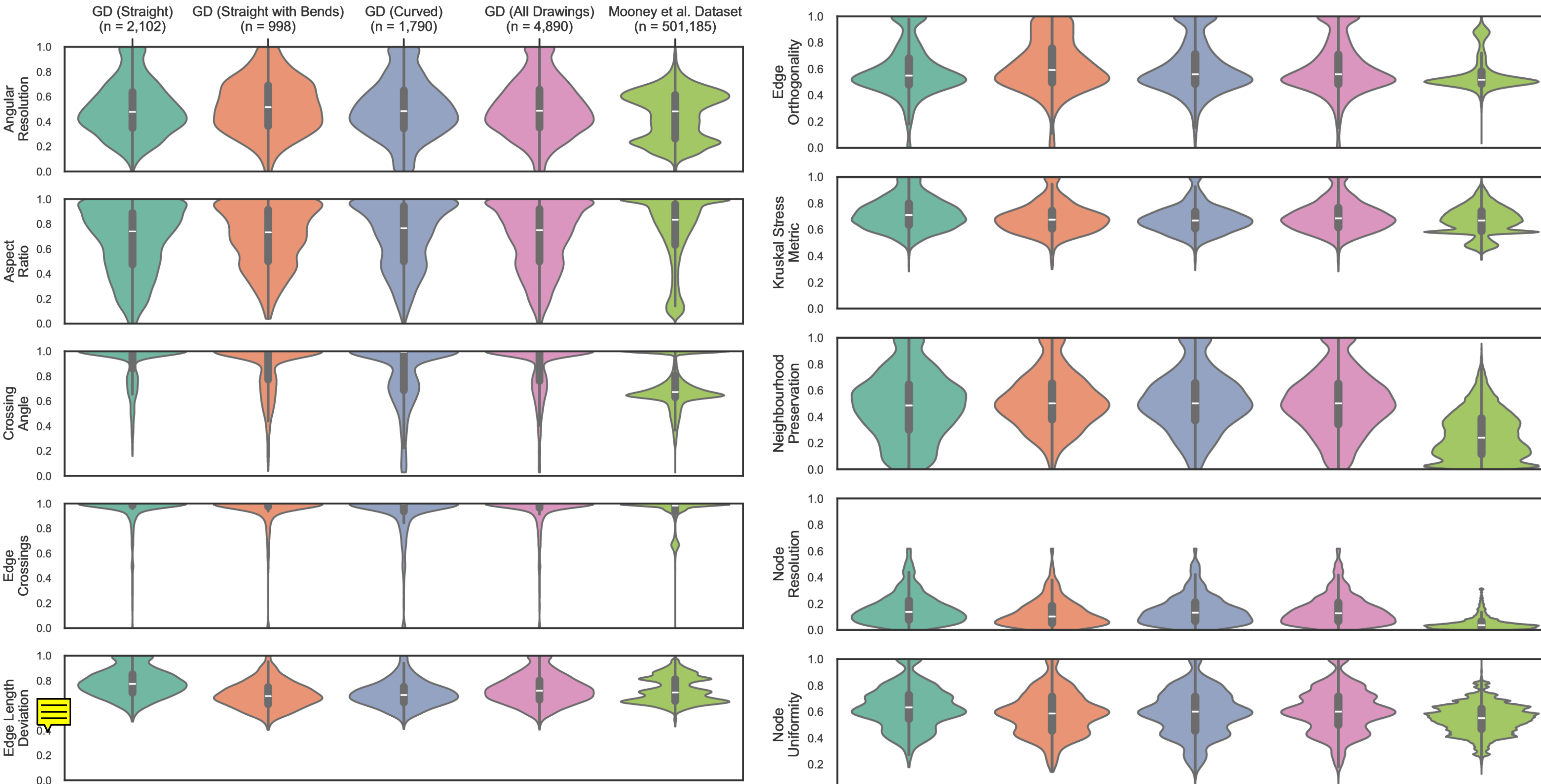


GD Proceedings: The Landscape



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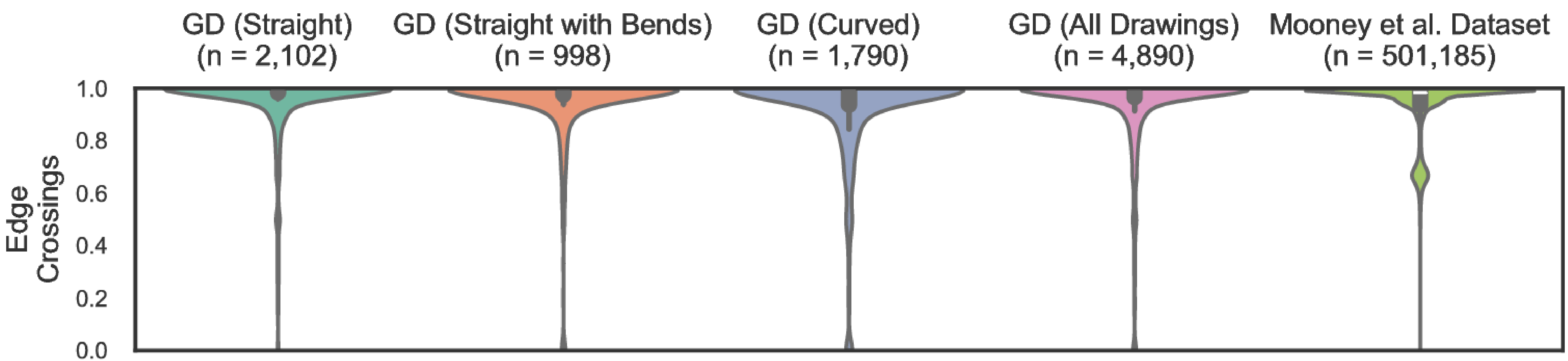

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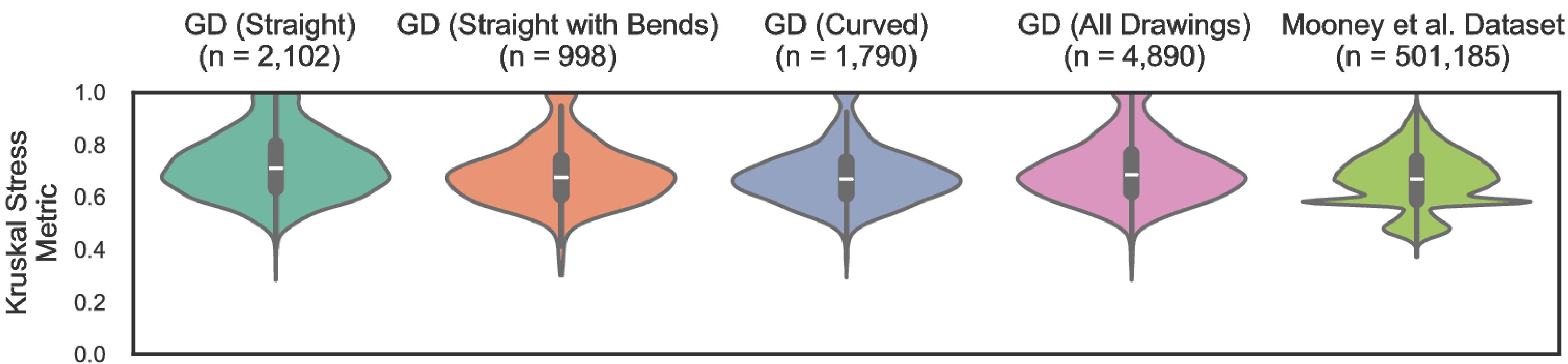

Conclusion

- Extended Graph Harvester to extract graph drawings from PDFs
- Redefined and implemented metrics, generalized to support complex edge geometries
- Explored the metrics landscape of the GD proceedings from 1998-2024

To support future analysis:

- Use vector images (not bitmap)
- Use vertex markers
- Avoid annotations that can be mistaken for edges

Metrics: `https://github.com/gavjmooney/geg`

Dataset: `https://github.com/hegetim/gd-collection`