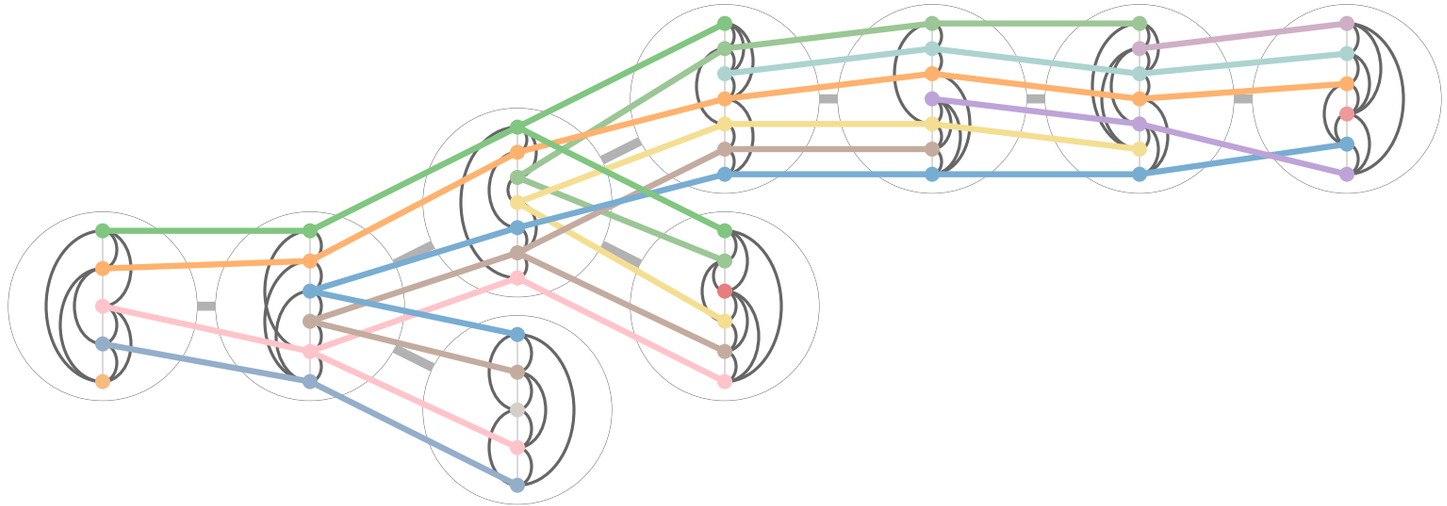

Visualizing Treewidth

Alvin Chiu[(1)], **Thomas Depian**[(2)], David Eppstein[(1)],
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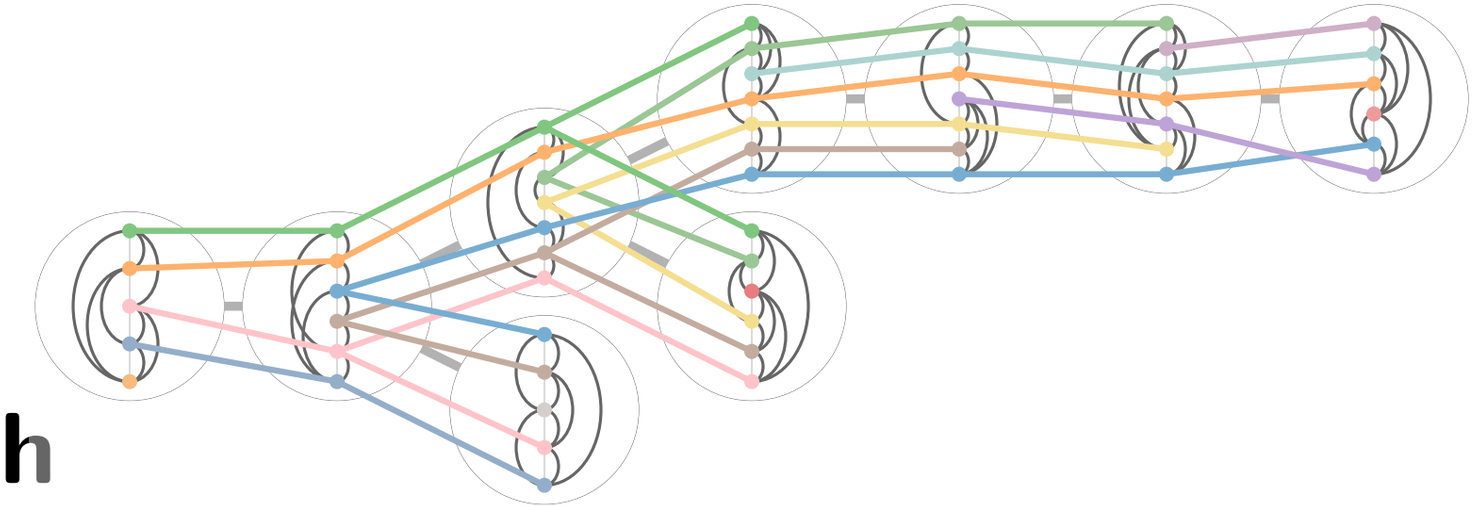
24. – 26. September · GD 2025

(1) UC Irvine

(2) TU WIEN ac ALGORITHMS AND COMPLEXITY GROUP

WWTF Vienna Science and Technology Fund



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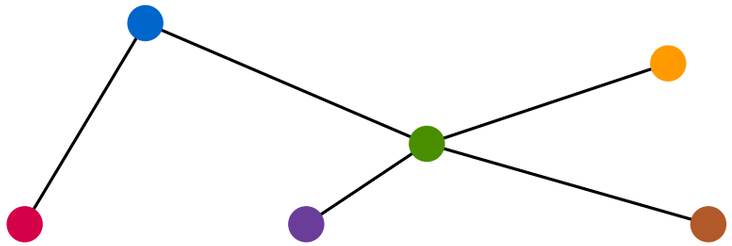

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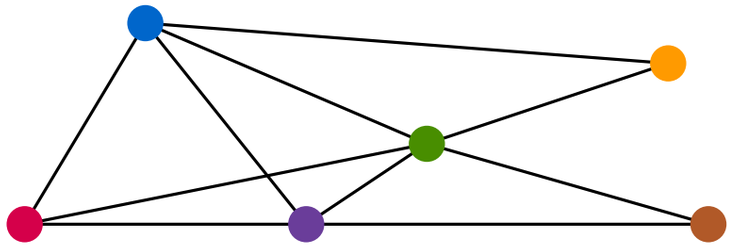

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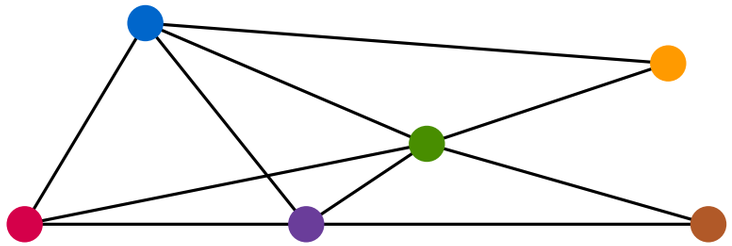

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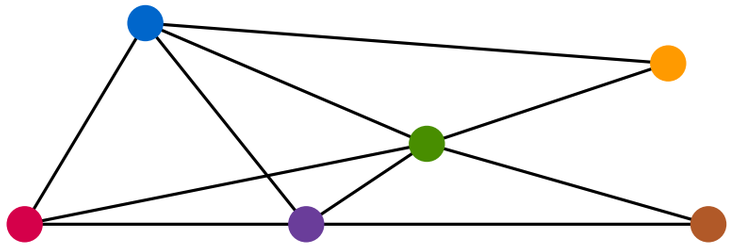

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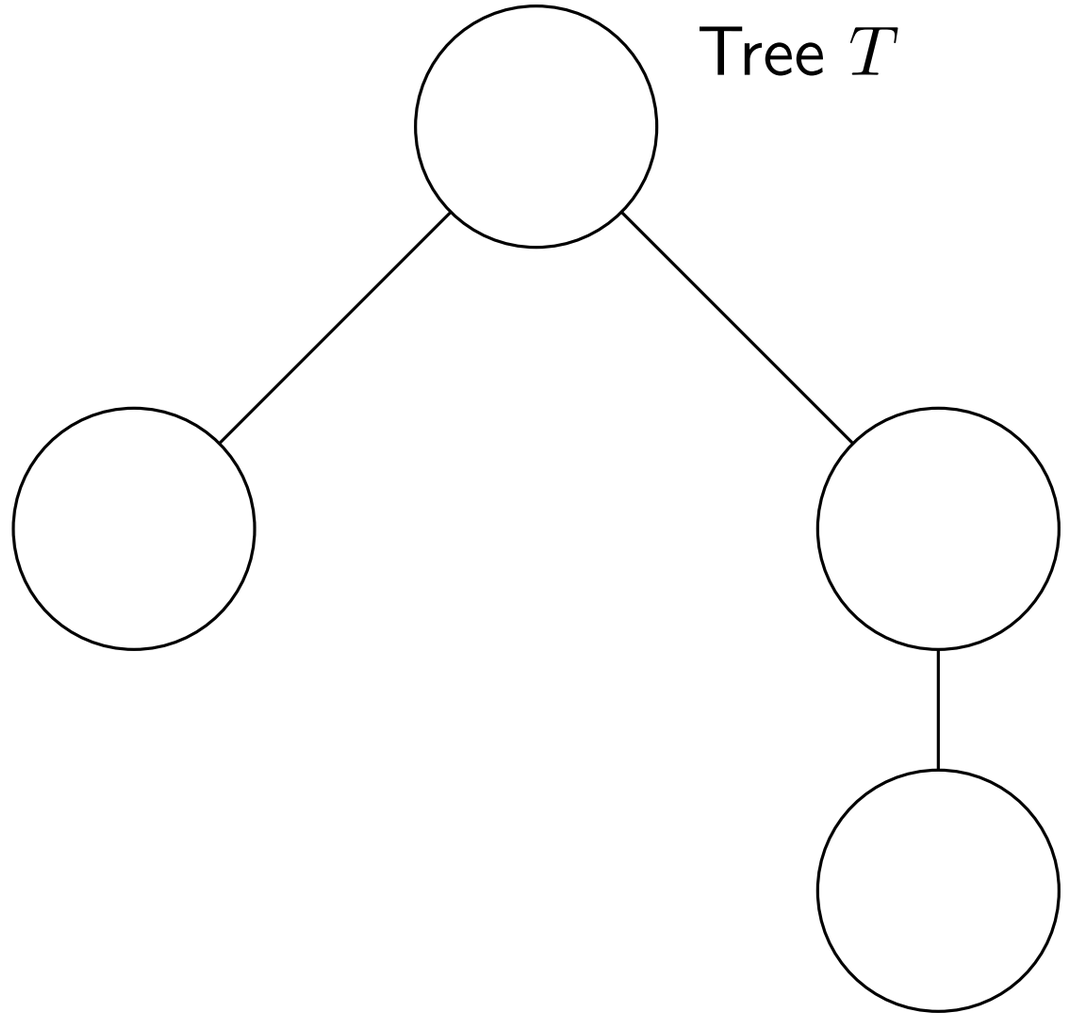

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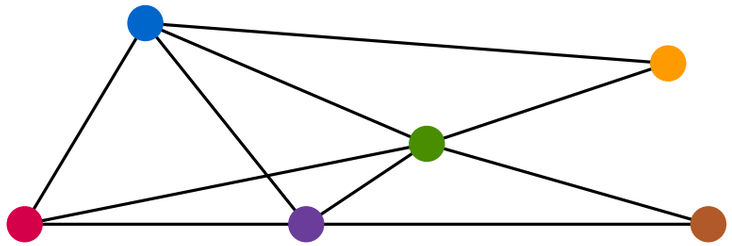

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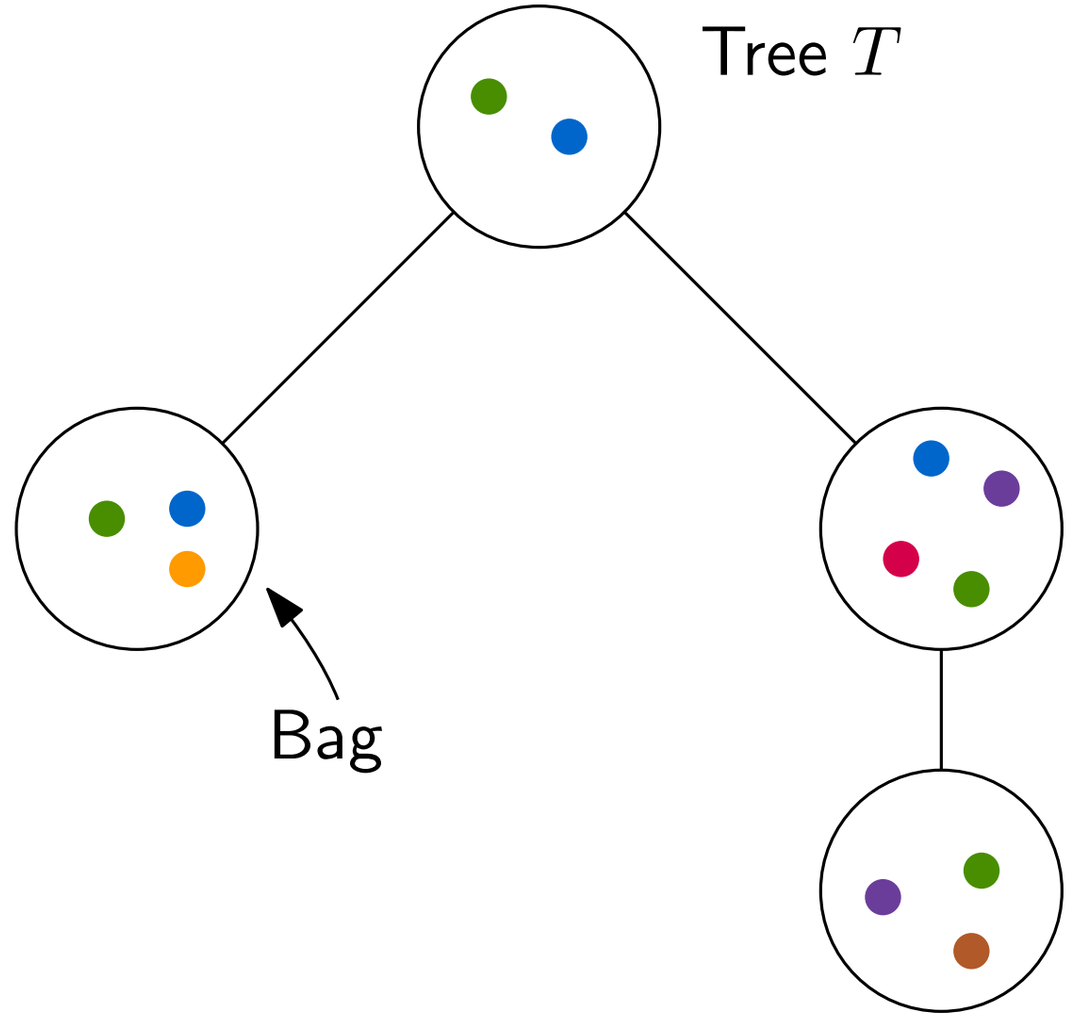

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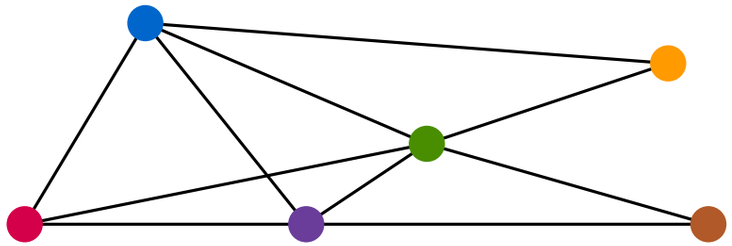

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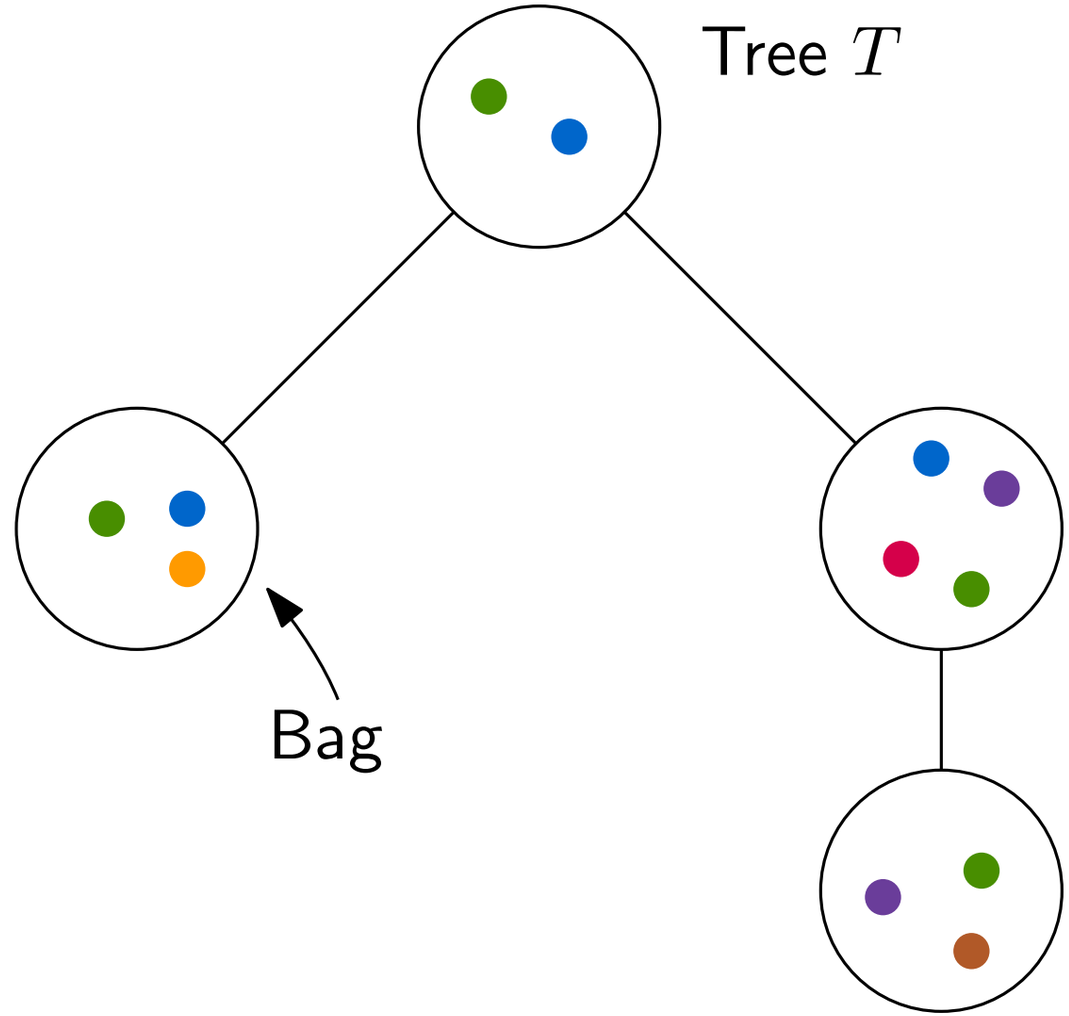

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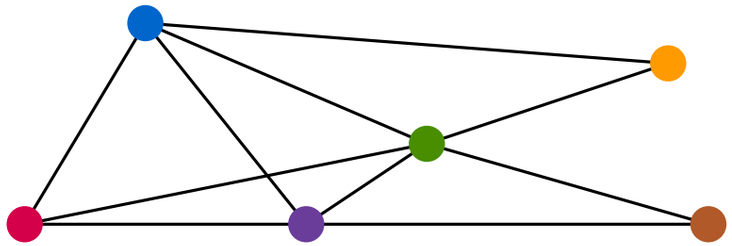

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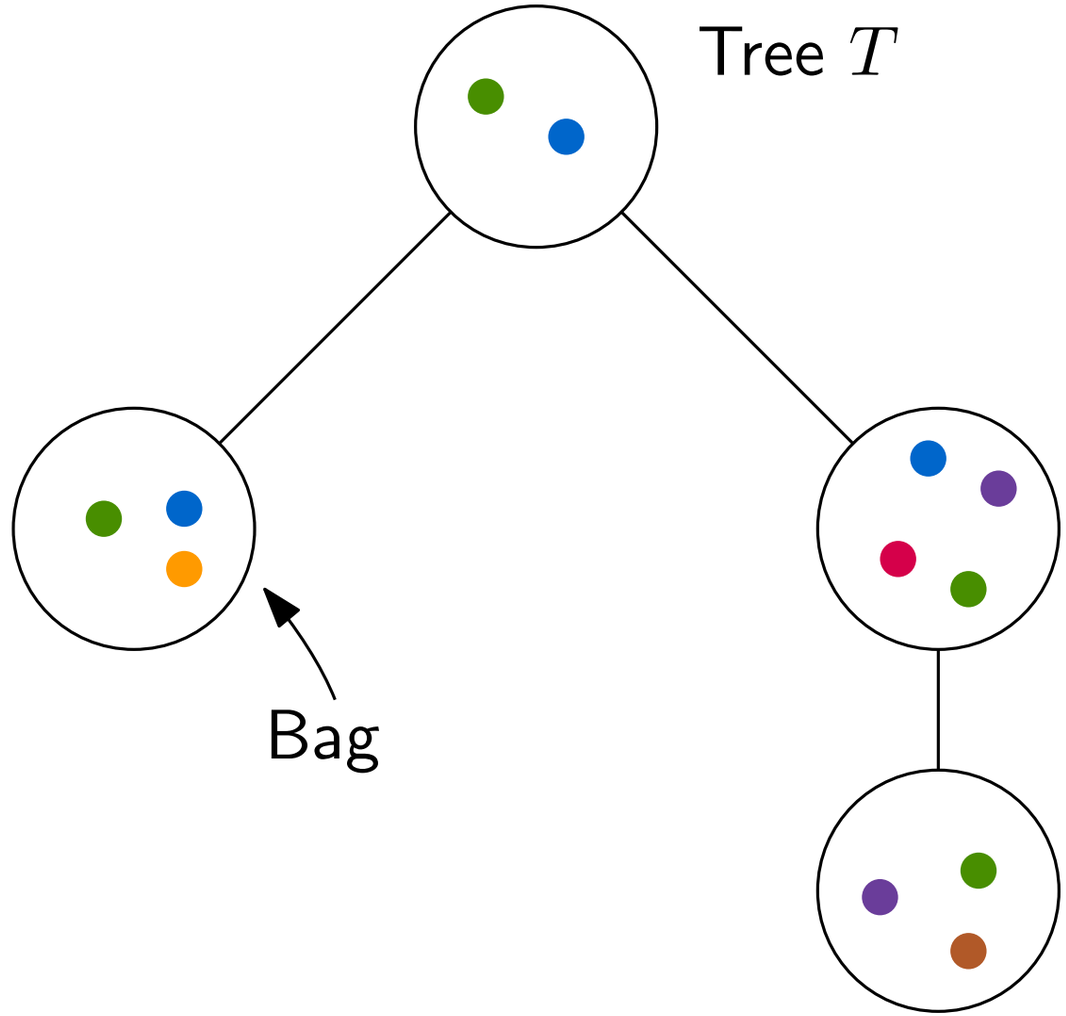

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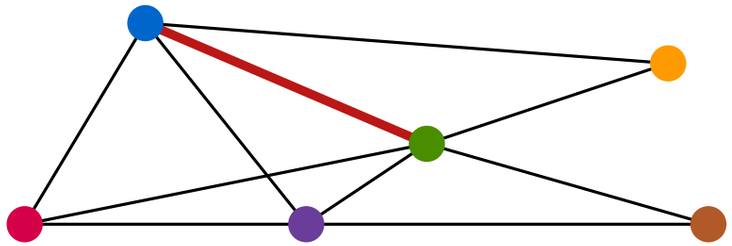

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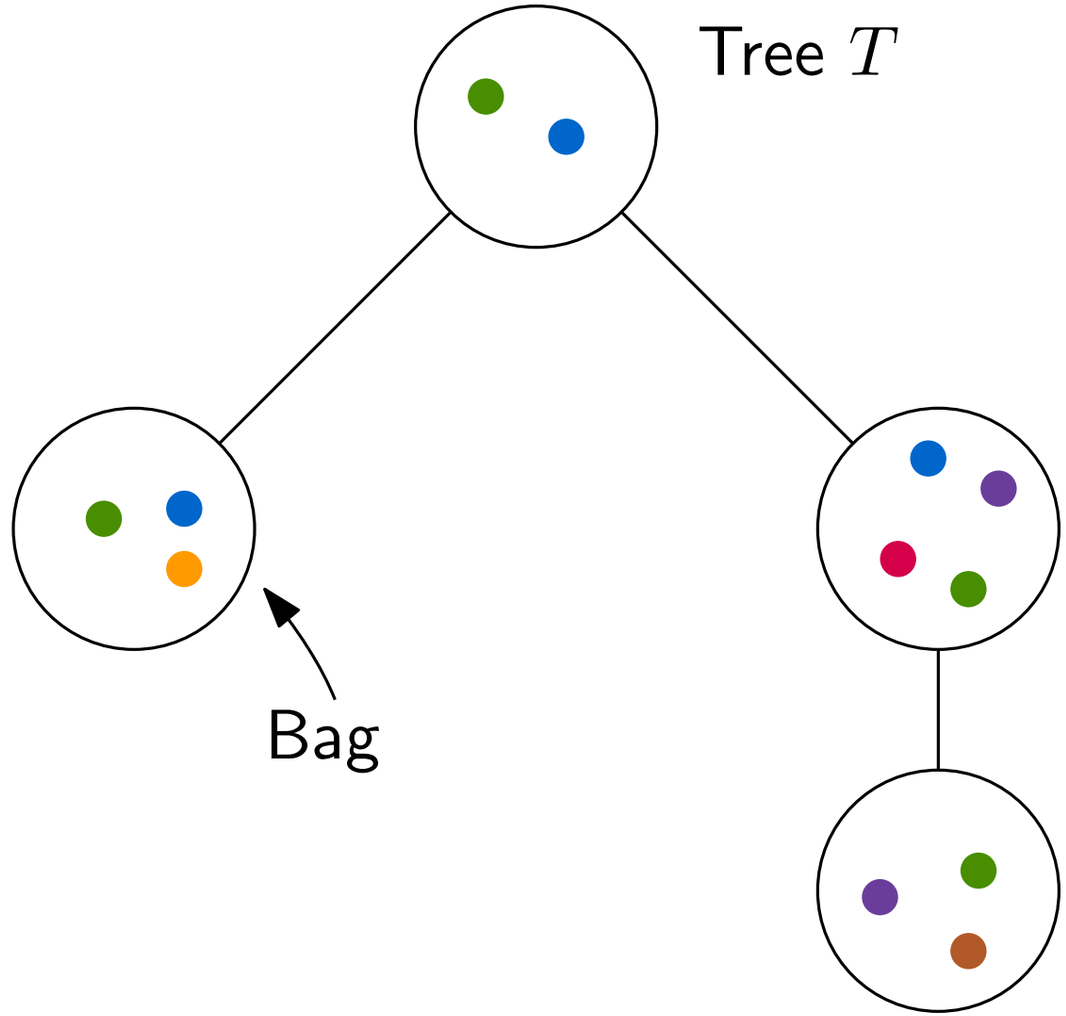

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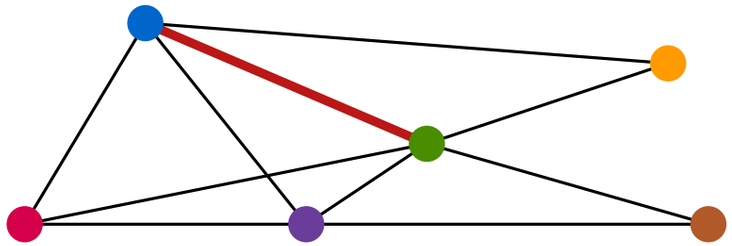

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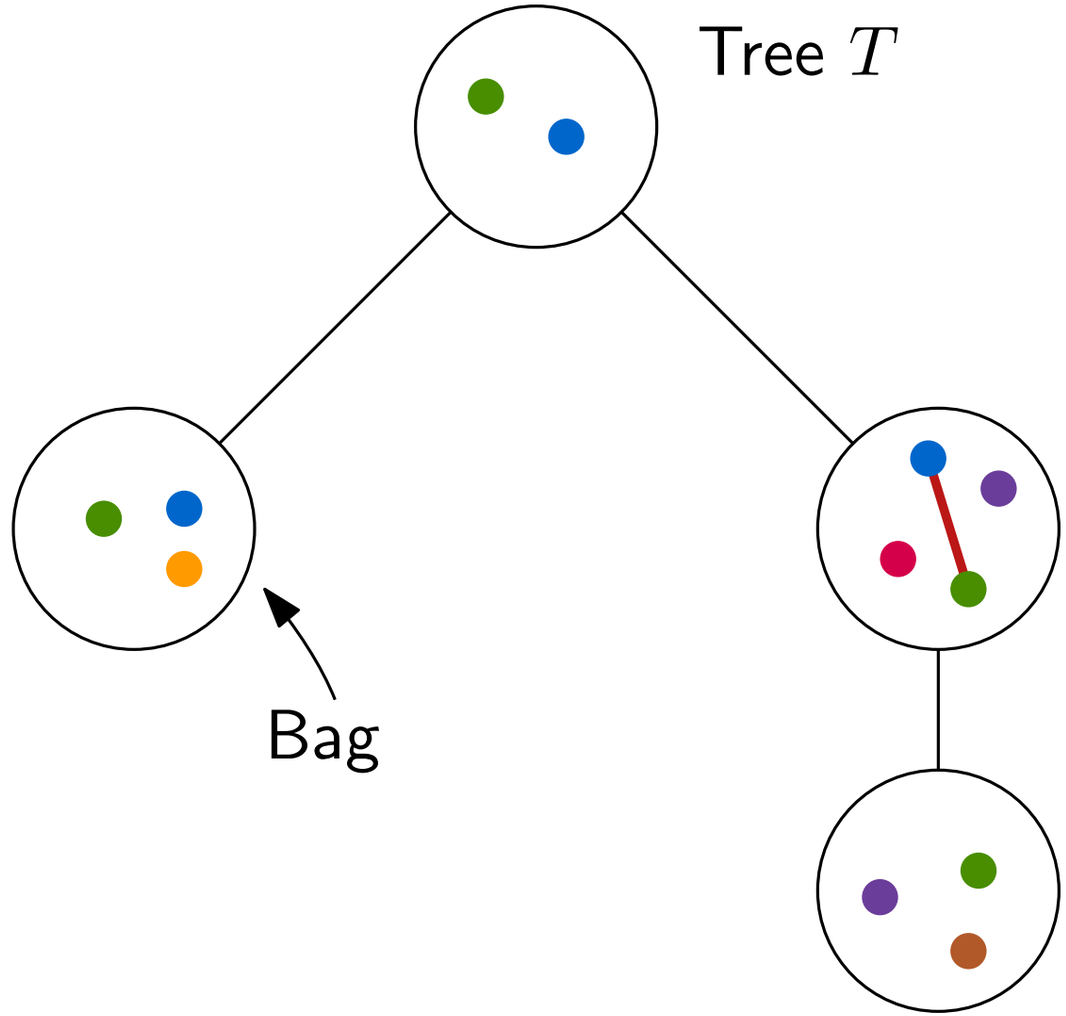

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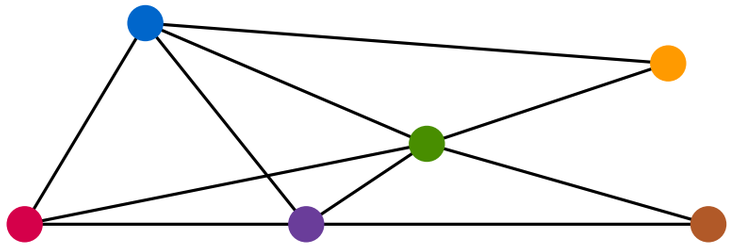

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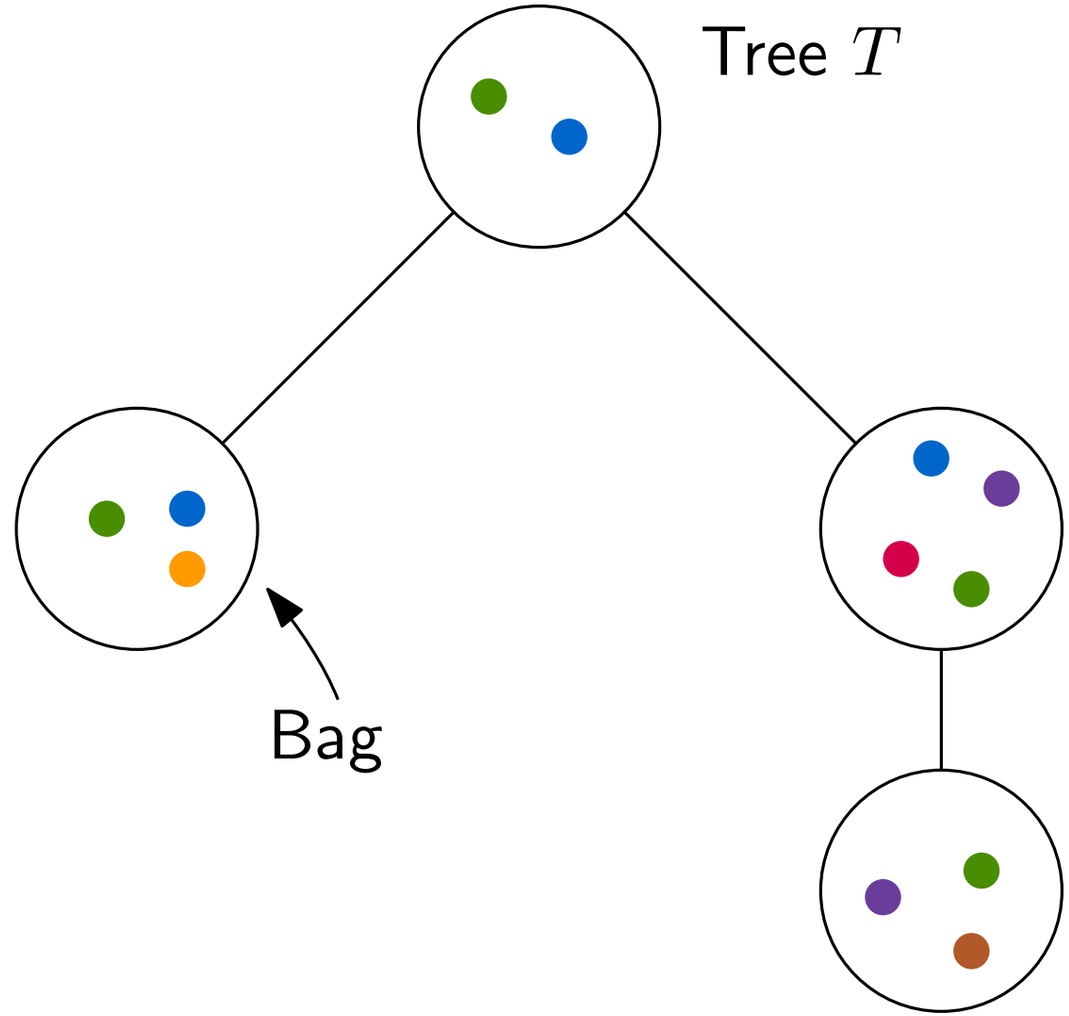

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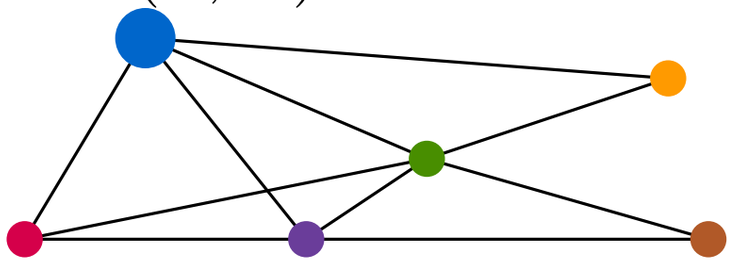

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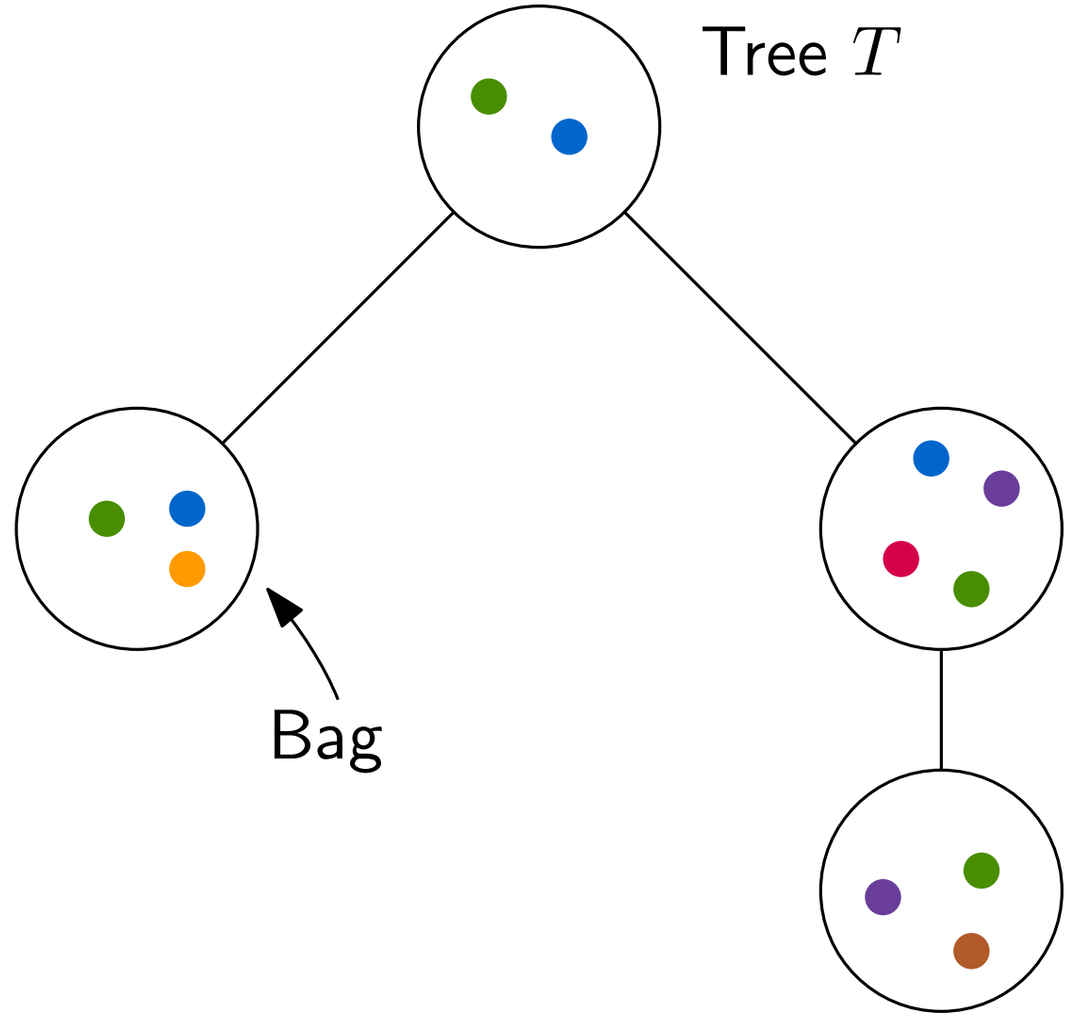

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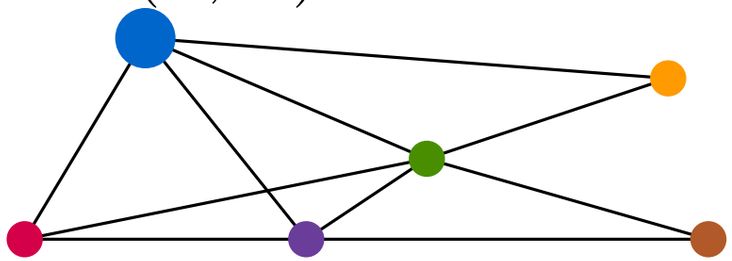

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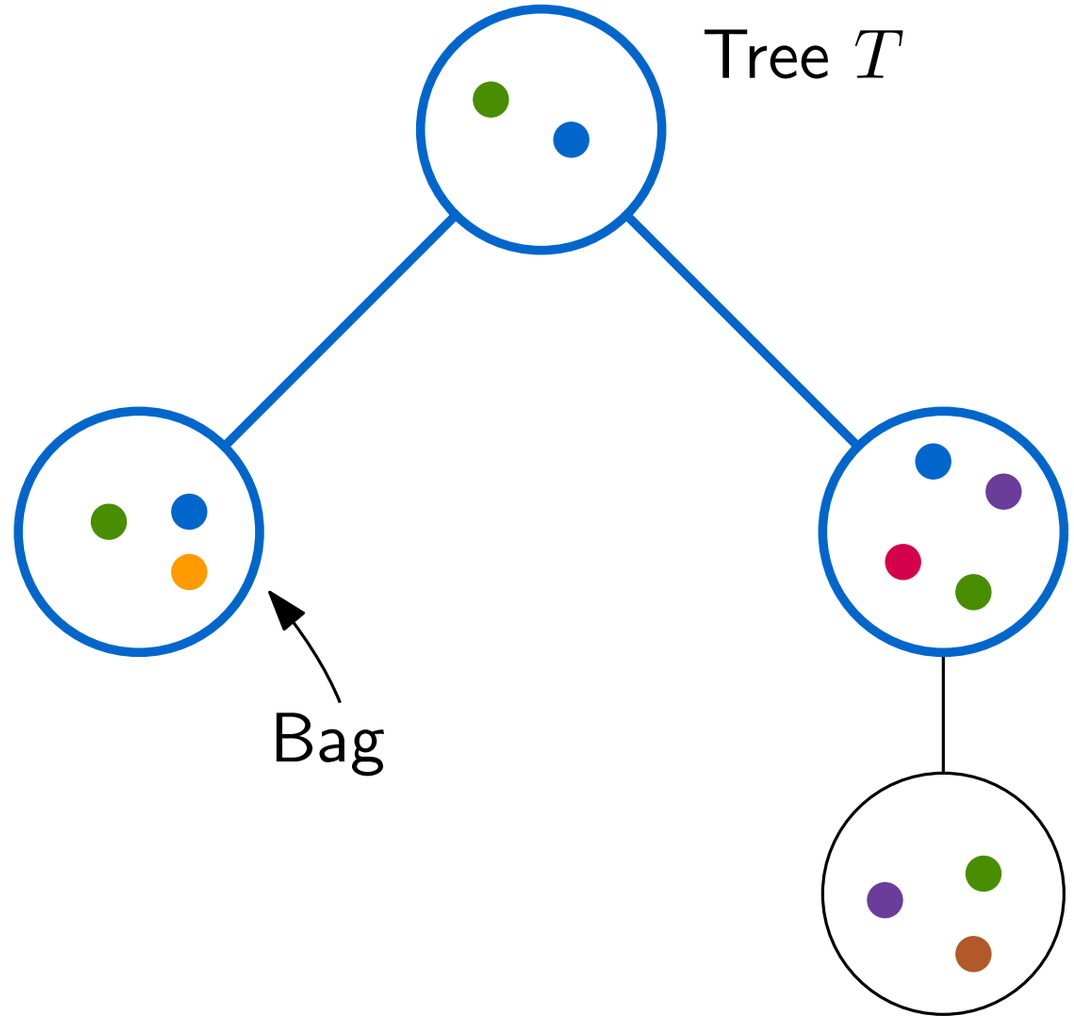

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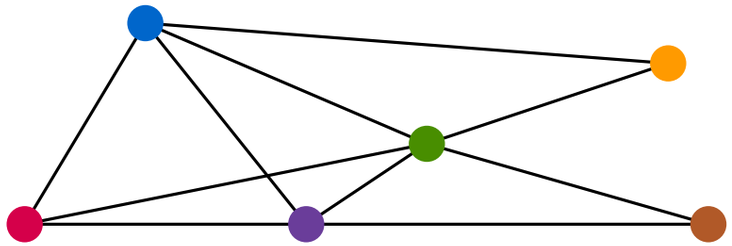

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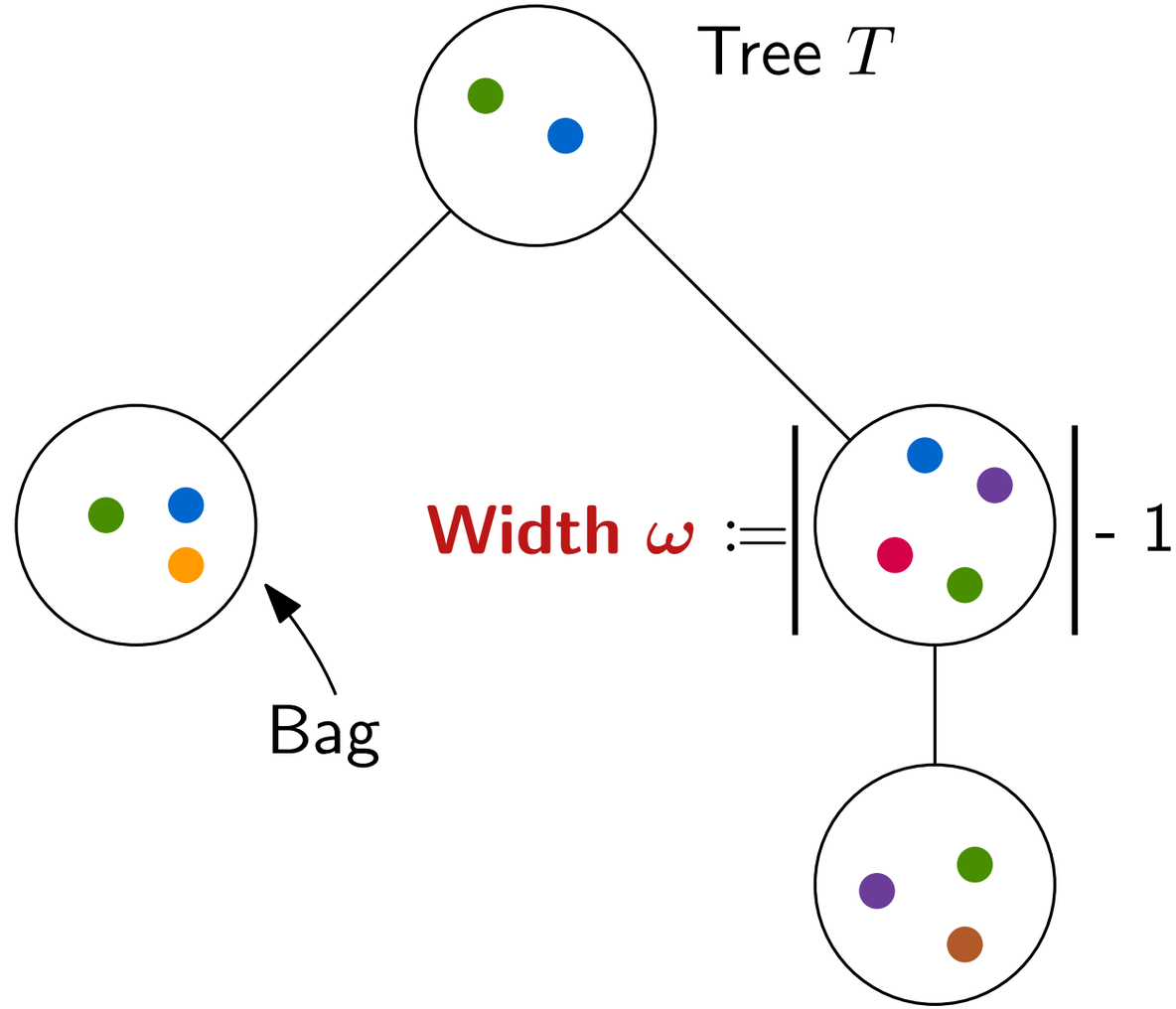

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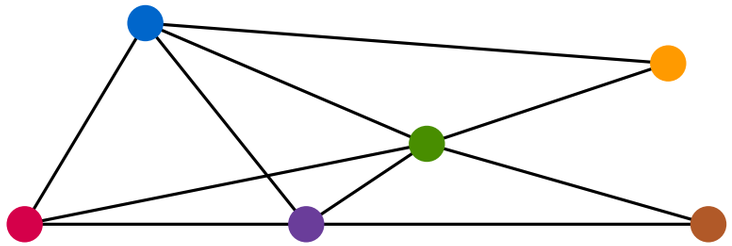

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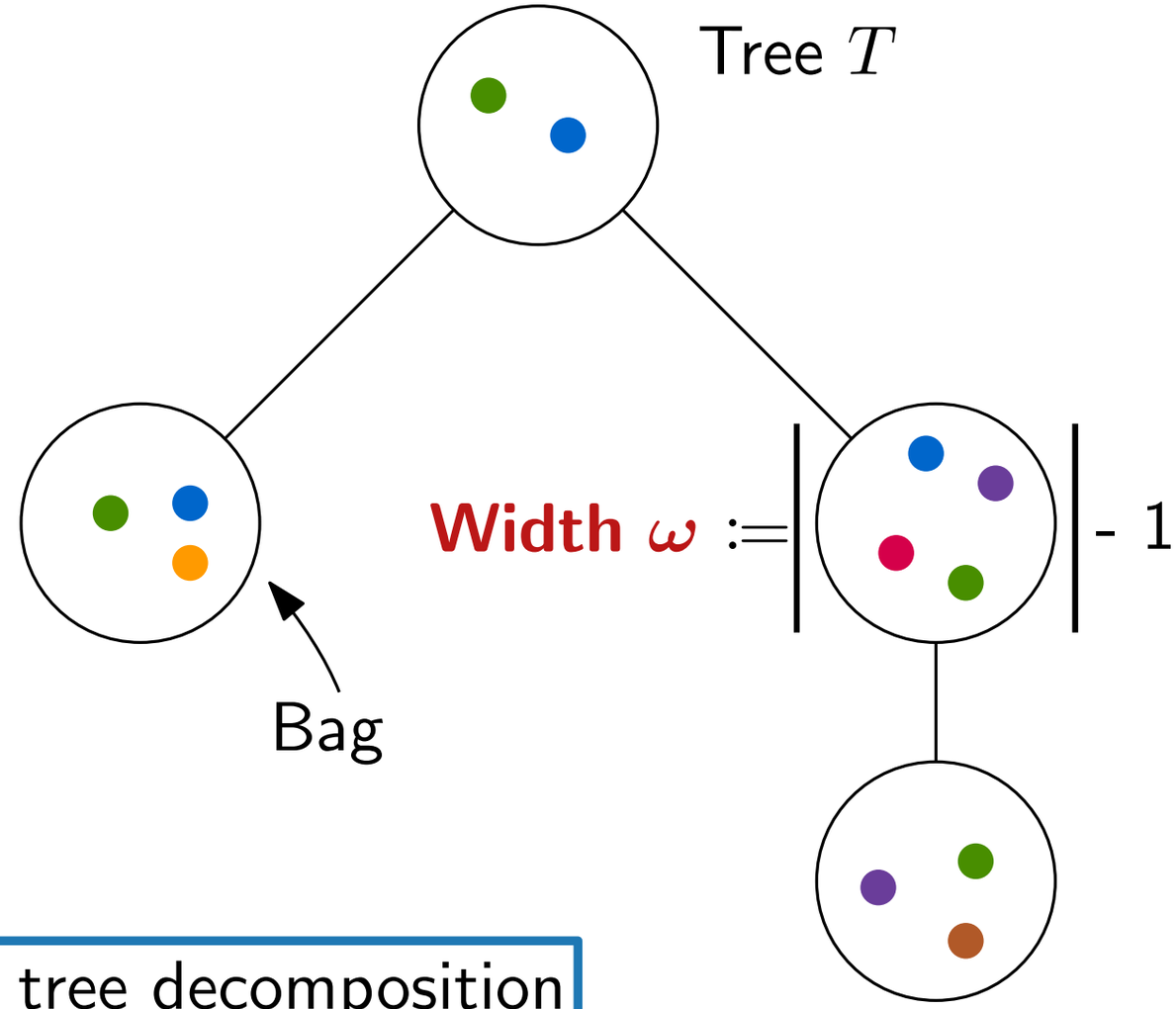

Treewidth of G: Smallest width of tree decomposition

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Many problems: NP-hard on general graphs, poly-time solvable on trees
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11:00 — 11:20	Md. Jawaherul Alam, Michael Bekos, Martin Gronemann and Michael Kaufmann. **The Page Number of Monotone Directed Acyclic Outerplanar Graphs is Four or Five** [T1]
11:20 — 11:40	Michael Bekos, Giordano Da Lozzo, Fabrizio Frati, Giuseppe Liotta and Antonios Symvonis. **Internally-Convex Drawings of Outerplanar Graphs in Small Area** [T1]
11:40 — 12:00	Rafał Pyzik. **Treewidth of Outer k-Planar Graphs** [T1]
12:00 — 12:20	Alvin Chiu, Thomas Depian, David Eppstein, Michael T. Goodrich and Martin Nöllenburg. **Visualizing Treewidth** [T2]
12:20 — 14:00	Lunch, *Trozelli Gallery*
14:00 — 15:00	**Invited Talk**, ***Hemerycksalen*** **Prof. Dr. Hans Bodlaender**. A Sketch of Parameterized Complexity, **Chair: Vida Dujmovic**
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Bounding the Treewidth of Outer k-Planar Graphs via Triangulations [GD'24]

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Universität Würzburg, Germany

Grzegorz Gutowski
Institute of Theoretical Computer Science, Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland

Myroslav Kryven
University of Manitoba, Canada

Yuto Okada
Nagoya University, Japan

Alexander Wolff
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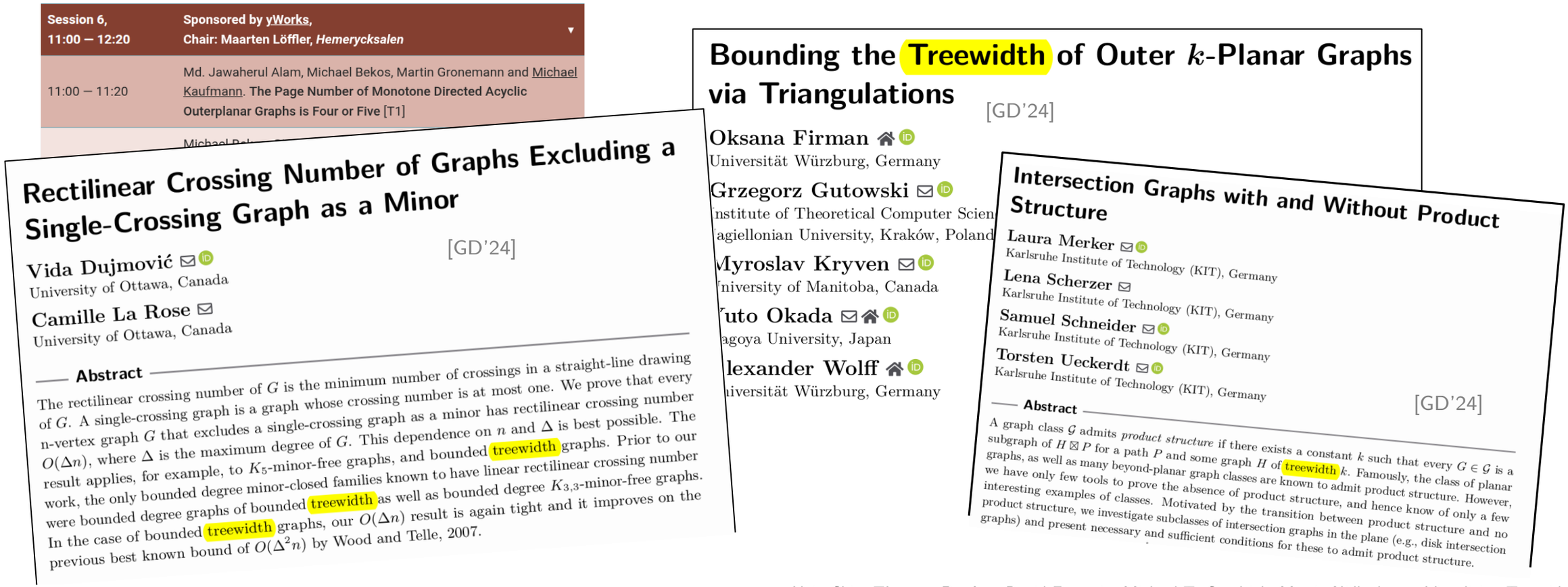

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Rectilinear Crossing Number of Graphs Excluding a Single-Crossing Graph as a Minor [GD'24]

Vida Dujmović
University of Ottawa, Canada

Camille La Rose
University of Ottawa, Canada

Abstract

The rectilinear crossing number of G is the minimum number of crossings in a straight-line drawing of G. A single-crossing graph is a graph whose crossing number is at most one. We prove that every n-vertex graph G that excludes a single-crossing graph as a minor has rectilinear crossing number $O(\Delta n)$, where Δ is the maximum degree of G. This dependence on n and Δ is best possible. The result applies, for example, to K_5-minor-free graphs, and bounded treewidth graphs. Prior to our work, the only bounded degree minor-closed families known to have linear rectilinear crossing number were bounded degree graphs of bounded treewidth as well as bounded degree $K_{3,3}$-minor-free graphs. In the case of bounded treewidth graphs, our $O(\Delta n)$ result is again tight and it improves on the previous best known bound of $O(\Delta^2 n)$ by Wood and Telle, 2007.

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Universität Würzburg, Germany

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Myroslav Kryven
niversity of Manitoba, Canada

uto Okada
agoya University, Japan

lexander Wolff
niversität Würzburg, Germany

Intersection Graphs with and Without Product Structure [GD'24]

Laura Merker
Karlsruhe Institute of Technology (KIT), Germany

Lena Scherzer
Karlsruhe Institute of Technology (KIT), Germany

Samuel Schneider
Karlsruhe Institute of Technology (KIT), Germany

Torsten Ueckerdt
Karlsruhe Institute of Technology (KIT), Germany

Abstract

A graph class $\mathcal{G}$ admits *product structure* if there exists a constant k such that every $G \in \mathcal{G}$ is a subgraph of $H \boxtimes P$ for a path P and some graph H of treewidth k. Famously, the class of planar graphs, as well as many beyond-planar graph classes are known to admit product structure. However, we have only few tools to prove the absence of product structure, and hence know of only a few interesting examples of classes. Motivated by the transition between product structure and no product structure, we investigate subclasses of intersection graphs in the plane (e.g., disk intersection graphs) and present necessary and sufficient conditions for these to admit product structure.

2-Layer k-Planar Graphs
Density, Crossing Lemma, Relationships, and Pathwidth

Patrizio Angelini[1], Giordano Da Lozzo[2], Henry Förster[3(✉)], and Thomas Schneck[3]

[1] John Cabot University, Rome, Italy
pangelini@johncabot.edu
[2] Roma Tre University, Rome, Italy
giordano.dalozzo@uniroma3.it
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{foersth,schneck}@informatik.uni-tuebingen.de

[GD'24]

Abstract. The 2-layer drawing model is a well-established paradigm to visualize bipartite graphs. Several beyond-planar graph classes have been studied under this model. Surprisingly, however, the fundamental class of k-planar graphs has been considered only for $k = 1$ in this context. We provide several contributions that address this gap in the literature. First, we show tight density bounds for the classes of 2-layer k-planar graphs with $k \in \{2, 3, 4, 5\}$. Based on these results, we provide a Crossing Lemma for 2-layer k-planar graphs, which then implies a general density bound for 2-layer k-planar graphs. We prove this bound to be almost optimal with a corresponding lower bound construction. Finally, we study relationships between k-planarity and h-quasiplanarity in the 2-layer model and show that 2-layer k-planar graphs have pathwidth at most $k+1$.

The Local Queue Number of Graphs with Bounded Treewidth

[GD'20]

Laura Merker(✉) and Torsten Ueckerdt

Institute of Theoretical Informatics, Karlsruhe Institute of Technology (KIT)
Karlsruhe, Germany
laura.merker@student.kit.edu, torsten.ueckerdt@kit.edu

Abstract. A queue layout of a graph G consists of a vertex ordering of G and a partition of the edges into so-called queues such that no two edges in the same queue nest, i.e., have their endpoints ordered in an ABBA-pattern. Continuing the research on local ordered covering numbers, we introduce the local queue number of a graph G as the minimum ℓ such that G admits a queue layout with each vertex having incident edges

Upward and Orthogonal Planarity are W[1]-Hard Parameterized by Treewidth

Bart M. P. Jansen[1], Liana Khazaliya[2(✉)], Philipp Kindermann[3], Giuseppe Liotta[4], Fabrizio Montecchiani[4], and Kirill Simonov[5]

[1] Eindhoven University of Technology, Eindhoven, The Netherlands
b.m.p.jansen@tue.nl
[2] Technische Universität Wien, Vienna, Austria
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[3] Universität Trier, Trier, Germany
kindermann@uni-trier.de
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{giuseppe.liotta,fabrizio.montecchiani}@unipg.it
[5] Hasso Plattner Institute, University of Potsdam, Potsdam, Germany

[GD'23]

Appl. Math.'93] . . .

Eppstein, Martin Gronemann and Michael

Outerplanar Graphs is Four or Five [T1]

Bounding the Treewidth of Outer k-Planar Graphs via Triangulations

[GD'24]

Oksana Firman
Universität Würzburg, Germany

Grzegorz Gutowski
nstitute of Theoretical Computer Scien
agiellonian University, Kraków, Poland

Myroslav Kryven
niversity of Manitoba, Canada

uto Okada
agoya University, Japan

lexander Wolff
niversität Würzburg, Germany

Rectilinear Crossing Number of Graphs Excluding a Single-Crossing Graph as a Minor

[GD'24]

Vida Dujmović
University of Ottawa, Canada

Camille La Rose
University of Ottawa, Canada

Abstract

The rectilinear crossing number of G is the minimum number of crossings in a straight-line drawing of G. A single-crossing graph is a graph whose crossing number is at most one. We prove that every n-vertex graph G that excludes a single-crossing graph as a minor has rectilinear crossing number $O(\Delta n)$, where Δ is the maximum degree of G. This dependence on n and Δ is best possible. The result applies, for example, to K_5-minor-free graphs, and bounded treewidth graphs. Prior to our work, the only bounded degree minor-closed families known to have linear rectilinear crossing number were bounded degree graphs of bounded treewidth as well as bounded degree $K_{3,3}$-minor-free graphs. In the case of bounded treewidth graphs, our $O(\Delta n)$ result is again tight and it improves on the previous best known bound of $O(\Delta^2 n)$ by Wood and Telle, 2007.

Intersection Graphs with and Without Product Structure

Laura Merker
Karlsruhe Institute of Technology (KIT), Germany

Lena Scherzer
Karlsruhe Institute of Technology (KIT), Germany

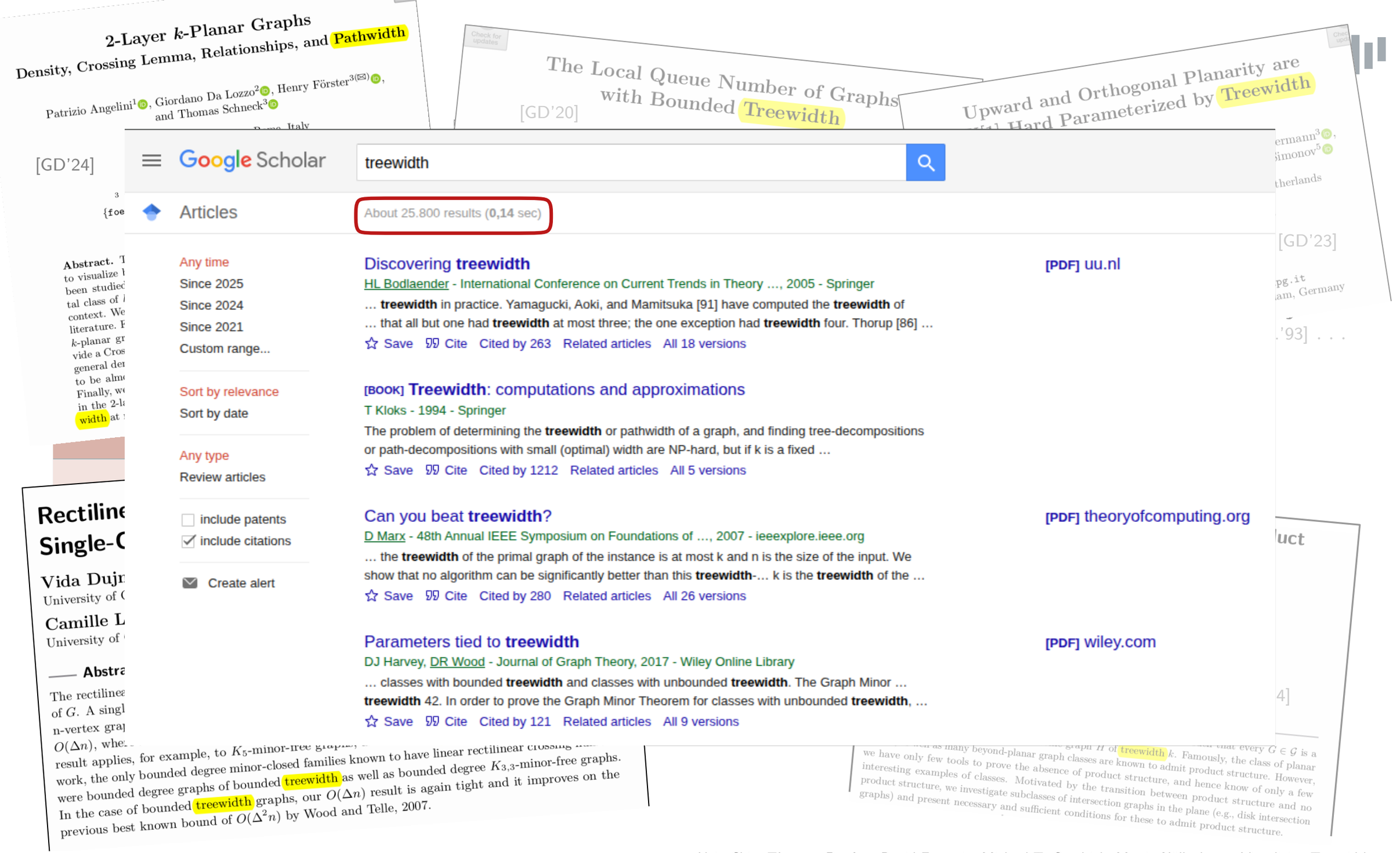
Samuel Schneider
Karlsruhe Institute of Technology (KIT), Germany

Torsten Ueckerdt
Karlsruhe Institute of Technology (KIT), Germany

[GD'24]

Abstract

A graph class $\mathcal{G}$ admits *product structure* if there exists a constant k such that every $G \in \mathcal{G}$ is a subgraph of $H \boxtimes P$ for a path P and some graph H of treewidth k. Famously, the class of planar graphs, as well as many beyond-planar graph classes are known to admit product structure. However, we have only few tools to prove the absence of product structure, and hence know of only a few interesting examples of classes. Motivated by the transition between product structure and no product structure, we investigate subclasses of intersection graphs in the plane (e.g., disk intersection graphs) and present necessary and sufficient conditions for these to admit product structure.


2-Layer k-Planar Graphs
Density, Crossing Lemma, Relationships, and Pathwidth
Patrizio Angelini[1], Giordano Da Lozzo[2], Henry Förster[3](✉), and Thomas Schneck[3]
[GD'24]
The Local Queue Number of Graphs with Bounded Treewidth
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Google Scholar
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About 25.800 results (0,14 sec)
Any time
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Custom range...
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Any type
Review articles
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include citations
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Discovering treewidth
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HL Bodlaender - International Conference on Current Trends in Theory ..., 2005 - Springer
... treewidth in practice. Yamagucki, Aoki, and Mamitsuka [91] have computed the treewidth of
... that all but one had treewidth at most three; the one exception had treewidth four. Thorup [86] ...
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[BOOK] Treewidth: computations and approximations
T Kloks - 1994 - Springer
The problem of determining the treewidth or pathwidth of a graph, and finding tree-decompositions or path-decompositions with small (optimal) width are NP-hard, but if k is a fixed ...
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... the treewidth of the primal graph of the instance is at most k and n is the size of the input. We show that no algorithm can be significantly better than this treewidth-... k is the treewidth of the ...
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DJ Harvey, DR Wood - Journal of Graph Theory, 2017 - Wiley Online Library
... classes with bounded treewidth and classes with unbounded treewidth. The Graph Minor ...
treewidth 42. In order to prove the Graph Minor Theorem for classes with unbounded treewidth, ...
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Rectilinear Single-C
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Why Visualizing Treewidth?

Explain and understand concepts and properties of graphs

[Bodlaender, WG'06] [Maniu et al., ICDT'19] [Eppstein, Notices of AMS'25]

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In graph theory, the **treewidth** of an undirected graph is an integer number which specifies, informally, how far the graph is from being a tree. The smallest treewidth is 1; the graphs with treewidth 1 are exactly the trees and the forests. An example of graphs with treewidth at most 2 are the series–parallel graphs. The maximal graphs with treewidth exactly k are called *k-trees*, and the graphs with treewidth at most k are called *partial k-trees*.[1] Many other well-studied graph families also have bounded treewidth.

Treewidth may be formally defined in several equivalent ways: in terms of the size of the largest vertex set in a tree decomposition of the graph, in terms of the size of the largest clique in a chordal completion of the graph, in terms of the maximum order of a haven describing a strategy for a pursuit–evasion game on the graph, or in terms of the maximum order of a bramble, a collection of connected subgraphs that all touch each other.

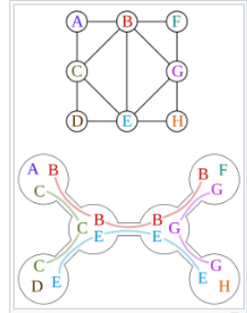
Treewidth is commonly used as a parameter in the parameterized complexity analysis of graph algorithms. Many algorithms that are NP-hard for general graphs, become easier when the treewidth is bounded by a constant.

The concept of treewidth was originally introduced by Umberto Bertelè and Francesco Brioschi (1972) under the name of *dimension*. It was later rediscovered by Rudolf Halin (1976), based on properties that it shares with a different graph parameter, the Hadwiger number. Later it was again rediscovered by Neil Robertson and Paul Seymour (1984) and has since been studied by many other authors.[2]

Definition [edit]

A tree decomposition of a graph $G = (V, E)$ is a tree T in which each node is associated with a subset of vertices called a "bag". (The term *node* is used to refer to a vertex of T to avoid confusion with vertices of G). The bags $X_1, \dots X_t$ must satisfy the following properties:[3]

1. Each graph vertex is contained in at least one bag: $\bigcup_i X_i = V$
2. If bags X_i and X_j both contain a vertex v, then all bags X_k associated with nodes in the (unique) path of T between X_i and X_j also contain v as well. Equivalently, the bags containing vertex v are associated with a connected subtree of T.
3. For every edge (v, w) in the graph, there is at least one bag X_i that contains both v and w. That is, vertices are adjacent in the graph only when their corresponding subtrees have a node in common. (However, two vertices may belong to a bag without being adjacent to each other.)



A graph with eight vertices, and a tree decomposition of it onto a tree with six nodes. Each graph edge connects two vertices that are listed together at some tree node,

The *width* of a tree decomposition is the size of its largest bag X_i minus one. The **treewidth** $\mathrm{tw}(G)$ of a graph G is the minimum width among all possible tree decompositions of G. In this definition, the size of the largest set is diminished by one in order to make the treewidth of a tree equal to one.

Equivalently, the treewidth of G is one less than the size of the largest clique in the chordal graph containing G with the smallest clique number. A chordal graph with this clique size may be obtained by adding to G an edge between every two vertices for which at least one bag contains both vertices.

Treewidth may also be characterized in terms of havens, functions describing an evasion strategy for a certain pursuit–evasion game defined on a graph. A graph G has treewidth k if and only if it has a haven of order $k + 1$ but of no higher order, where a haven of order $k + 1$ is a function β that maps each set X of at most k vertices in G into one of the connected components of $G \setminus X$ and that obeys the monotonicity property that $\beta(Y) \subset \beta(X)$ whenever $X \subset Y$

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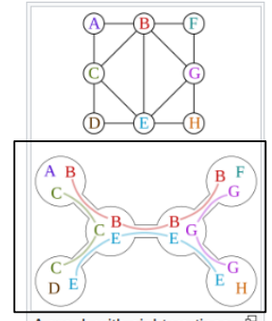

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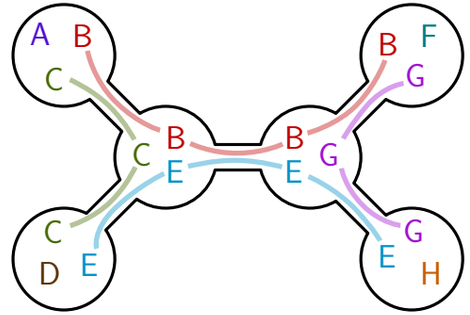

[Eppstein, Wikipedia'10]

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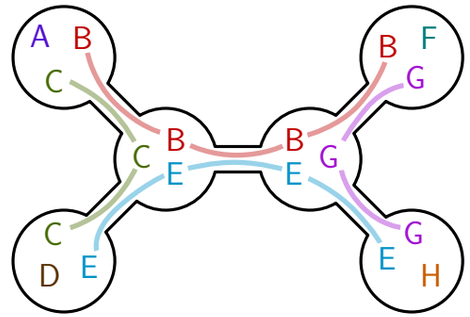

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Provide a **visual proof** for bounded treewidth graphs

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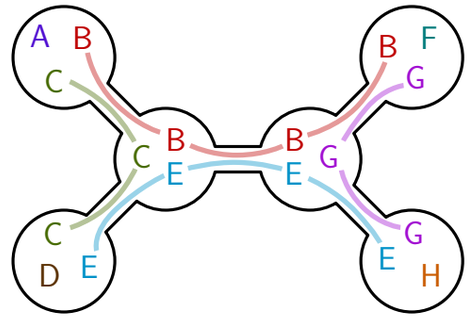

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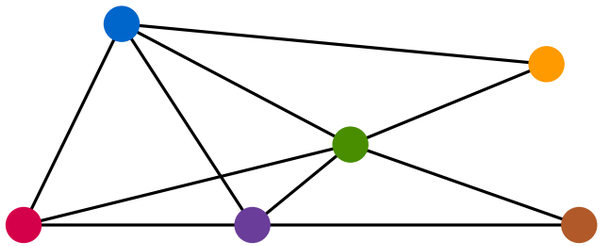

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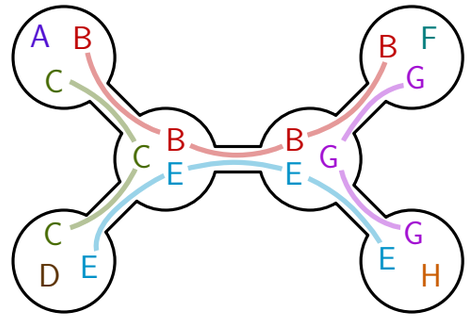

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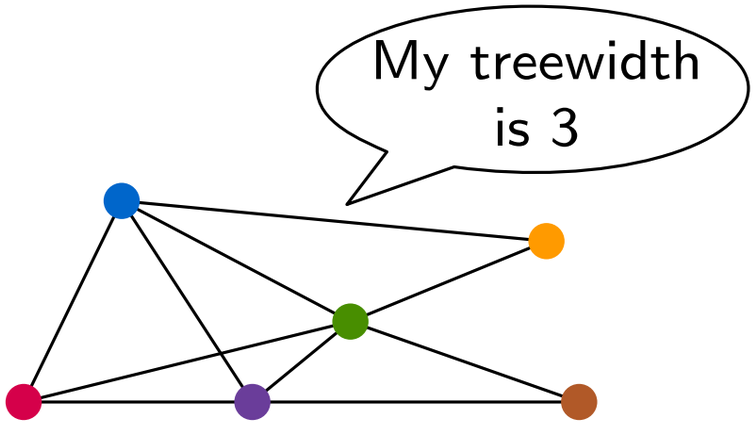

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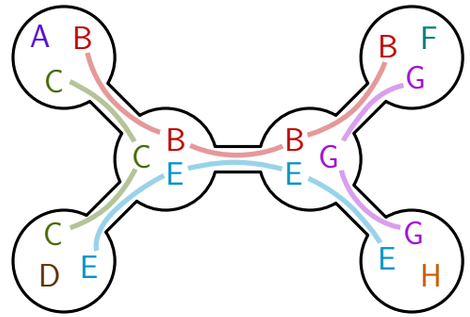

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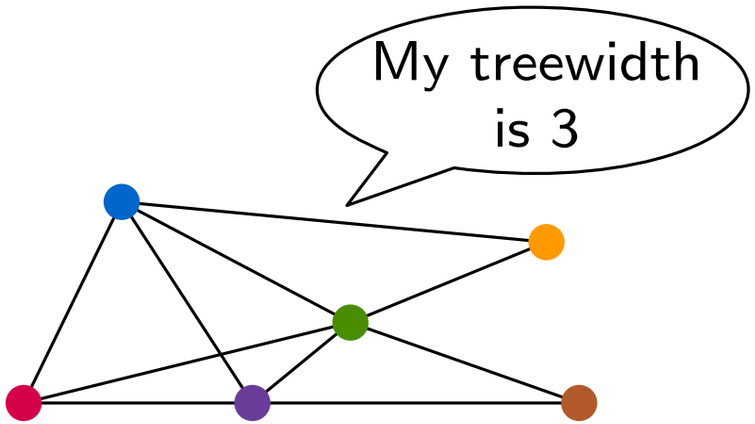

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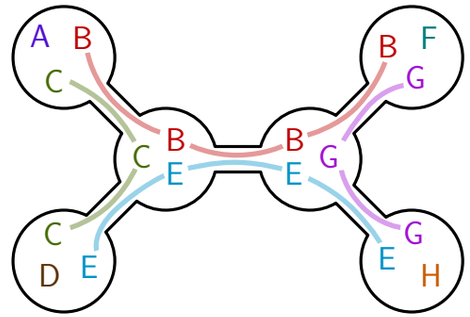

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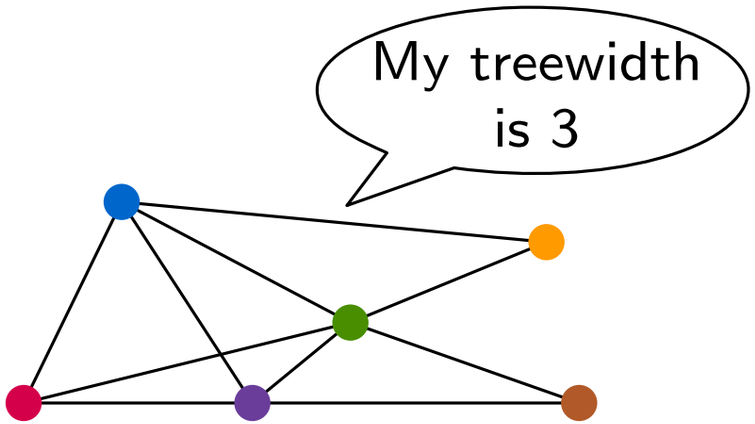

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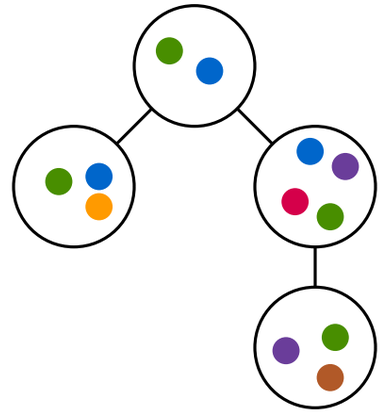

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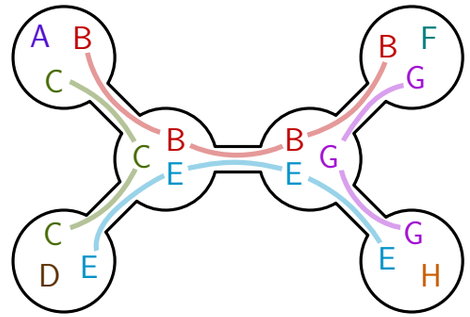

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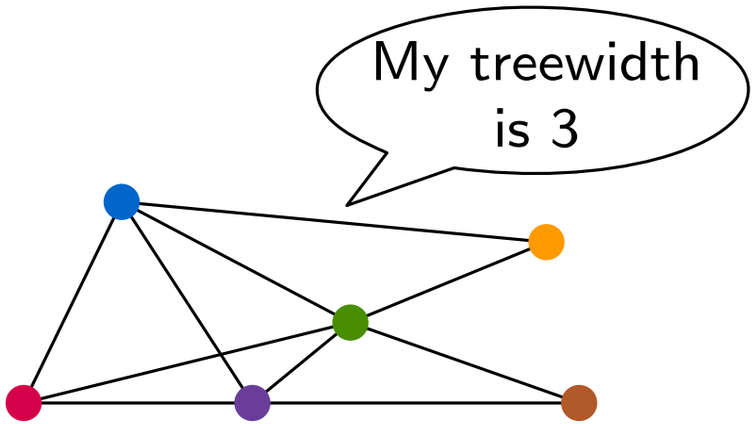

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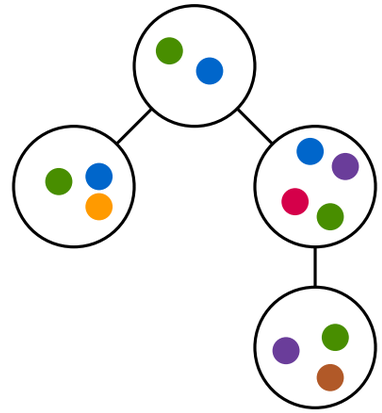

$\Rightarrow$ GraphTrials Framework [Förster et al., GD'24]

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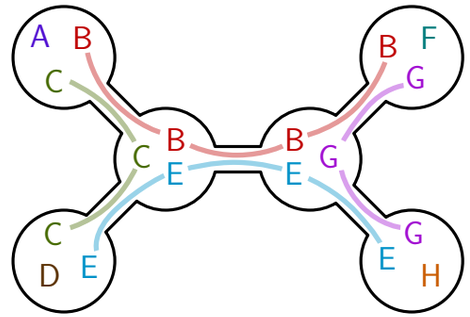

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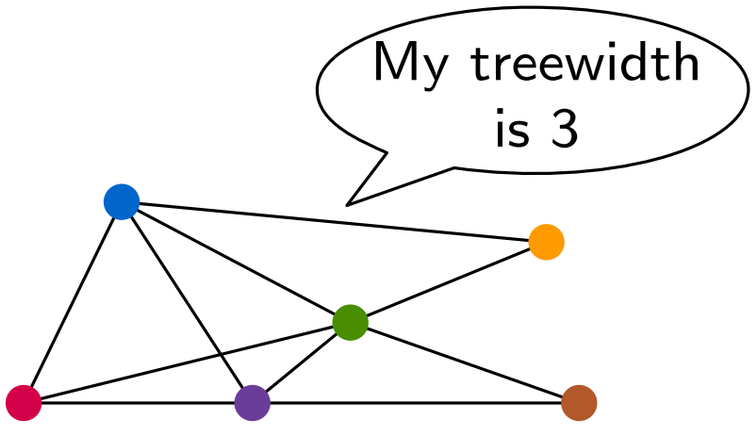

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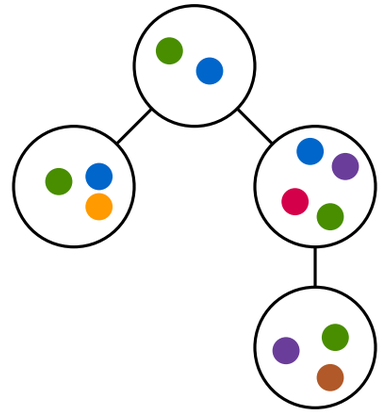


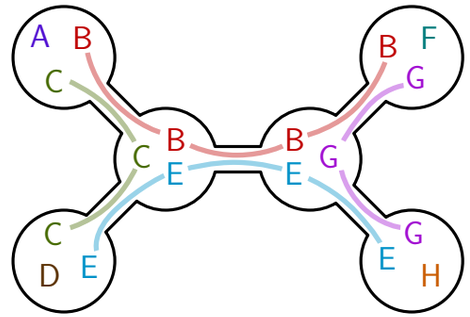
$\Rightarrow$ GraphTrials Framework [Förster et al., GD'24]

Algorithmic problem, related to crossing minimization in metro maps, book drawings, ...

[Argyriou et al., JGAA'10]
[Klawitter et al., GD'18]

Our Contributions

Drawing Paradigms for Treewidth



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Our Contributions

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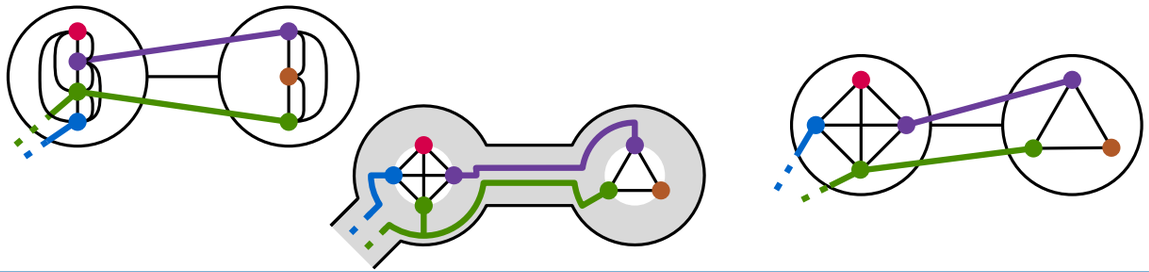

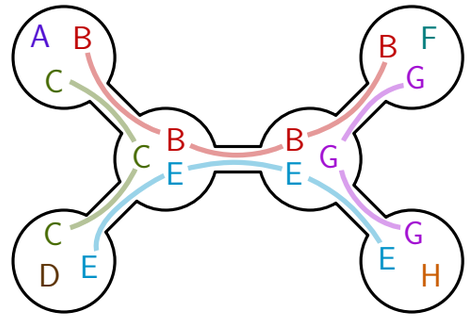

[Eppstein, Wikipedia'10]

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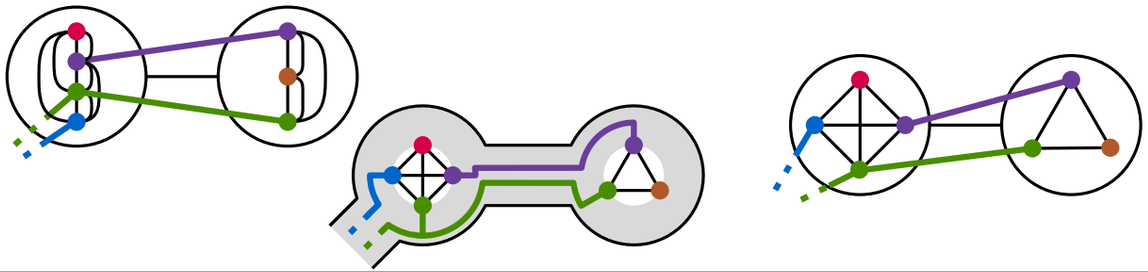

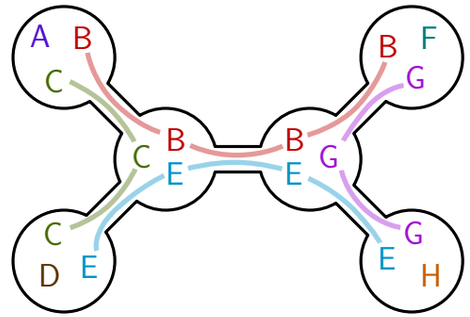

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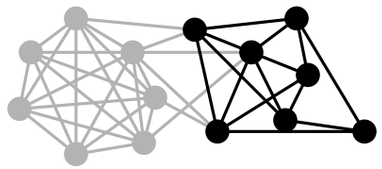
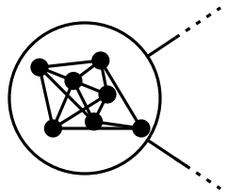
Complexity and Algorithms for Minimizing Crossings





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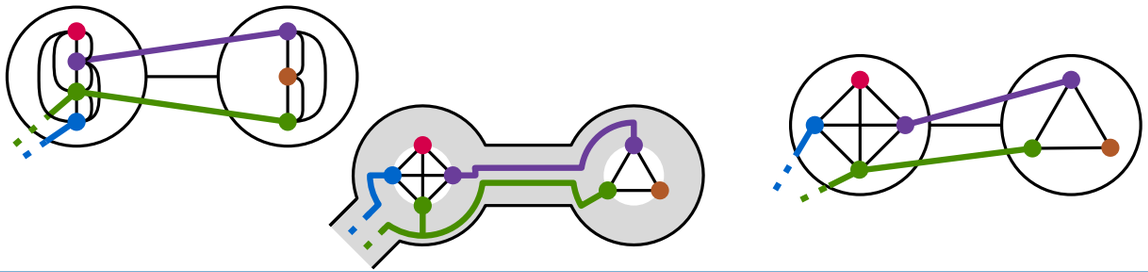

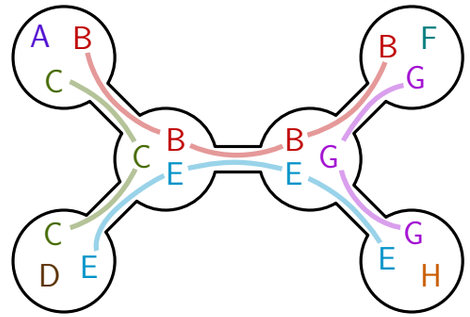

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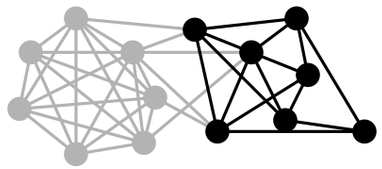
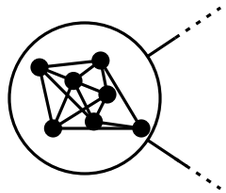
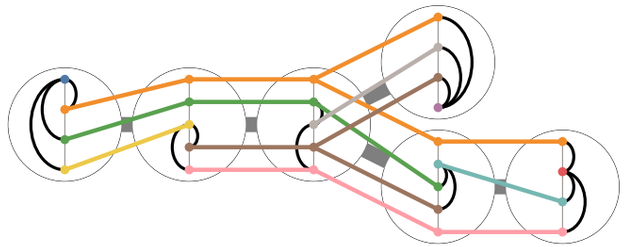




Experimental Evaluation



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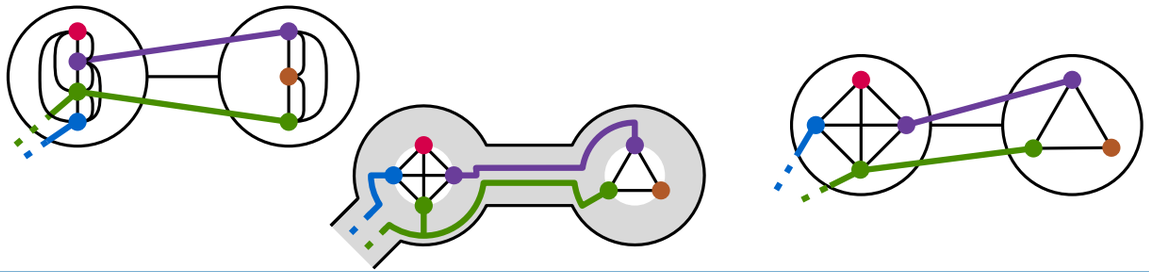

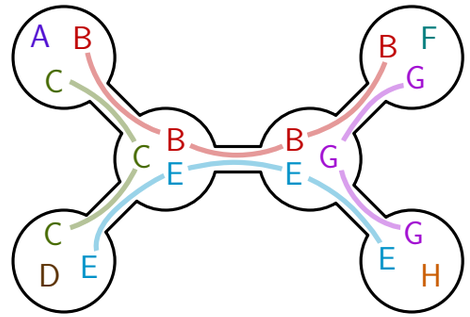

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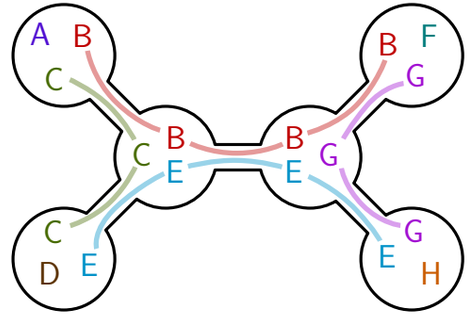
Witness Drawing Styles



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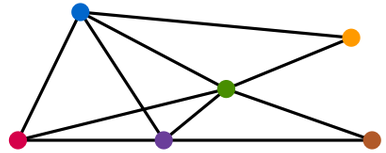

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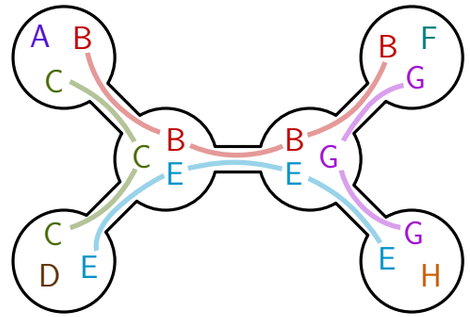

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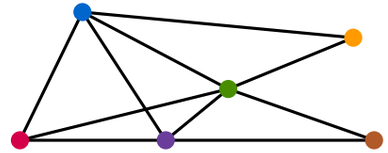

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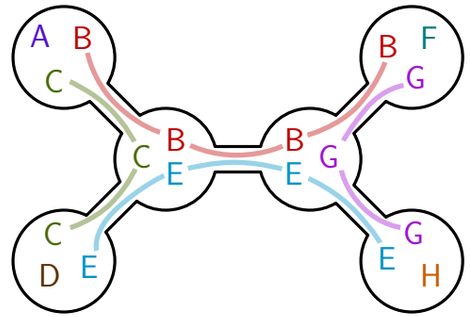

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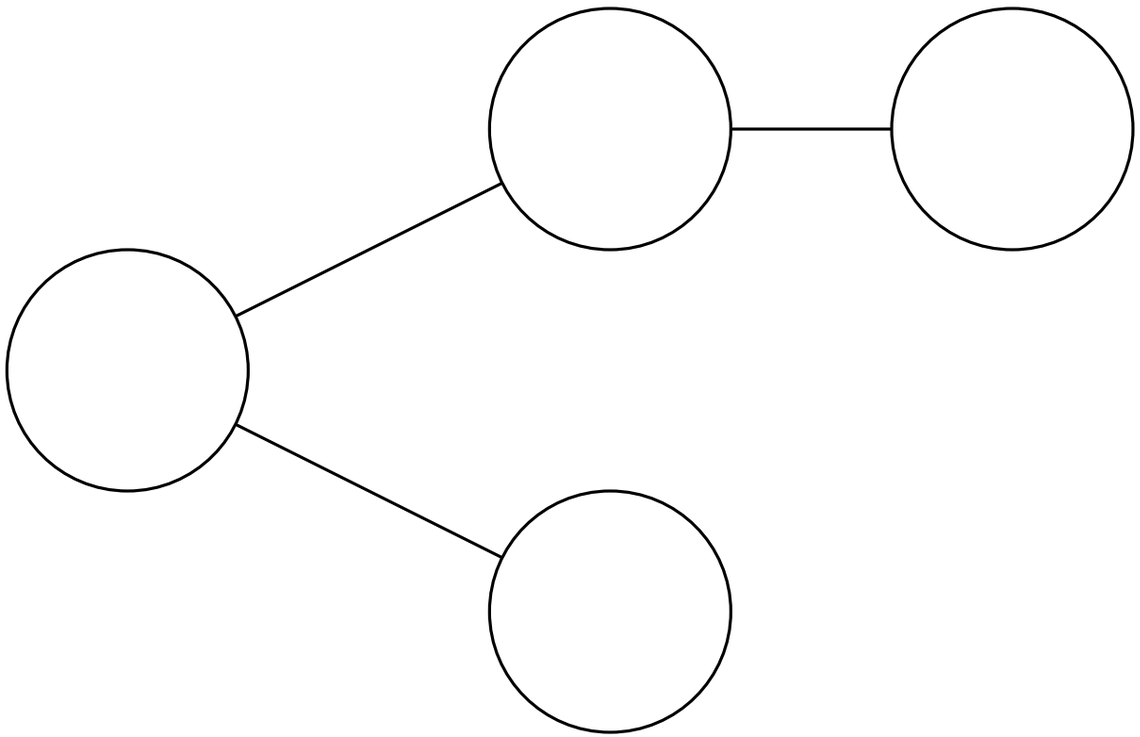

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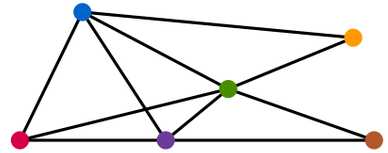

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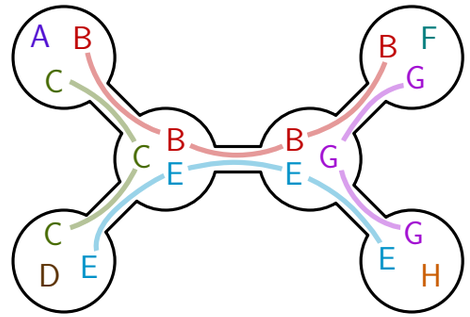

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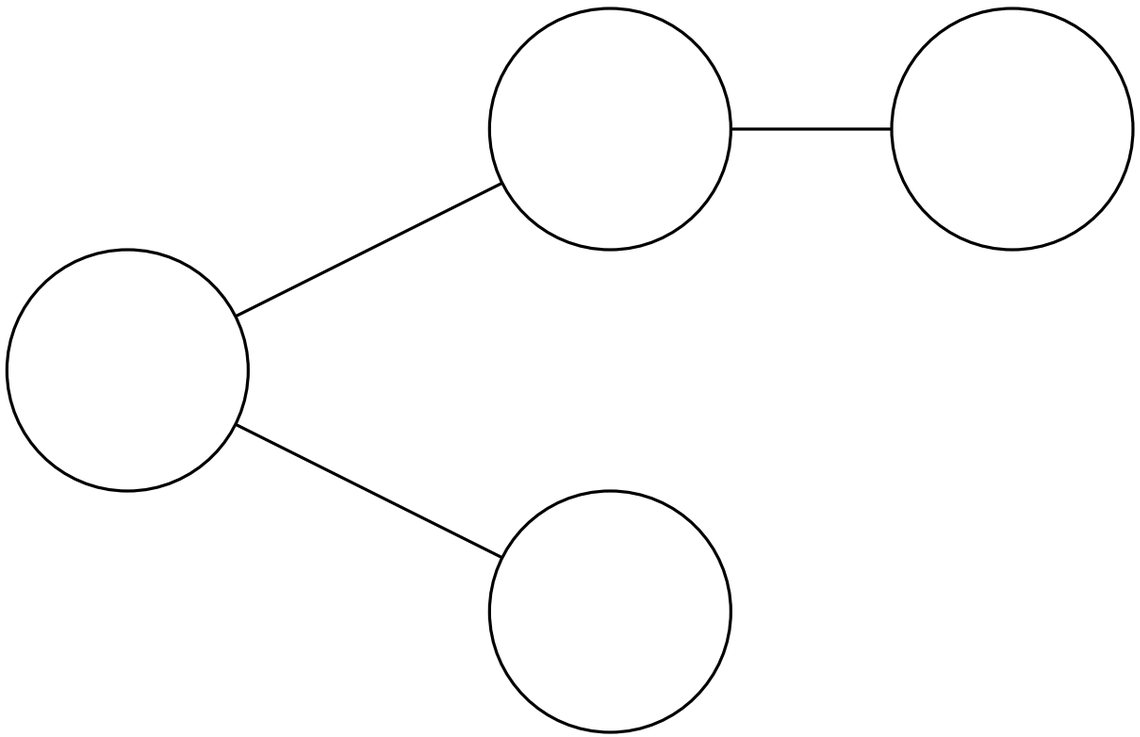

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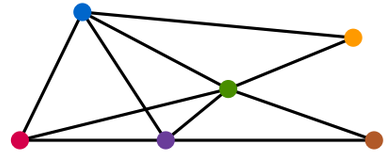

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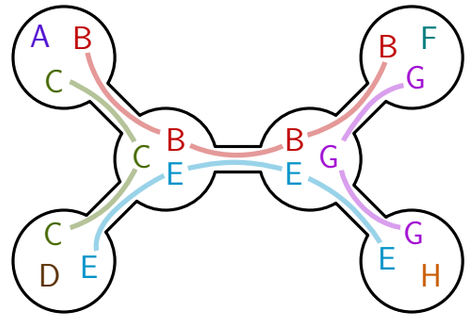

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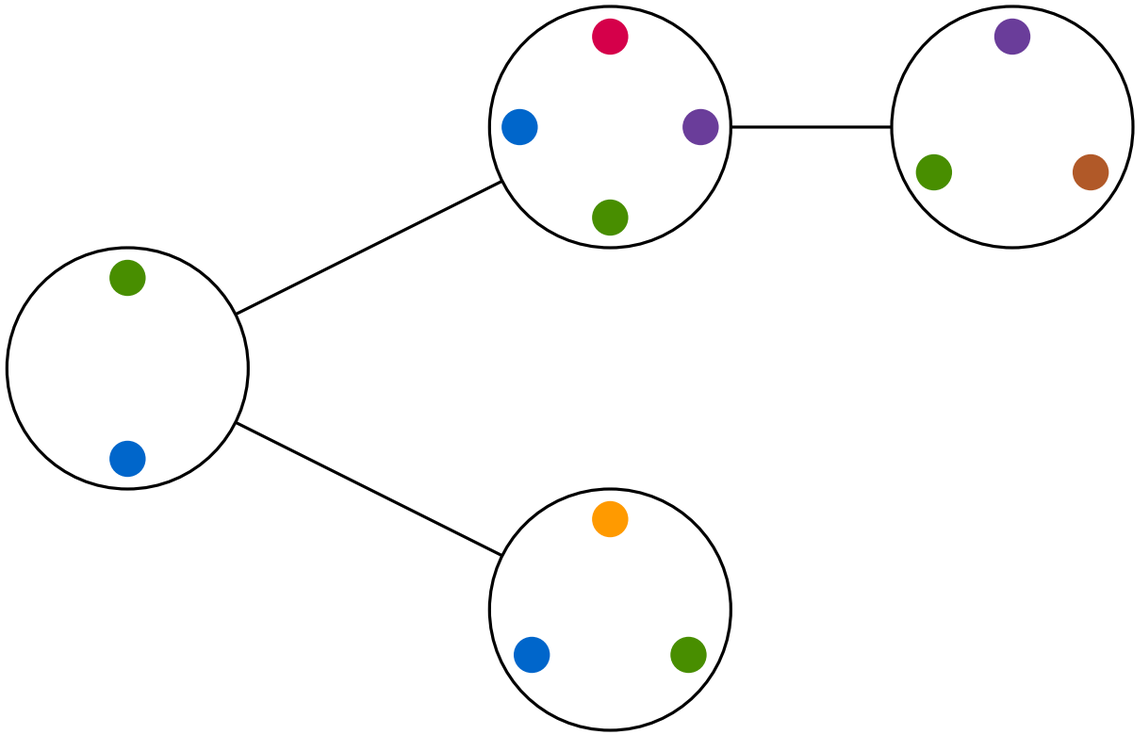

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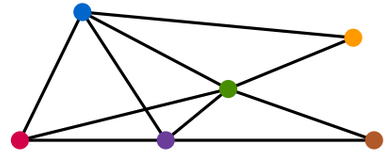

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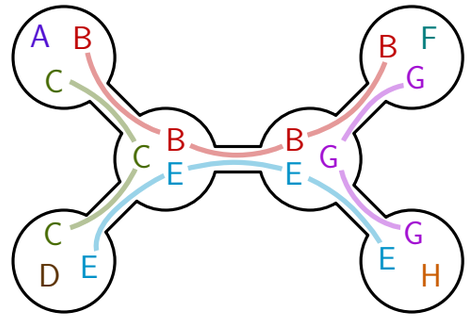

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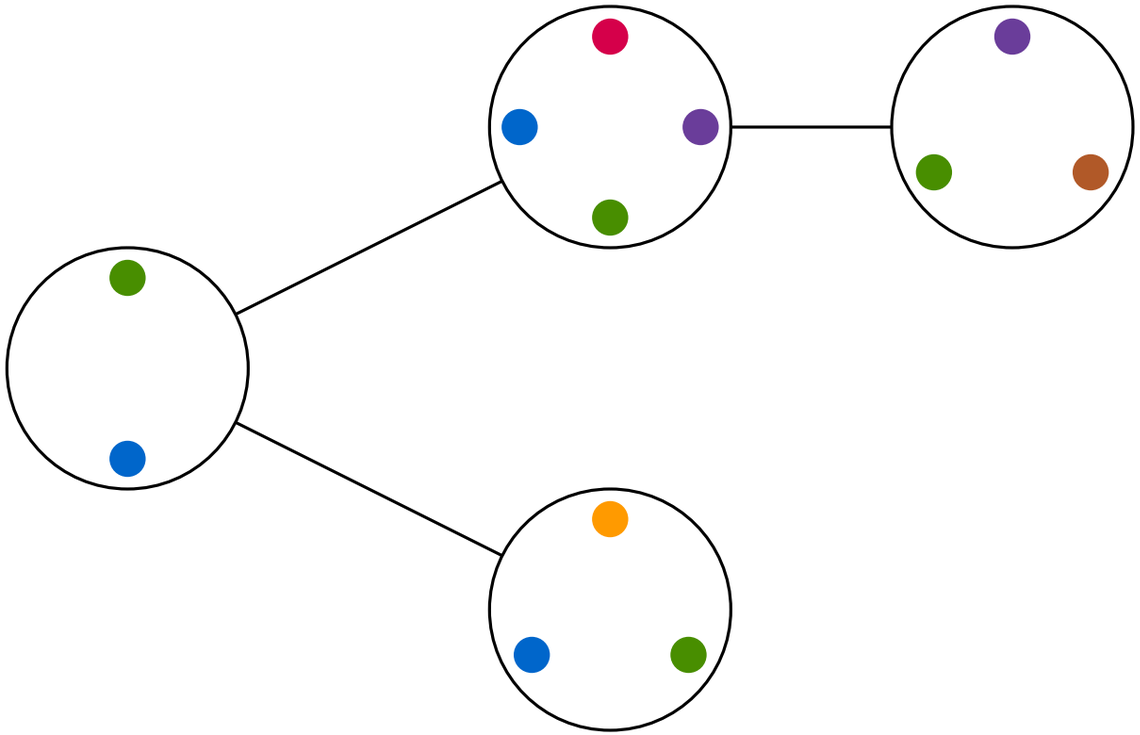

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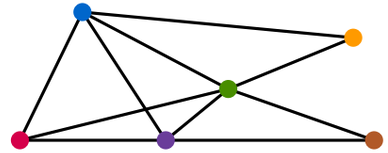

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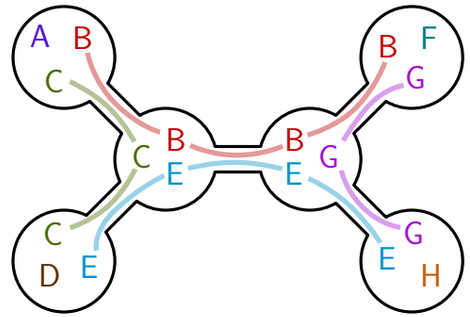

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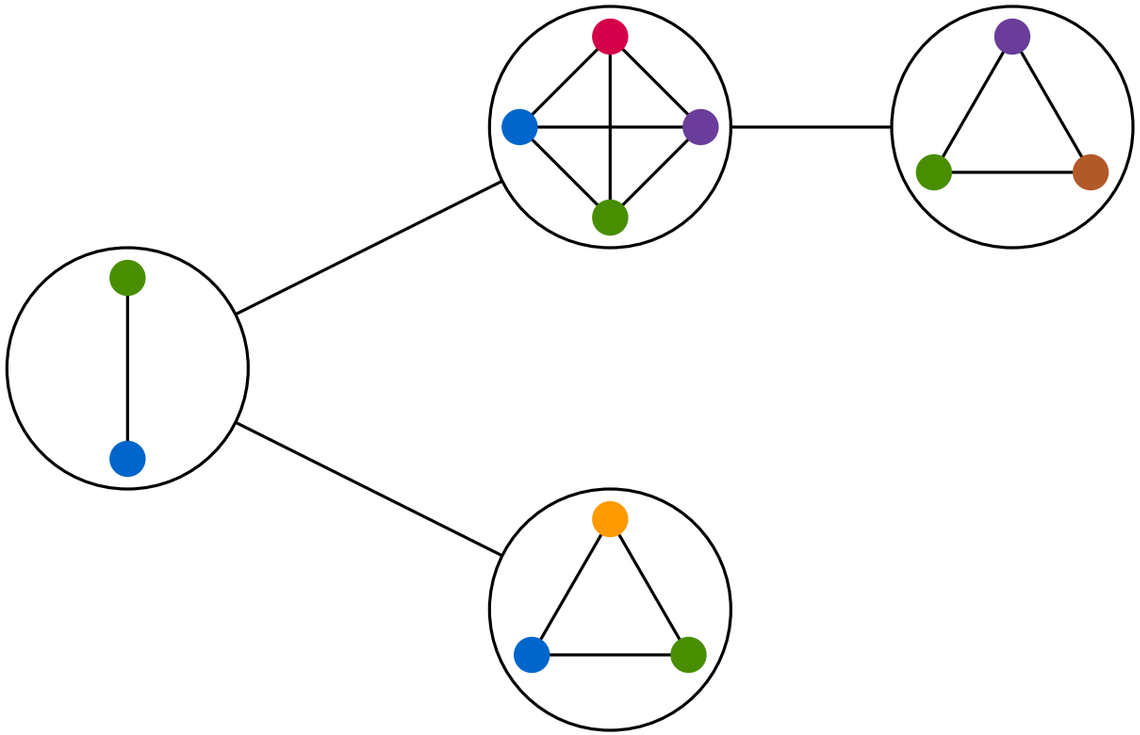

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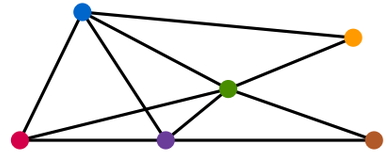

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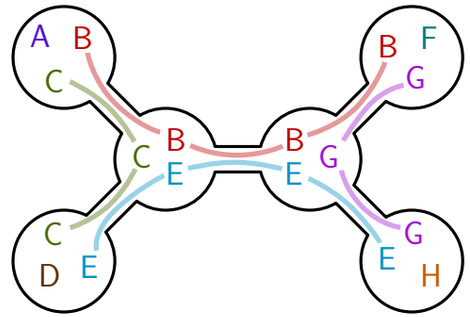

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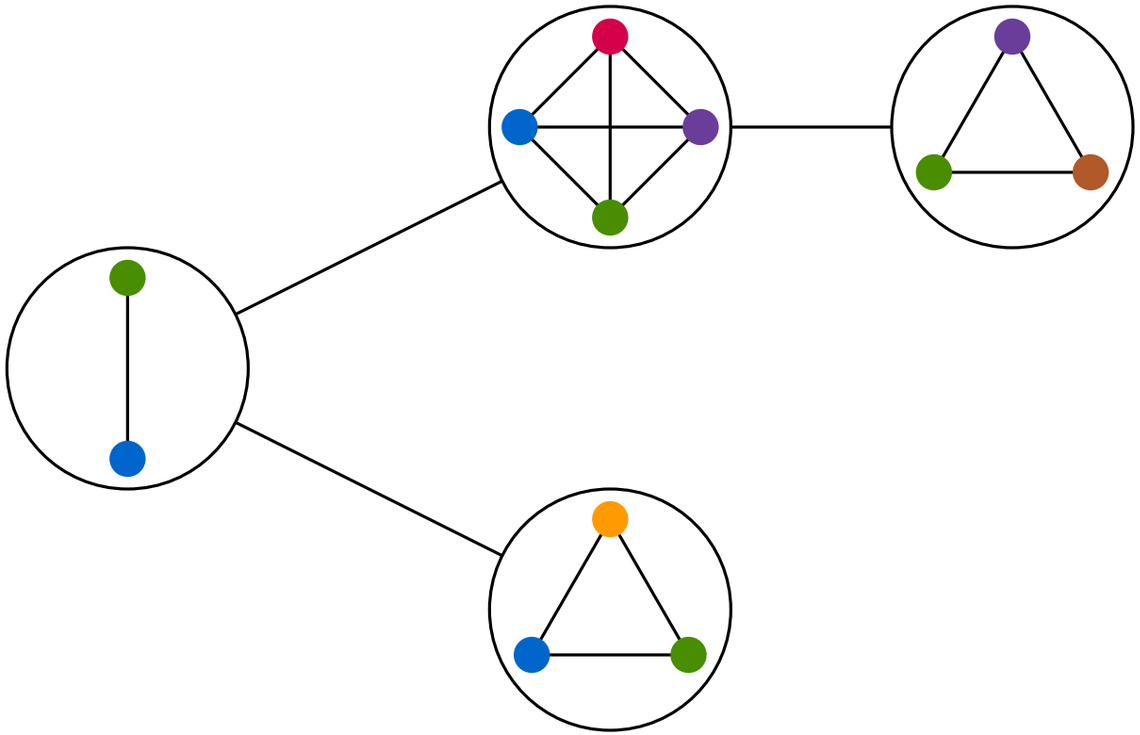

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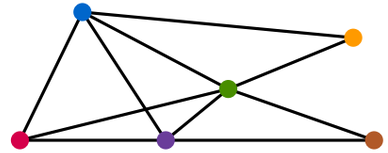

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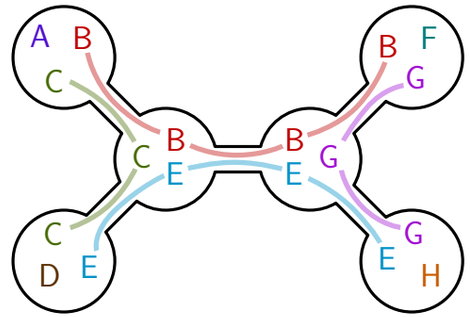

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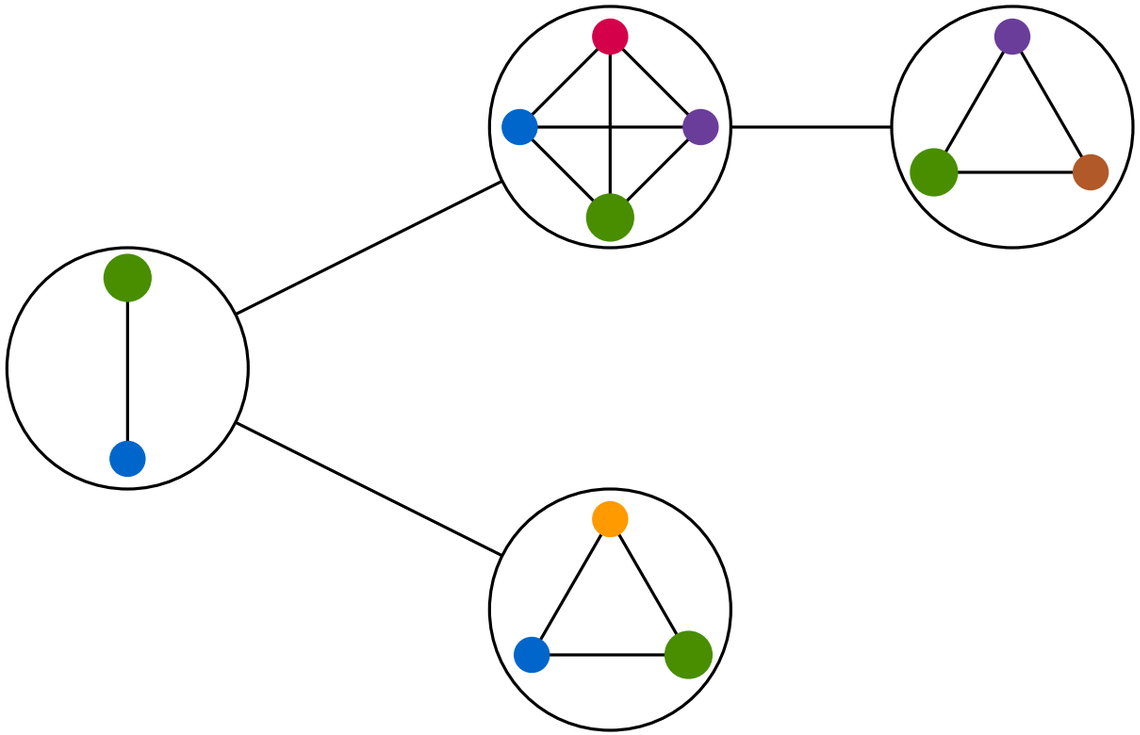

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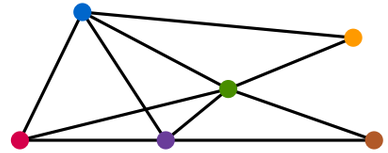

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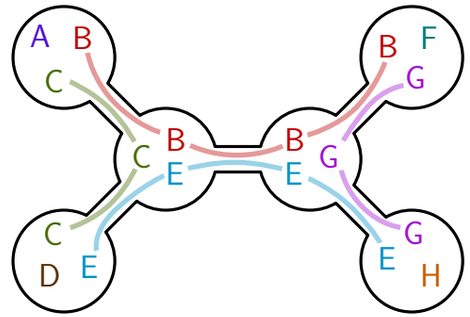

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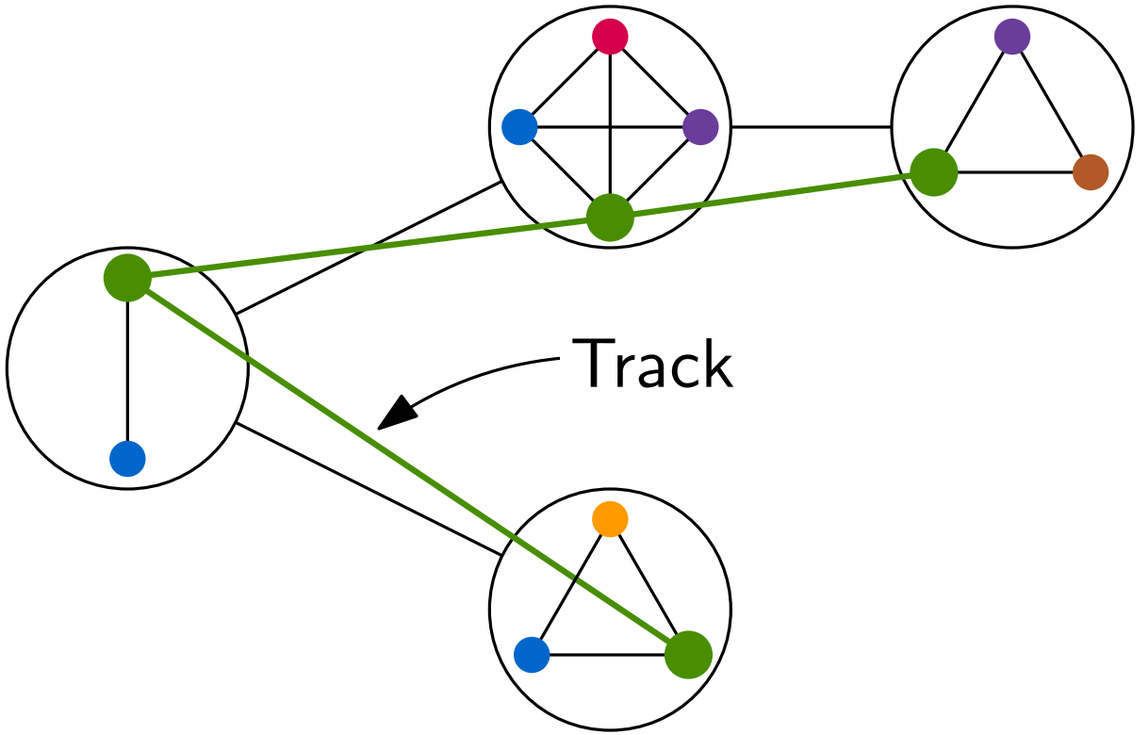

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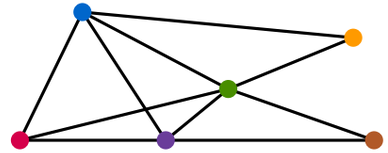

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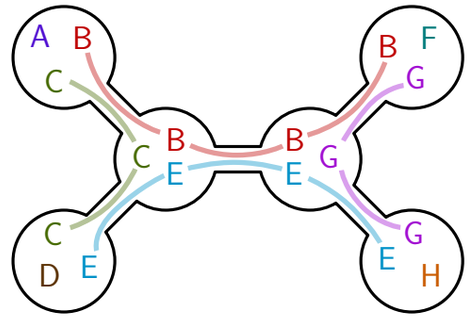

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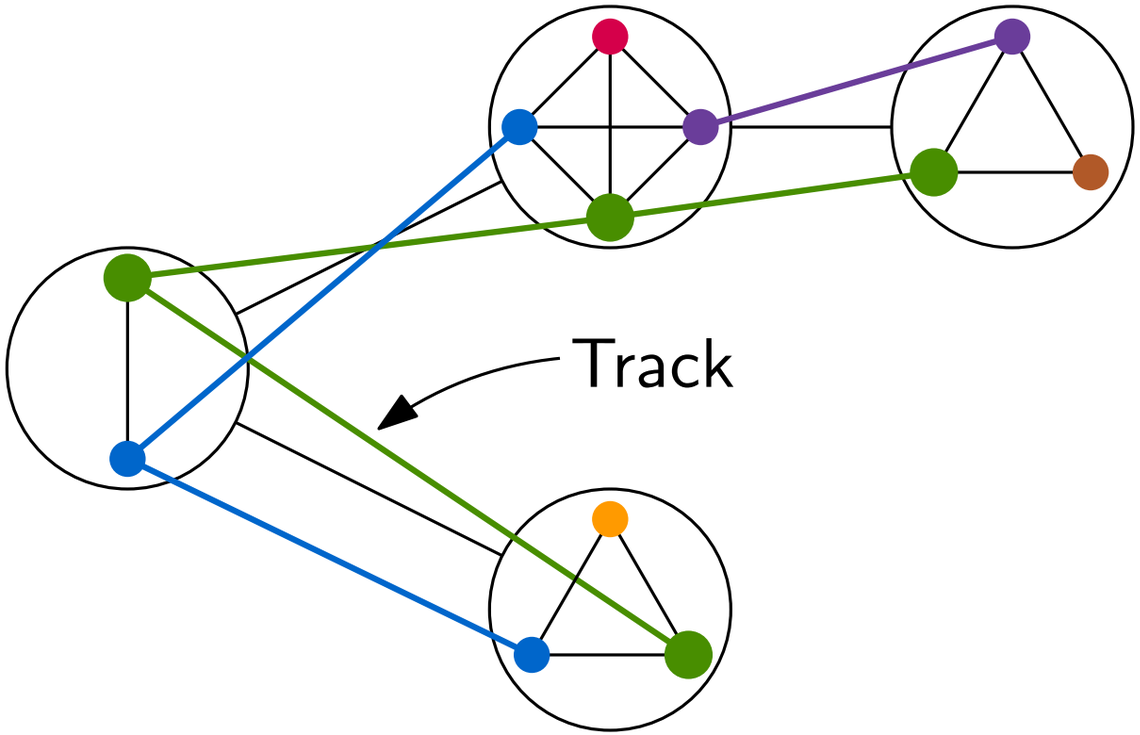

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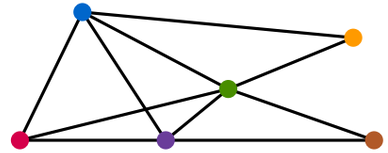

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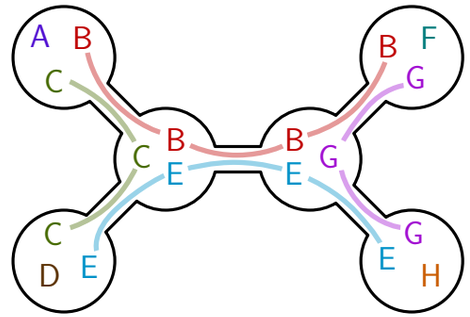

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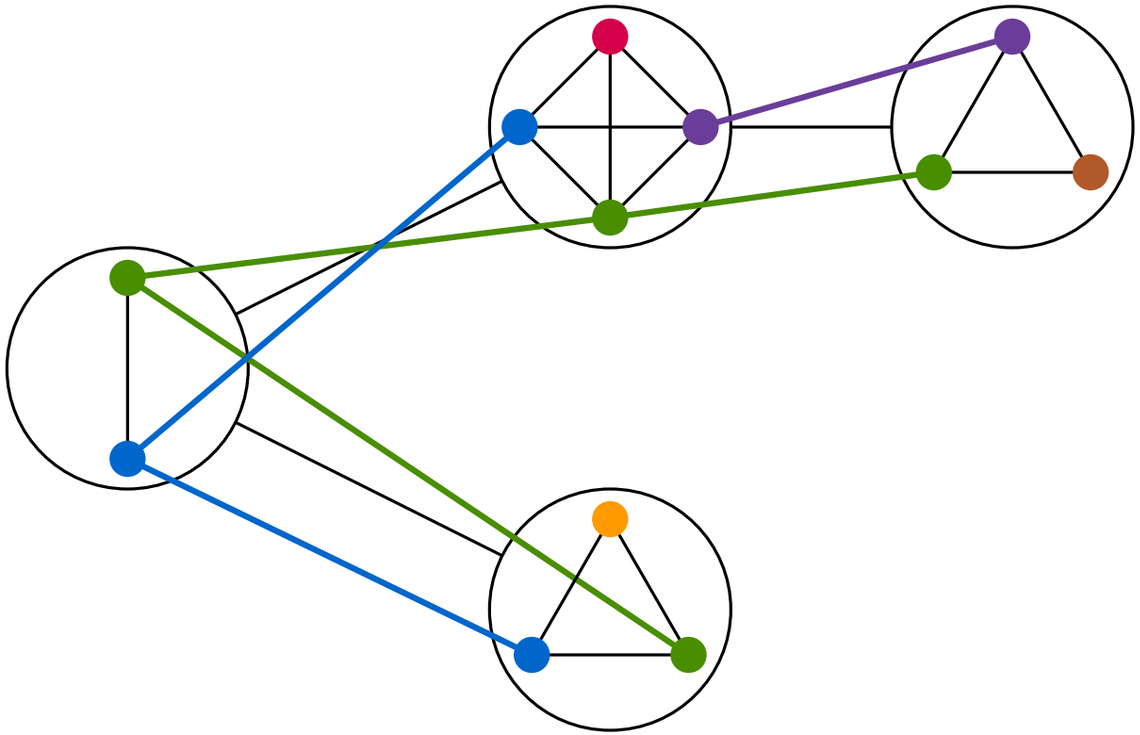

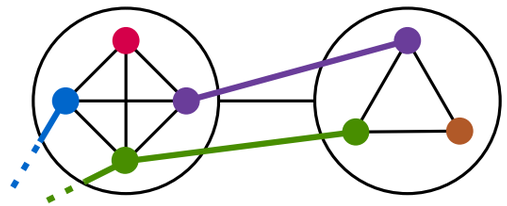

Circular

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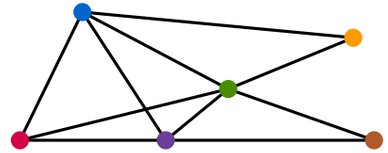

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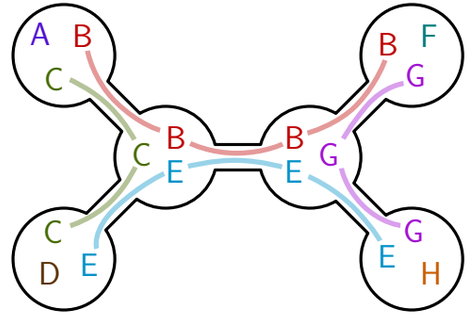

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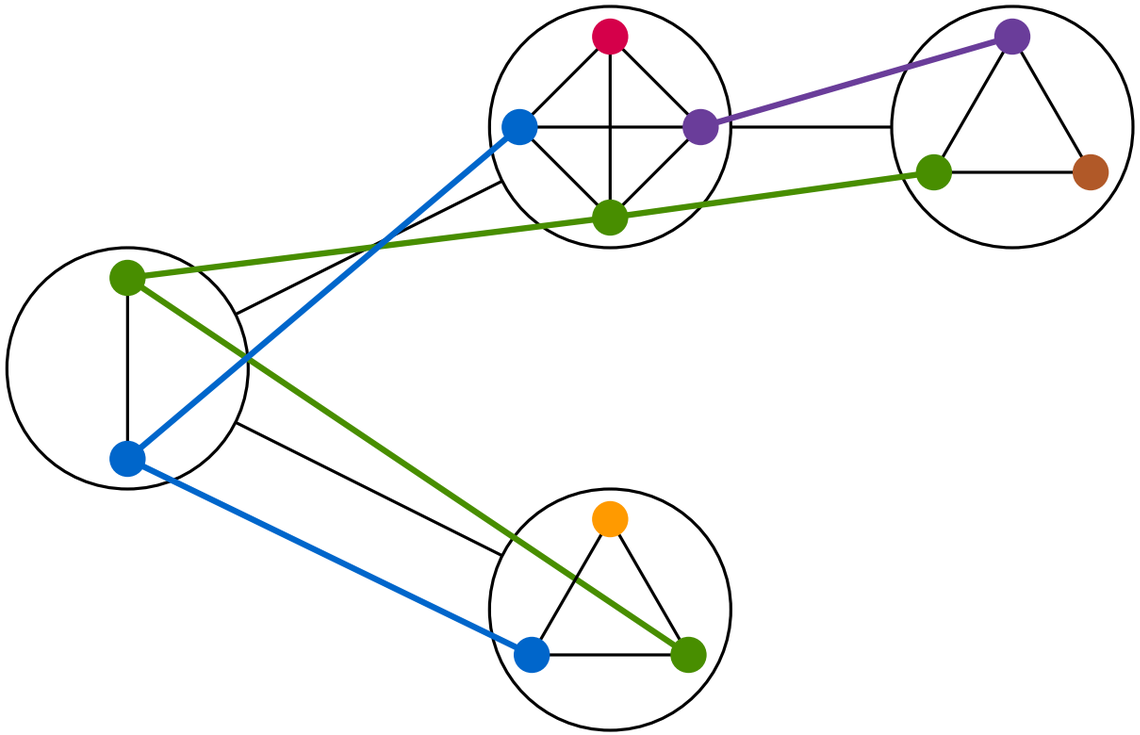
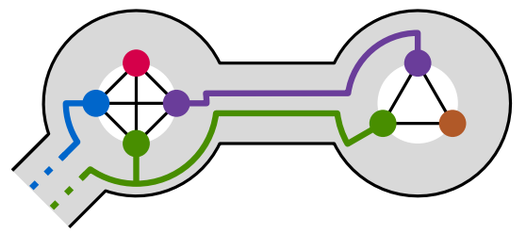
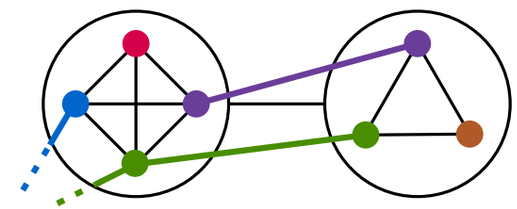




Orbital



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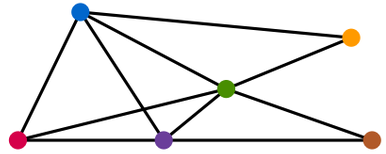

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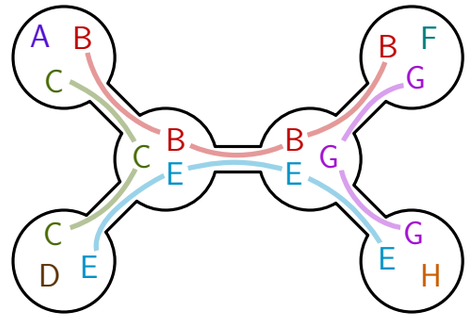

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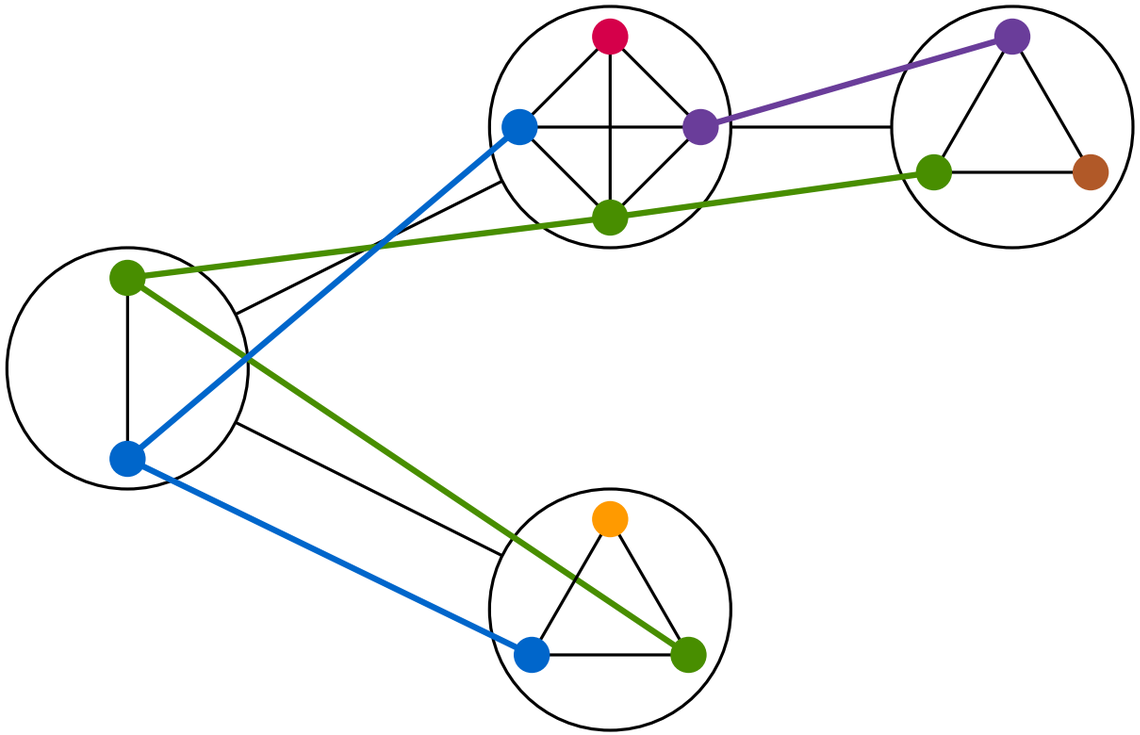
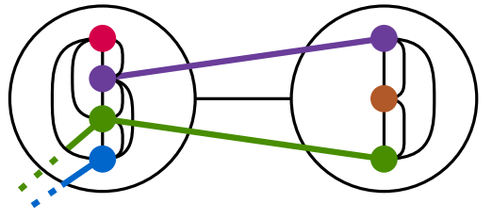
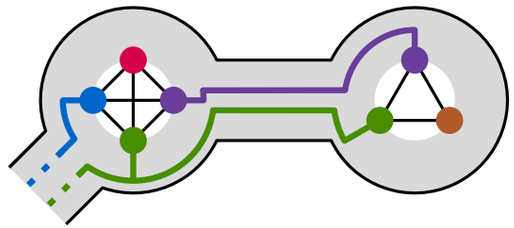
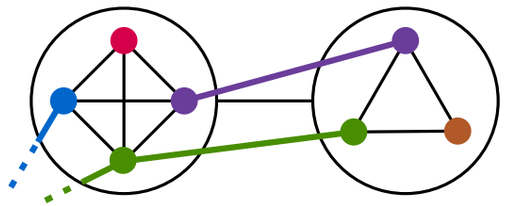




Linear

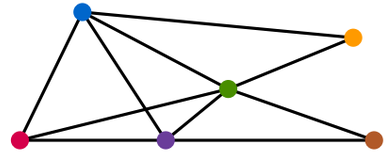


Orbital



Circular

Witness Drawing Styles



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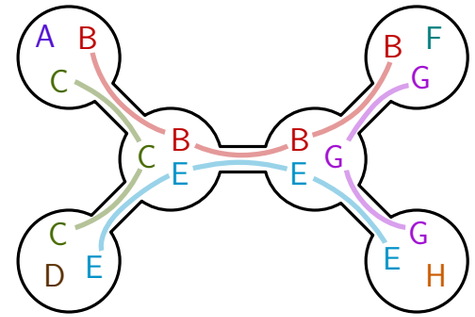

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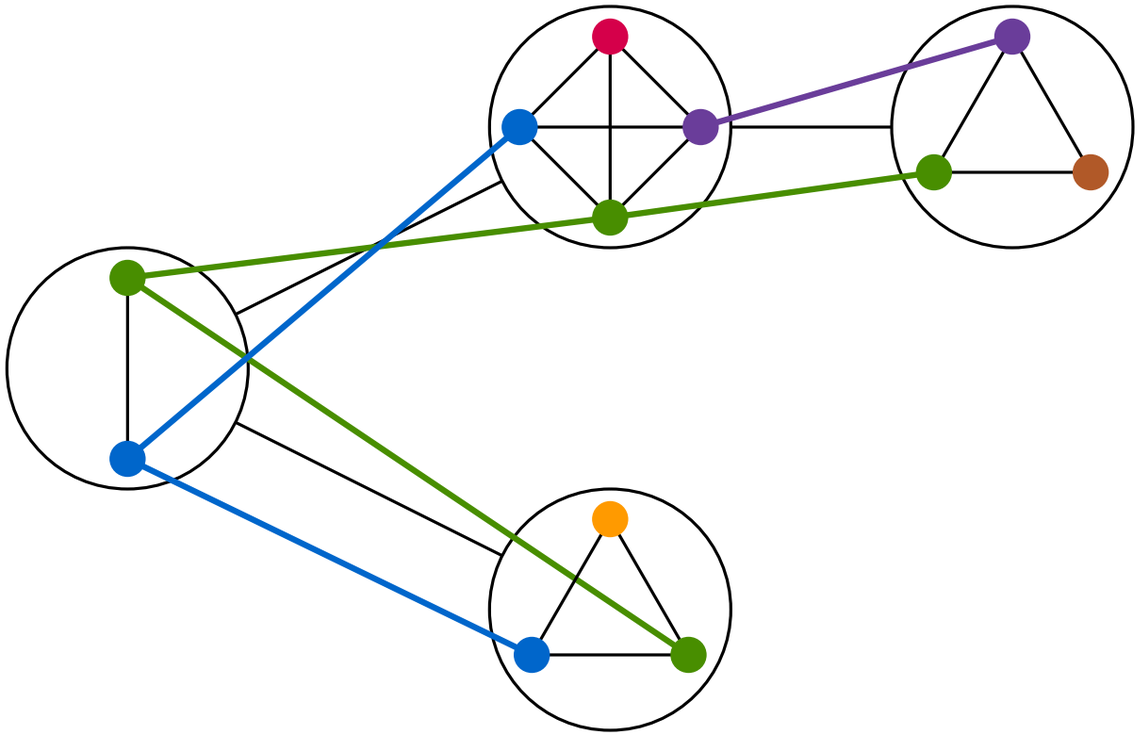

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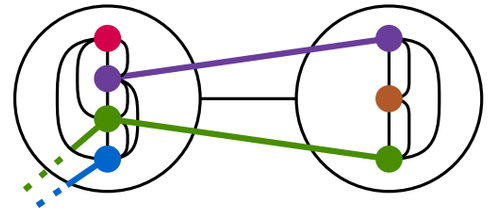

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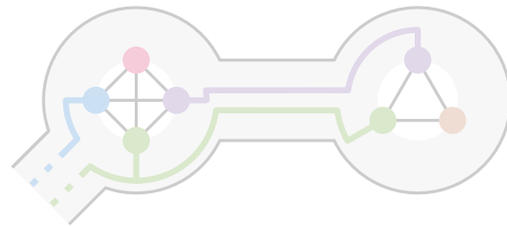

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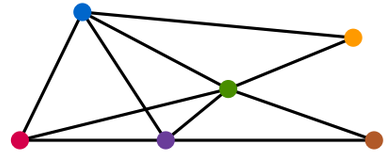

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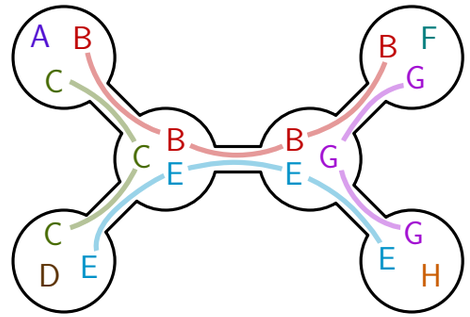

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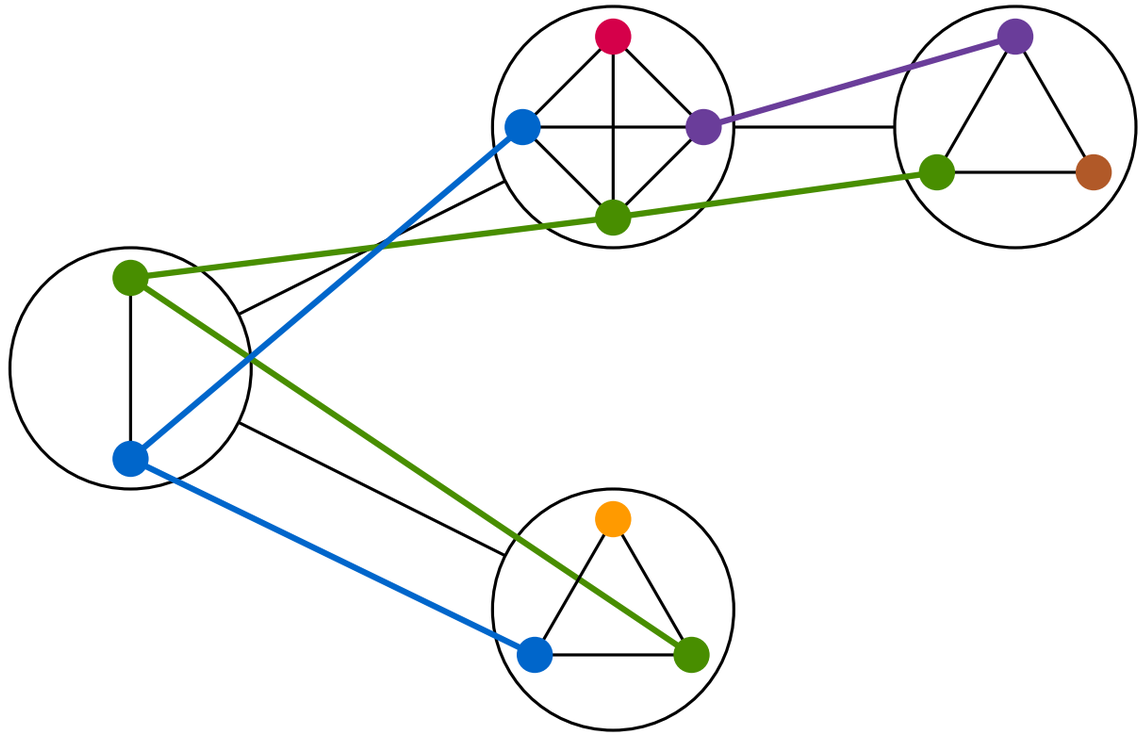

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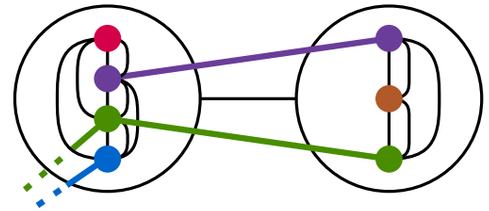

This talk:

Crossing minimization in linear drawings

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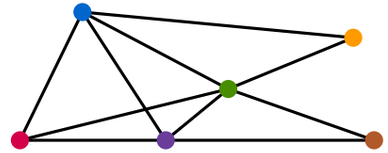

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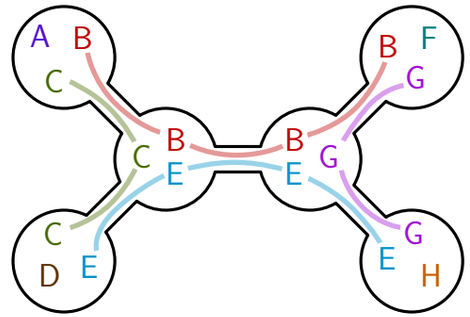

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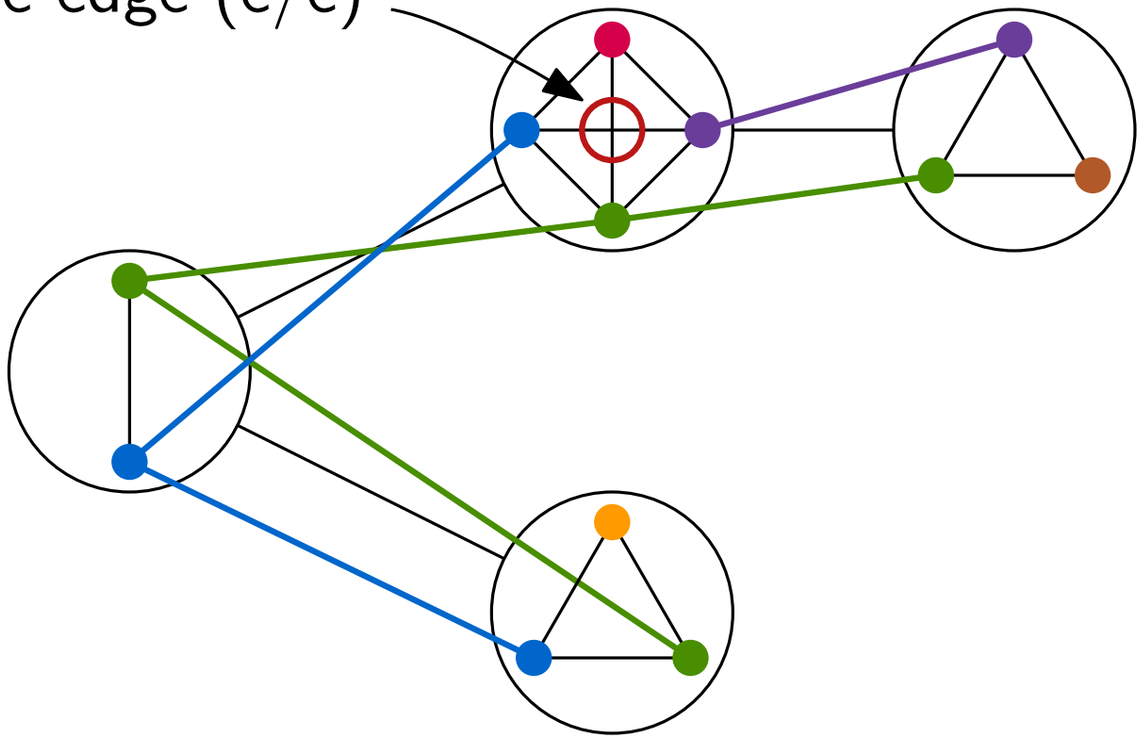

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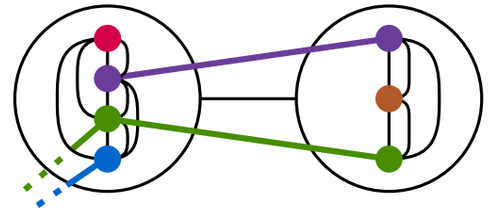

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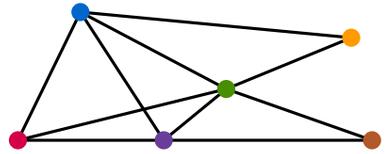

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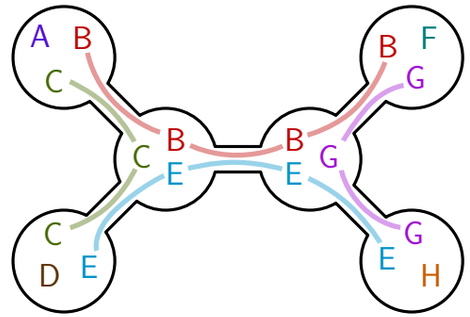

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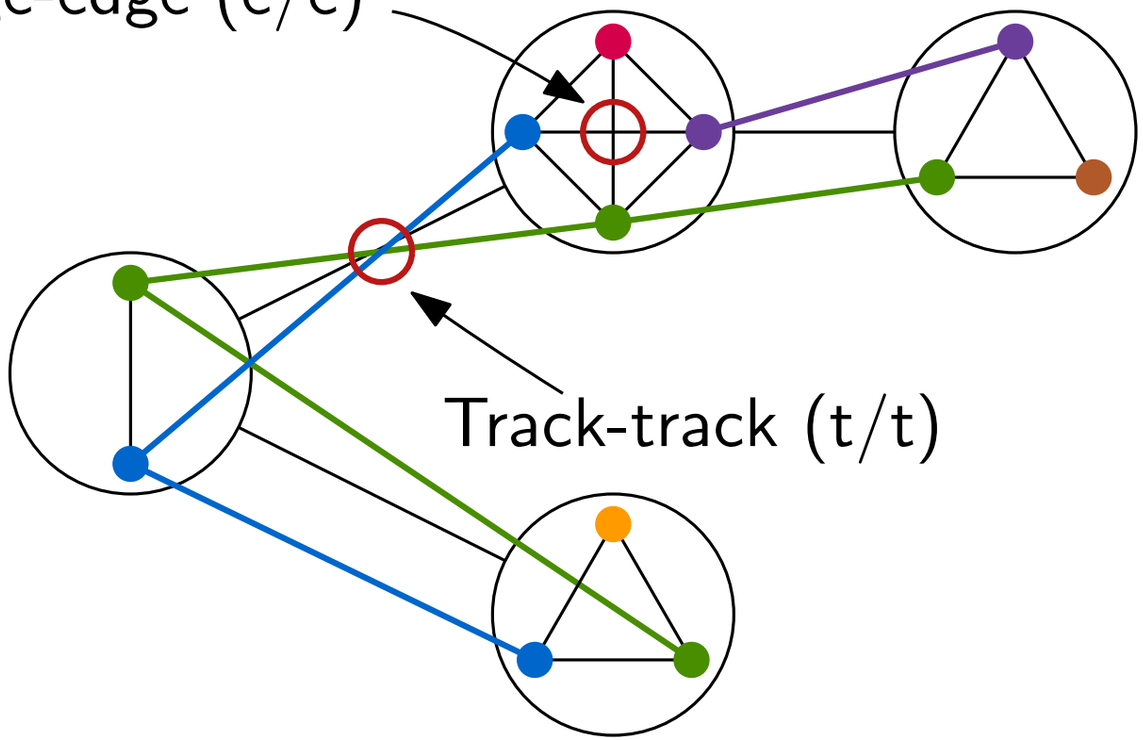

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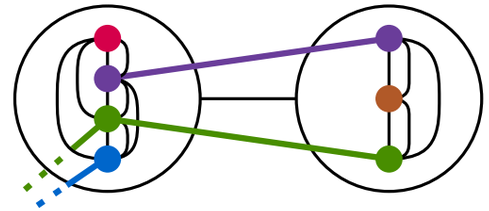

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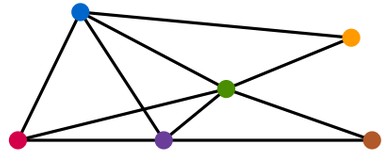

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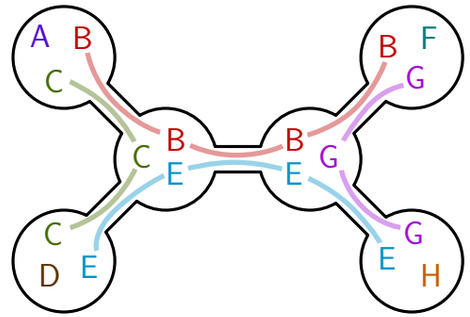

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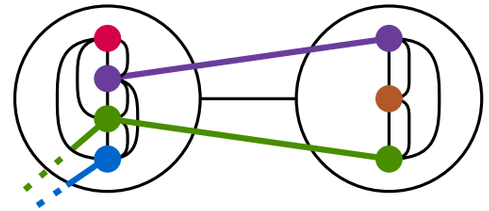
Track-track (t/t)

Track-edge (t/e)



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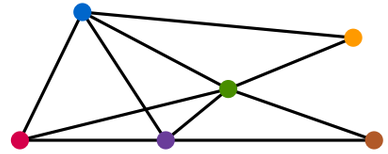

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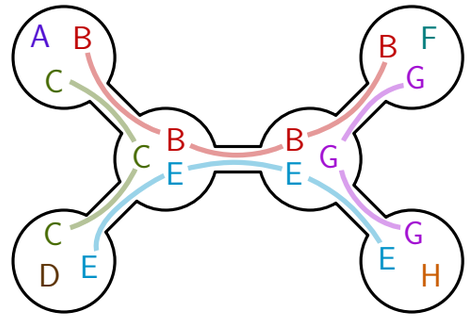

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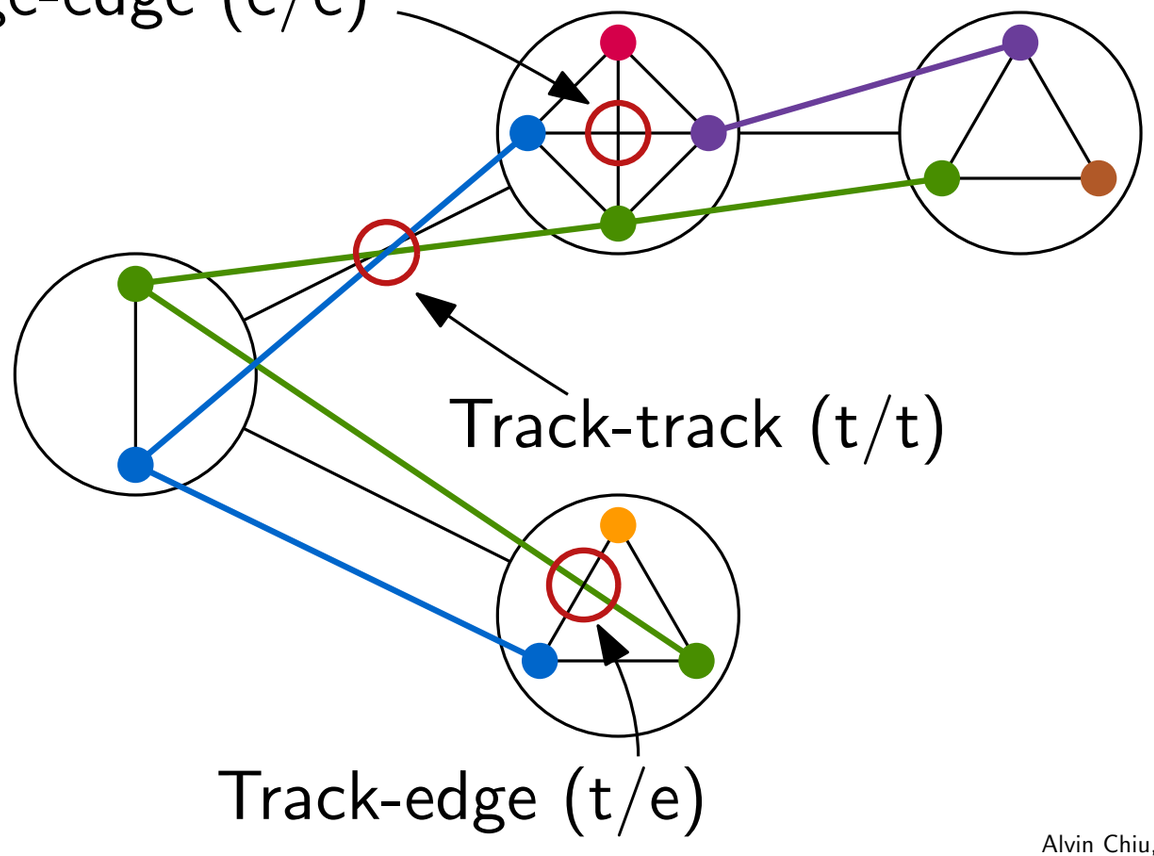

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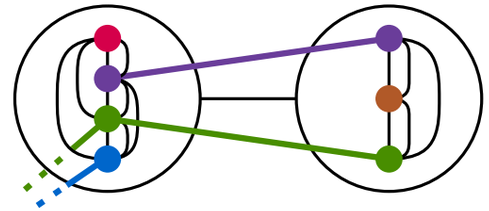


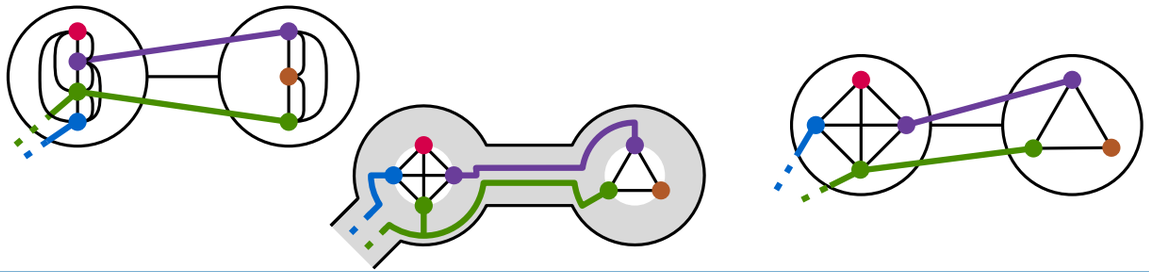
This talk:
Crossing minimization in linear drawings

Assumptions:
$\mathcal{O}(|V|)$ bags
Tree has maximum degree 3

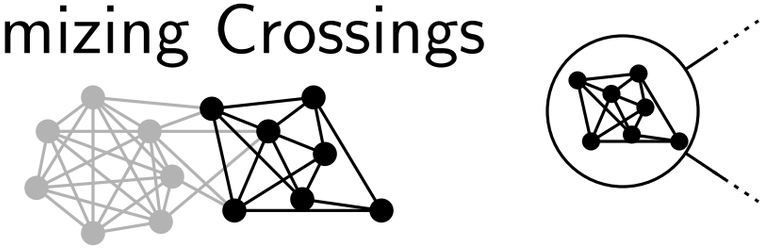
[Bodlaender & Kloks, J.Alg.'96]

Our Contributions

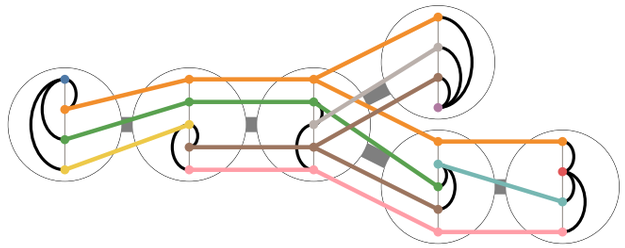
Drawing Paradigms for Treewidth



Complexity and Algorithms for Minimizing Crossings



Experimental Evaluation



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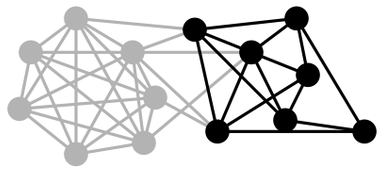

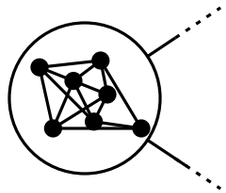

Crossing Minimization for Linear Drawings

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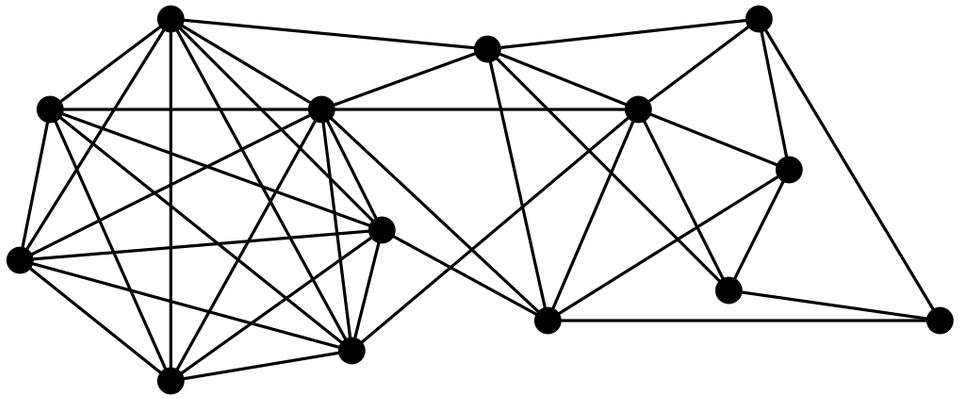

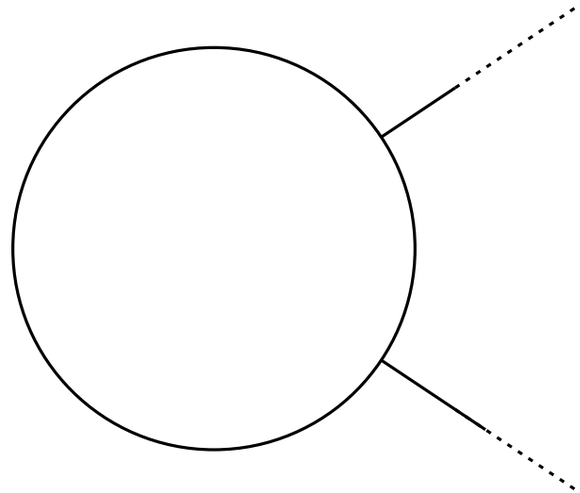

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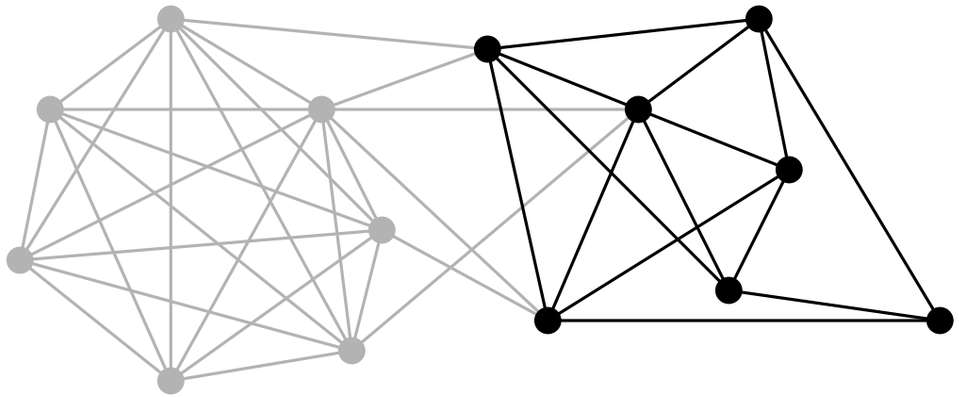

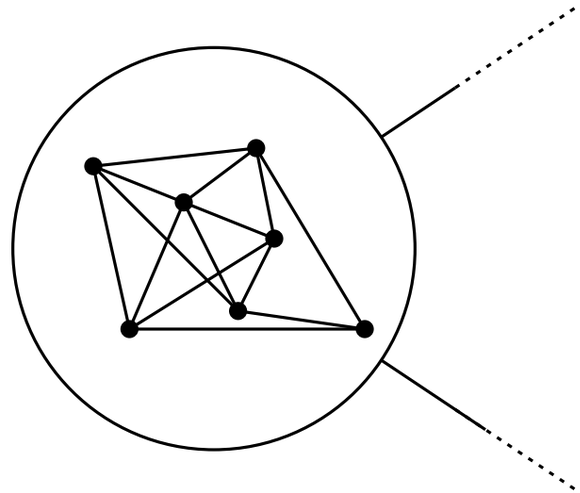

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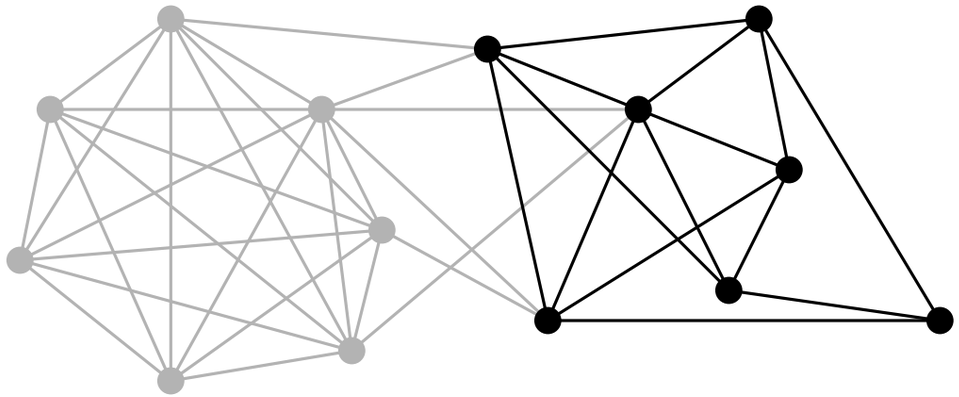
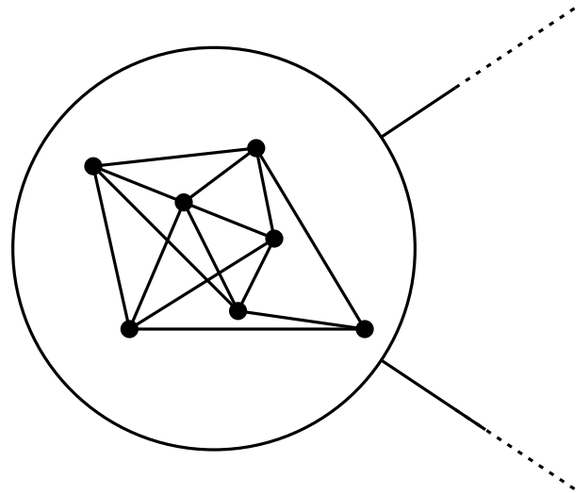
[Masuda et al., Symp. Circuits and Systems'87]





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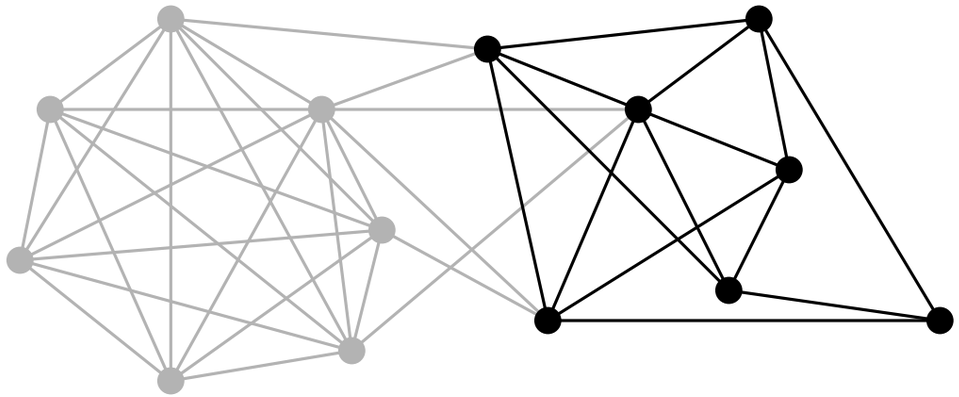

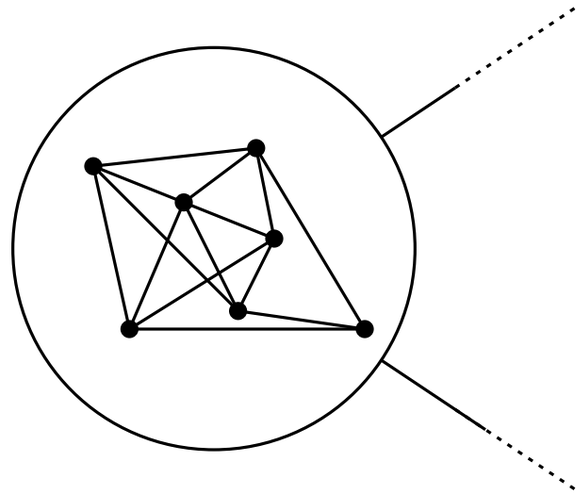

Theorem:
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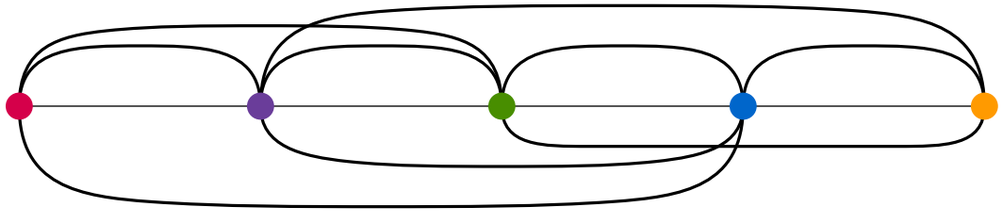
$f(\omega)$ exponential in $\omega \Rightarrow$ Heuristics!

Heuristics for Linear Witness Drawings

Building Block: Book drawing heuristic `conGreedy+` [Klawitter, Mchedlidze, & Nöllenburg, GD'17]



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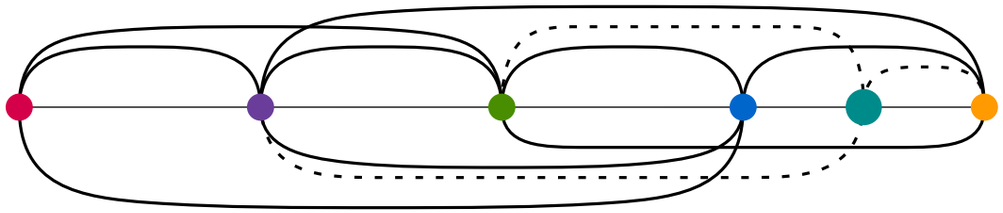

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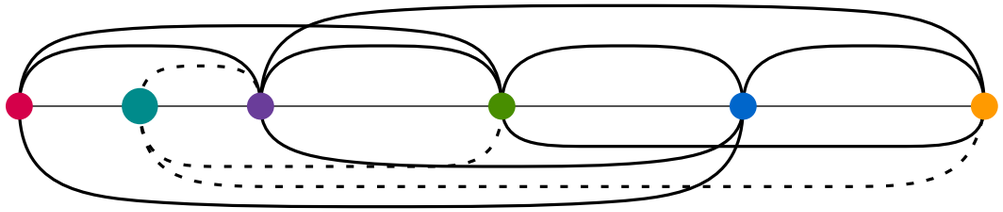

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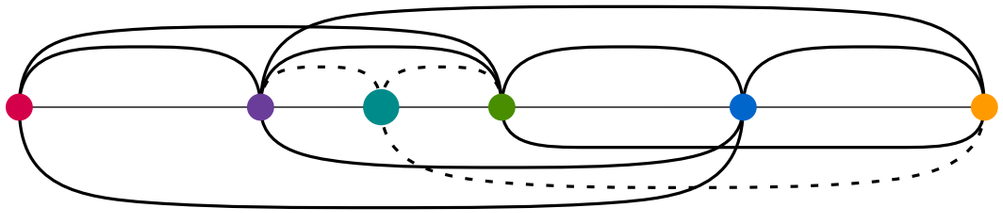

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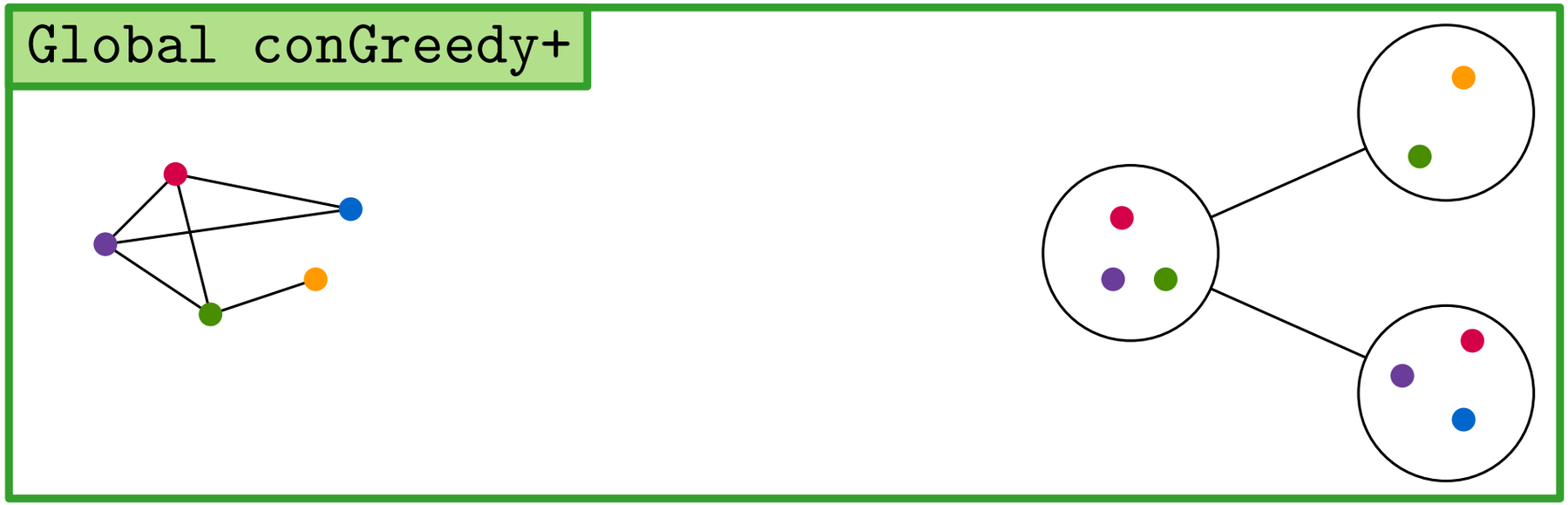

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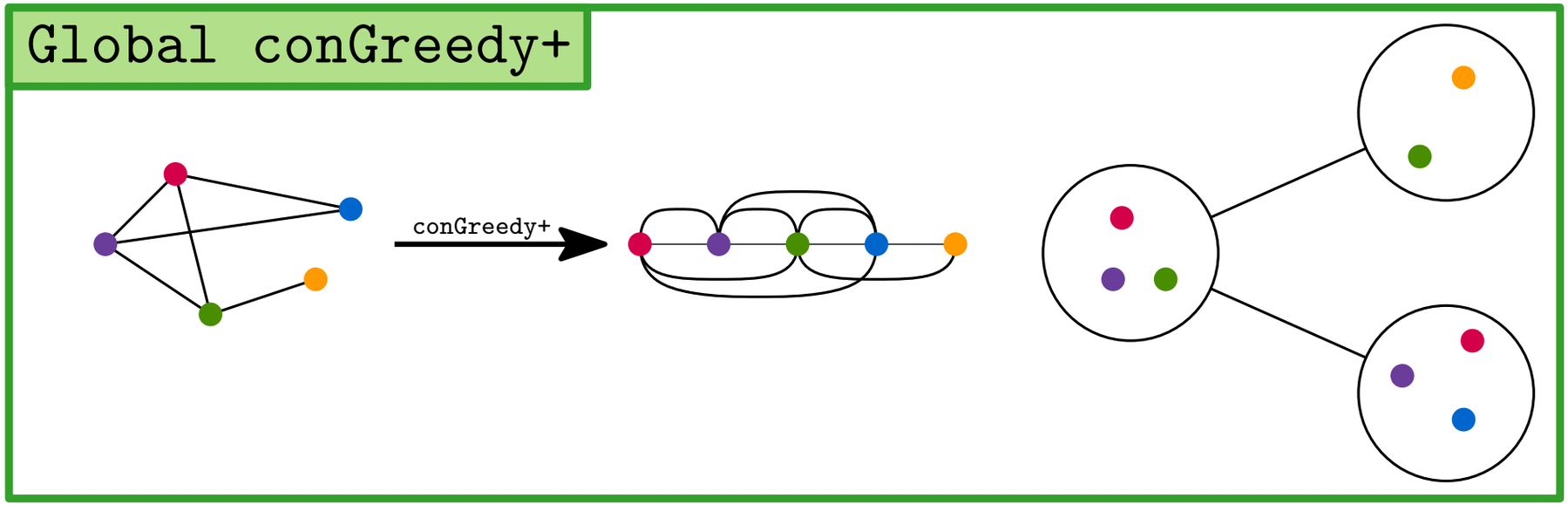

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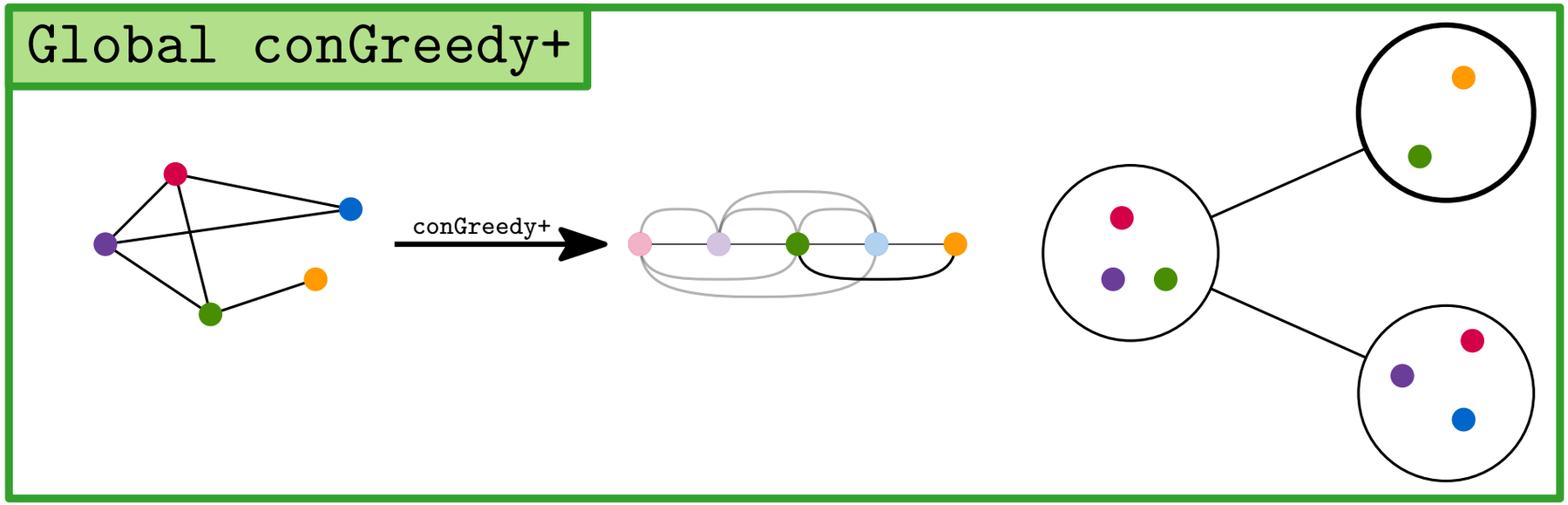

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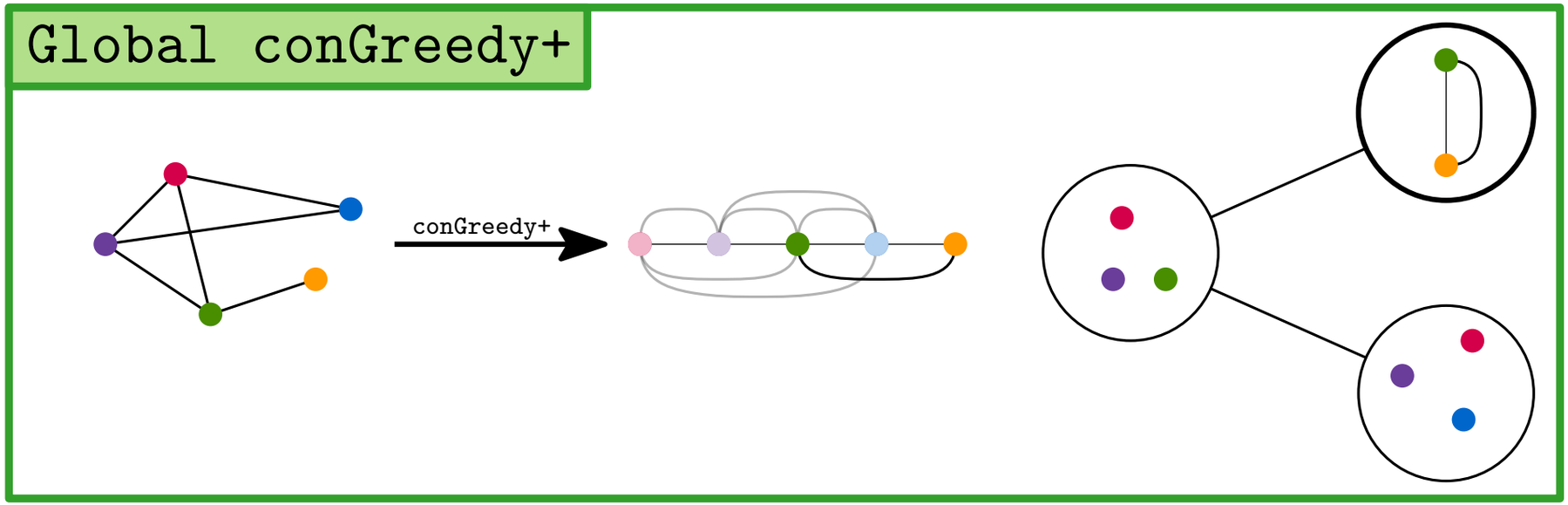

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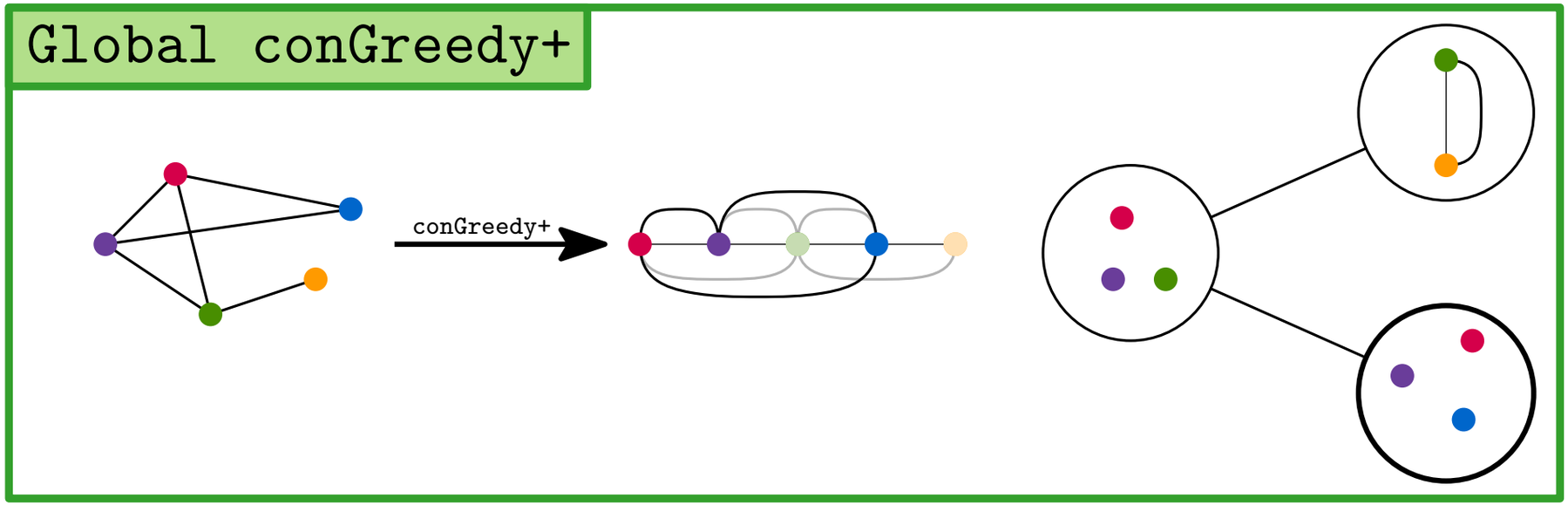

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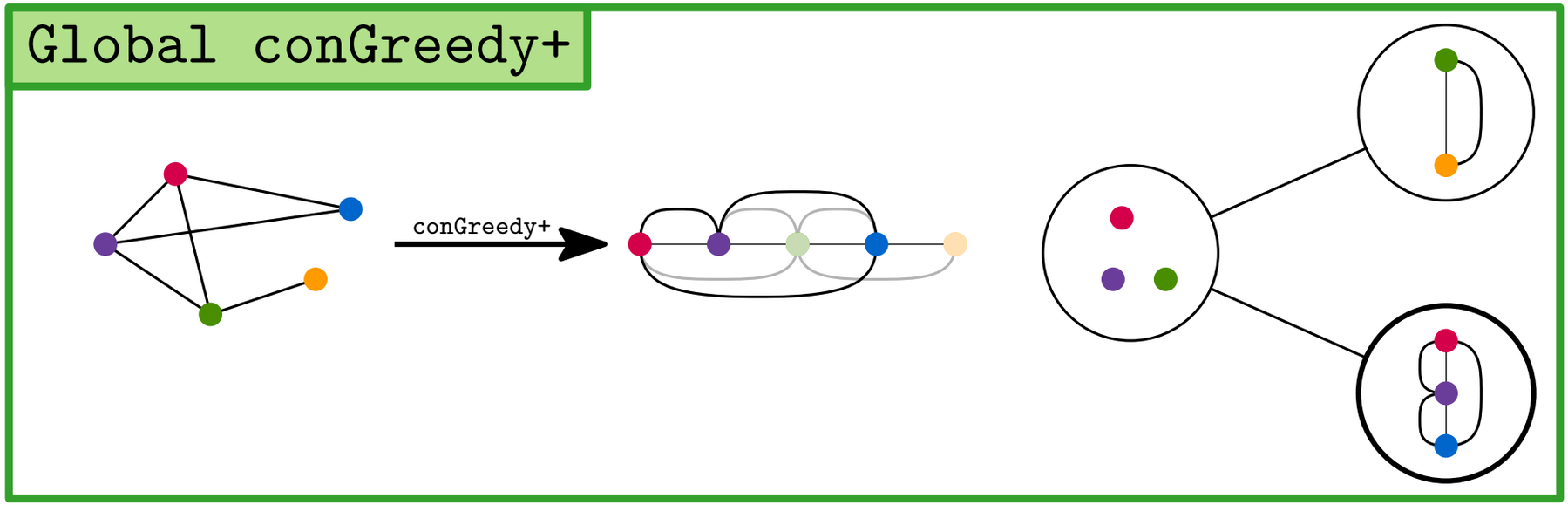

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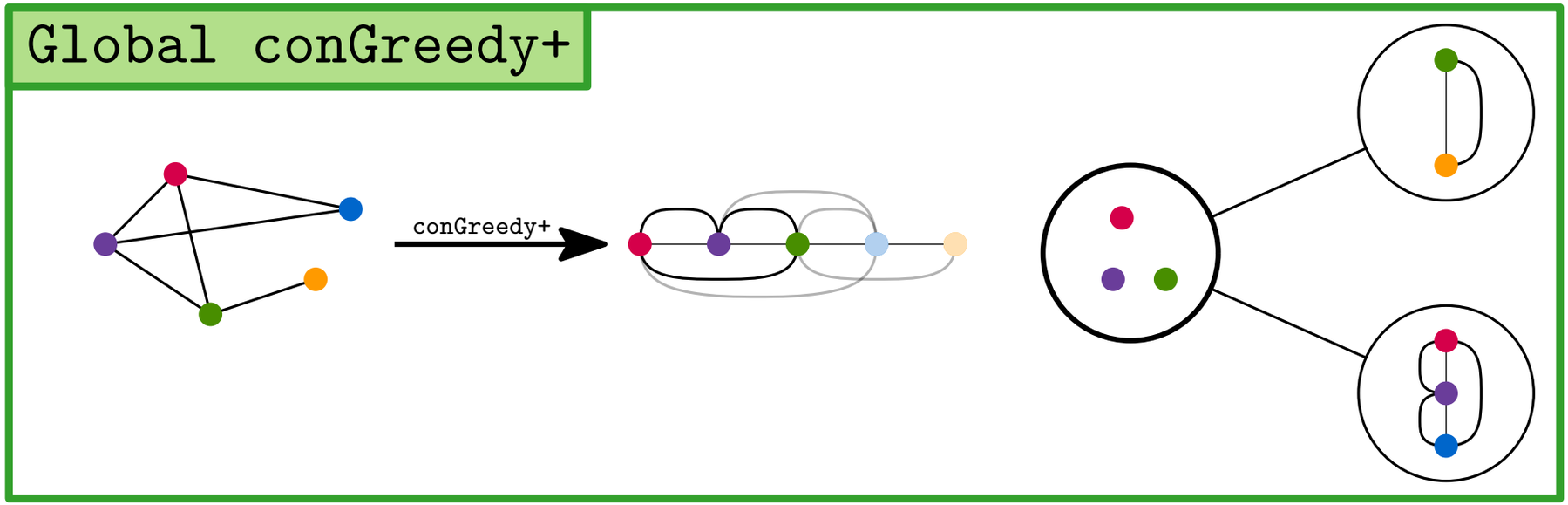

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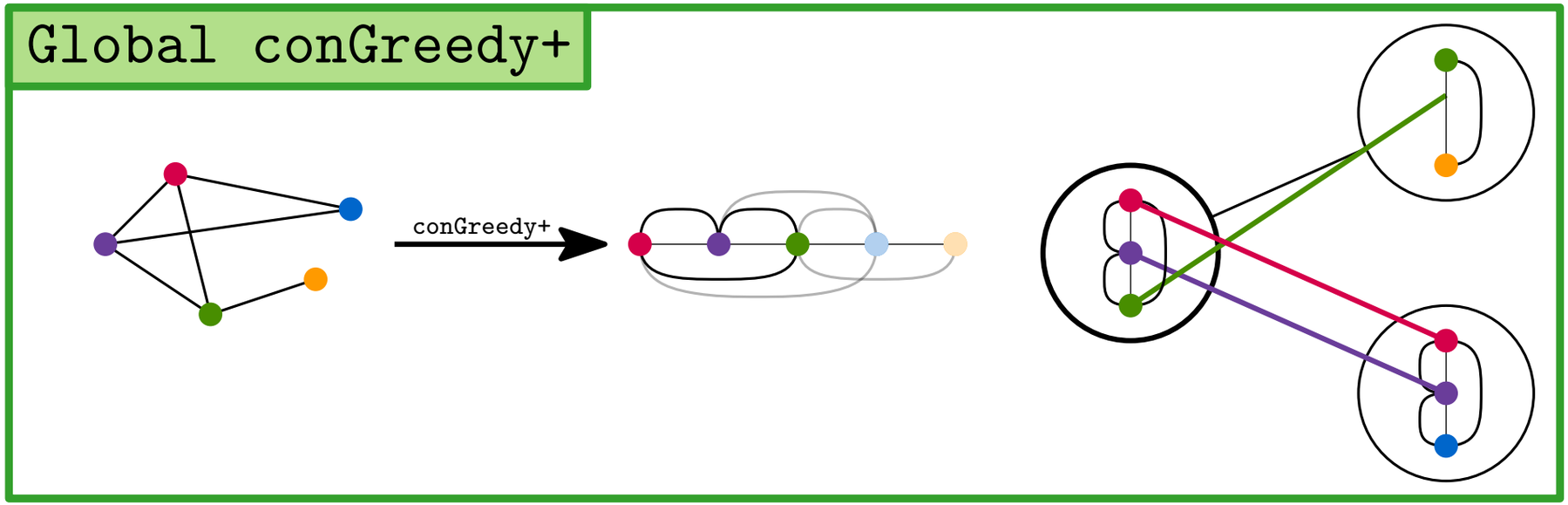

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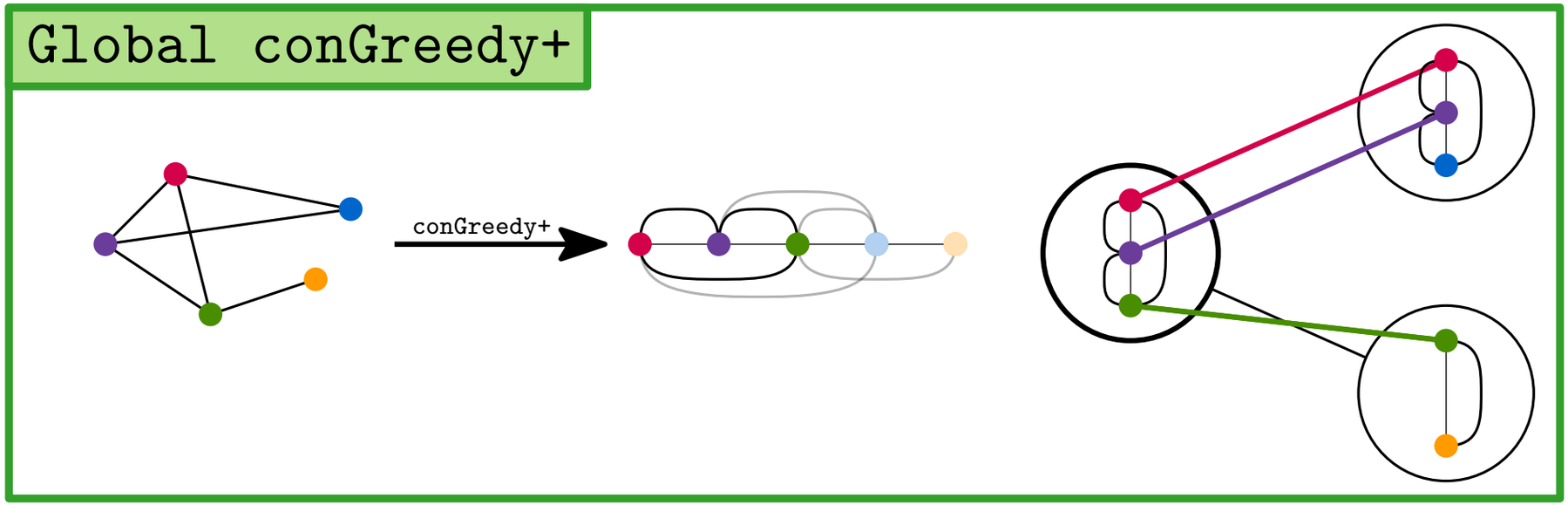
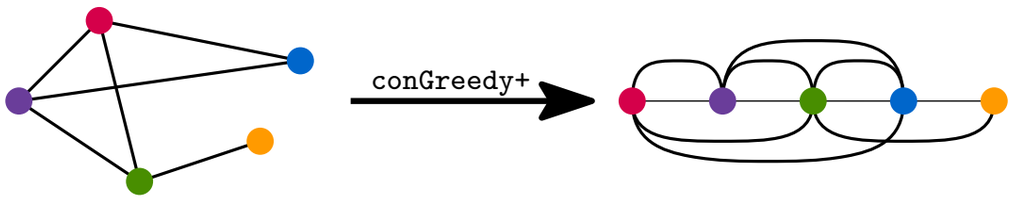
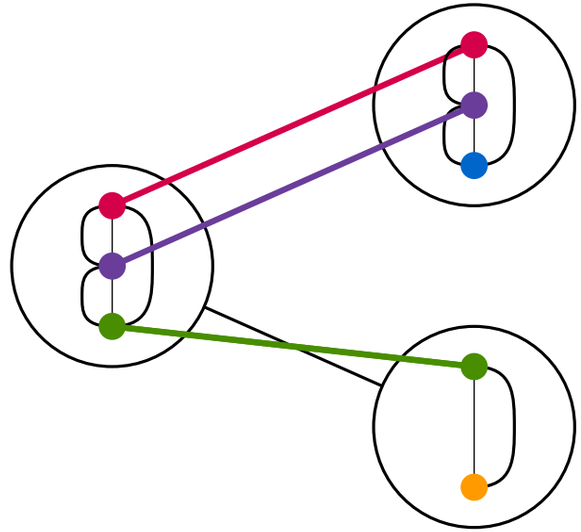


Heuristics for Linear Witness Drawings

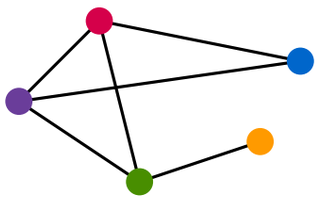
Building Block: Book drawing heuristic conGreedy+ [Klawitter, Mchedlidze, & Nöllenburg, GD'17]

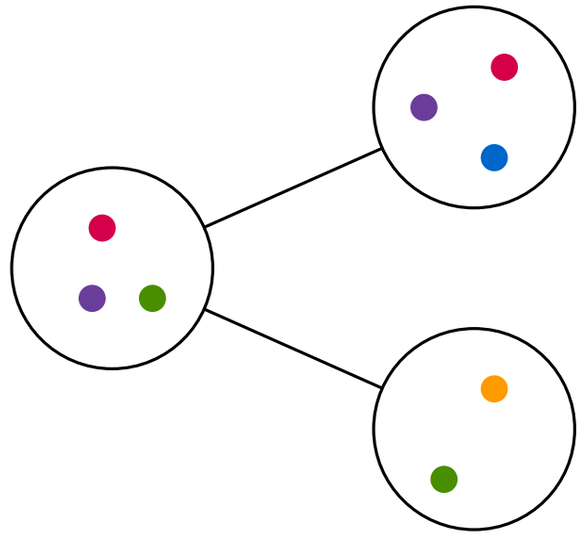
Global conGreedy+





Local conGreedy+





Heuristics for Linear Witness Drawings

Building Block: Book drawing heuristic conGreedy+

[Klawitter, Mchedlidze, & Nöllenburg, GD'17]

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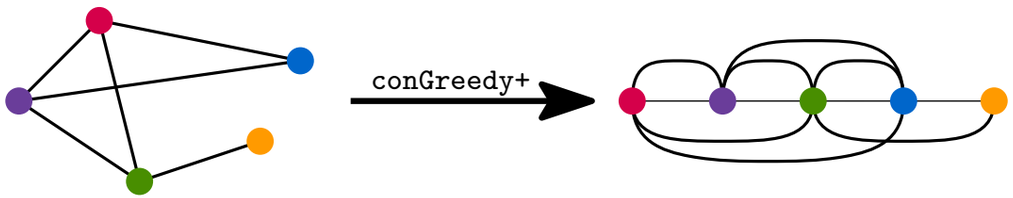

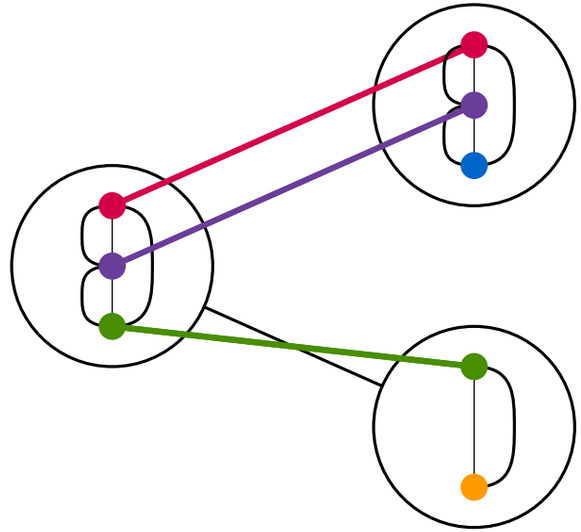

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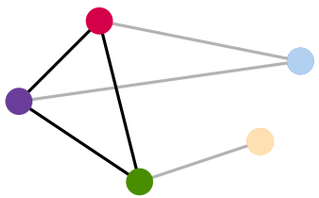
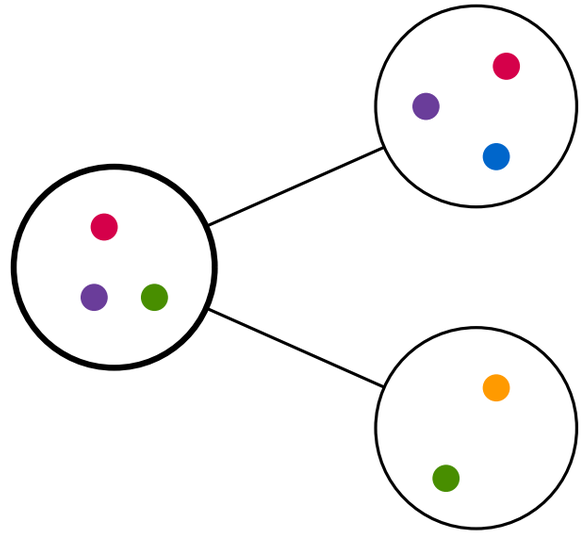
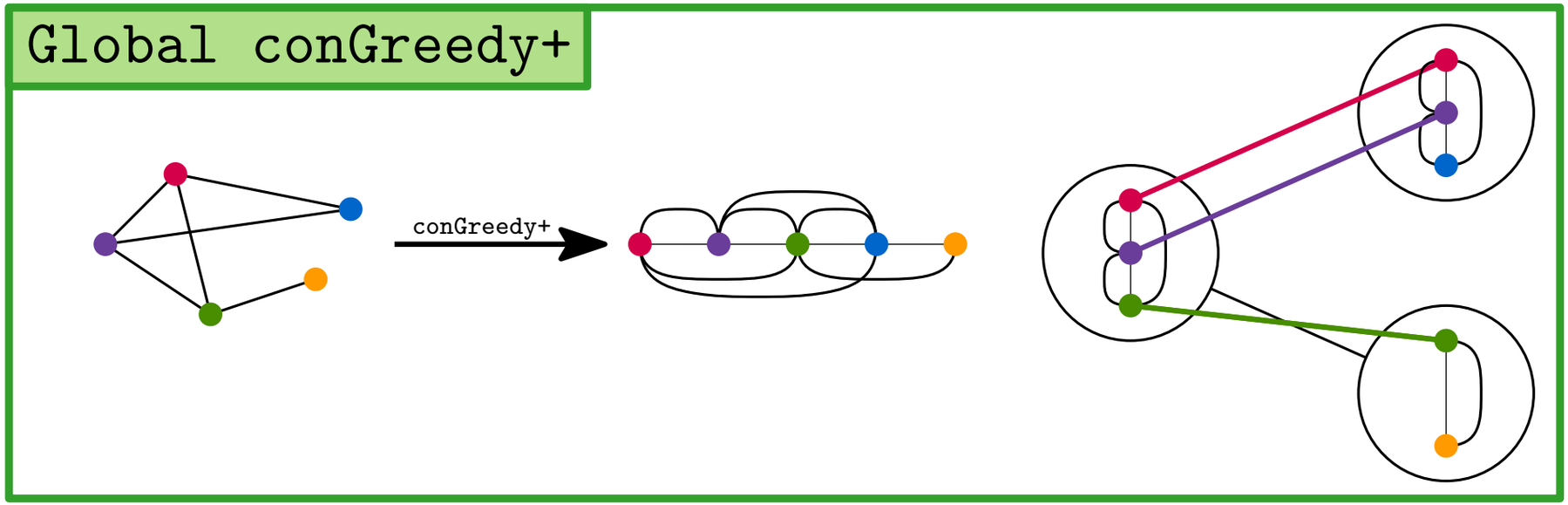
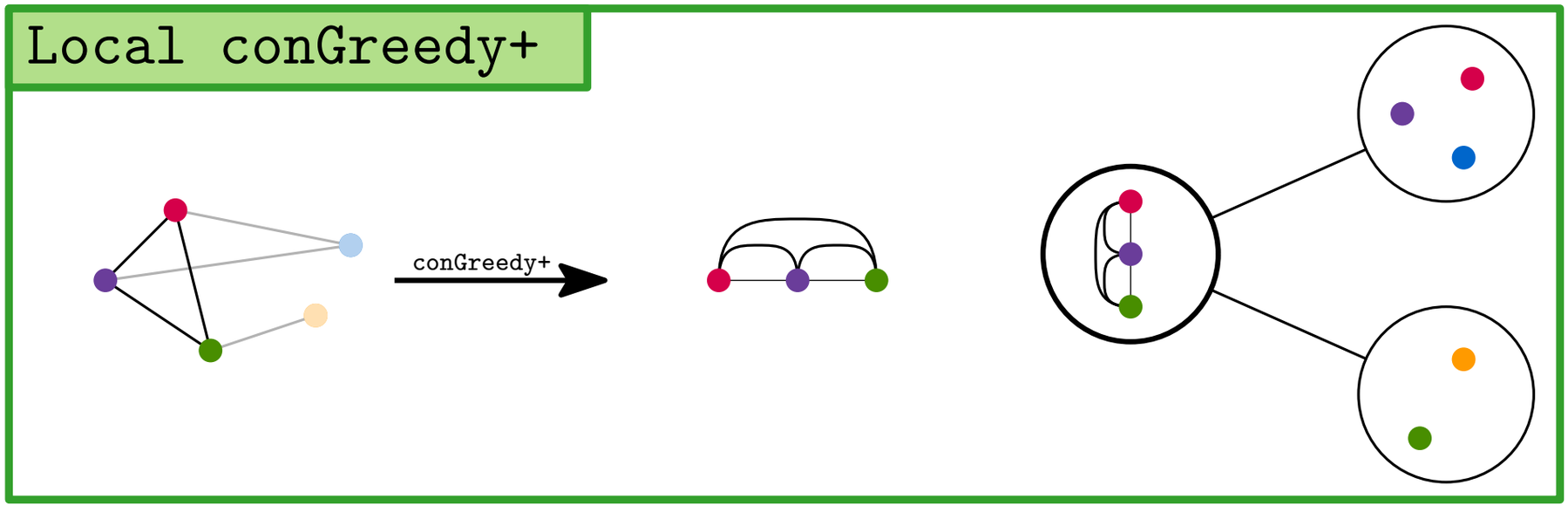




Heuristics for Linear Witness Drawings

Building Block: Book drawing heuristic `conGreedy+` [Klawitter, Mchedlidze, & Nöllenburg, GD'17]



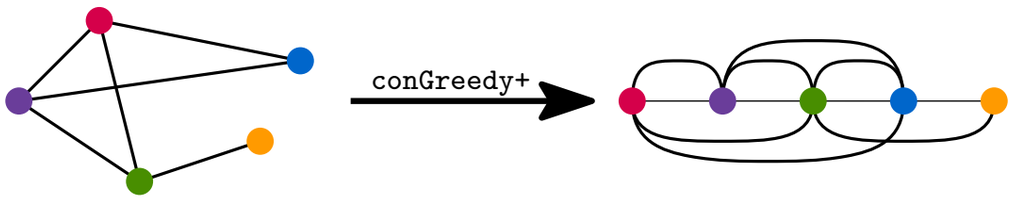


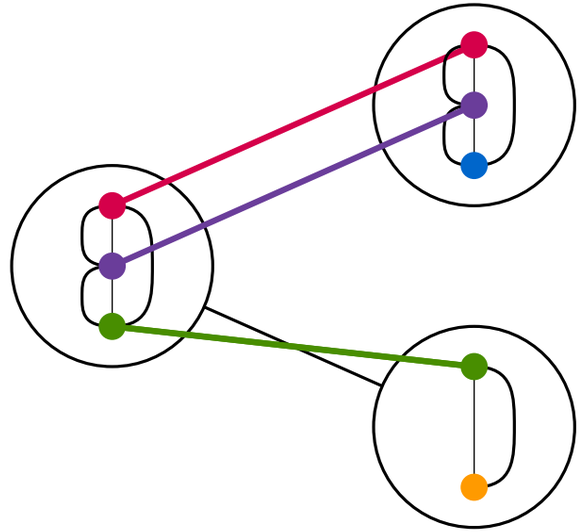
Heuristics for Linear Witness Drawings

Building Block: Book drawing heuristic conGreedy+

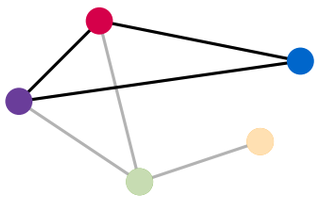
[Klawitter, Mchedlidze, & Nöllenburg, GD'17]

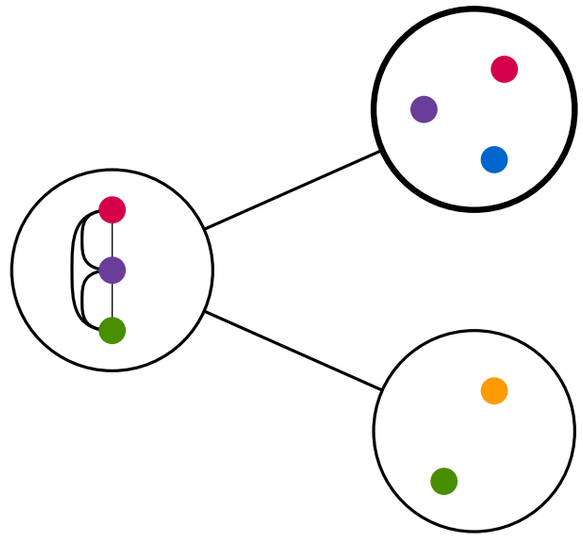
Global conGreedy+





Local conGreedy+



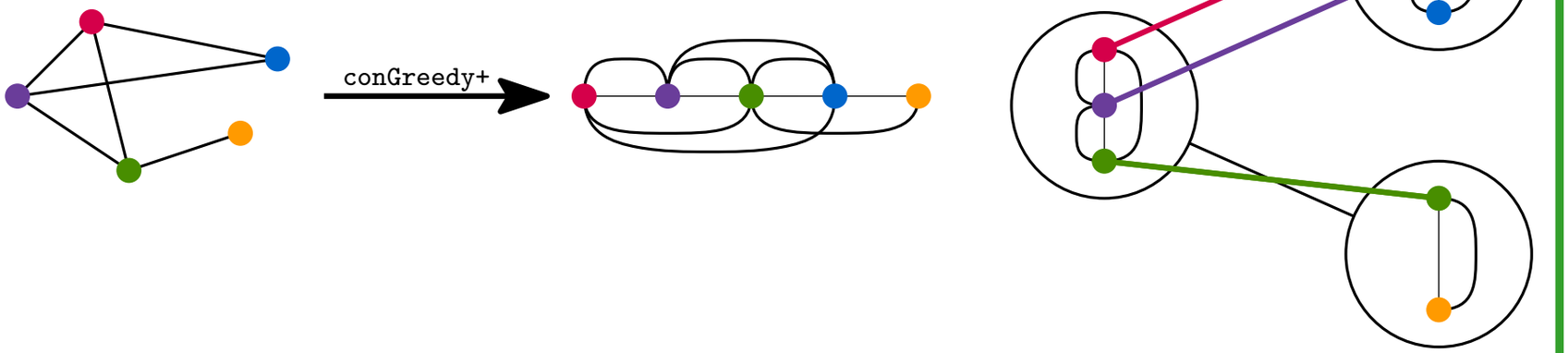


Heuristics for Linear Witness Drawings

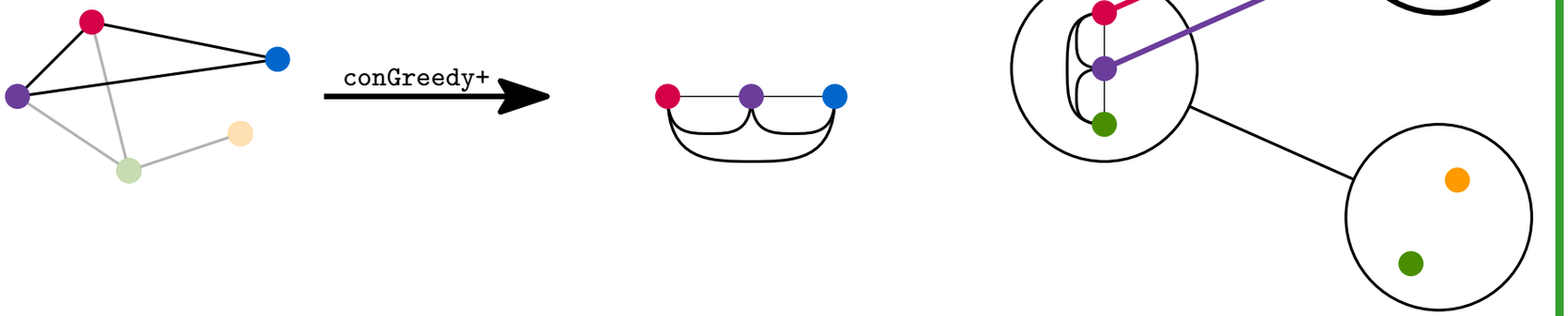
Building Block: Book drawing heuristic `conGreedy+`

[Klawitter, Mchedlidze, & Nöllenburg, GD'17]

`Global conGreedy+`



`Local conGreedy+`



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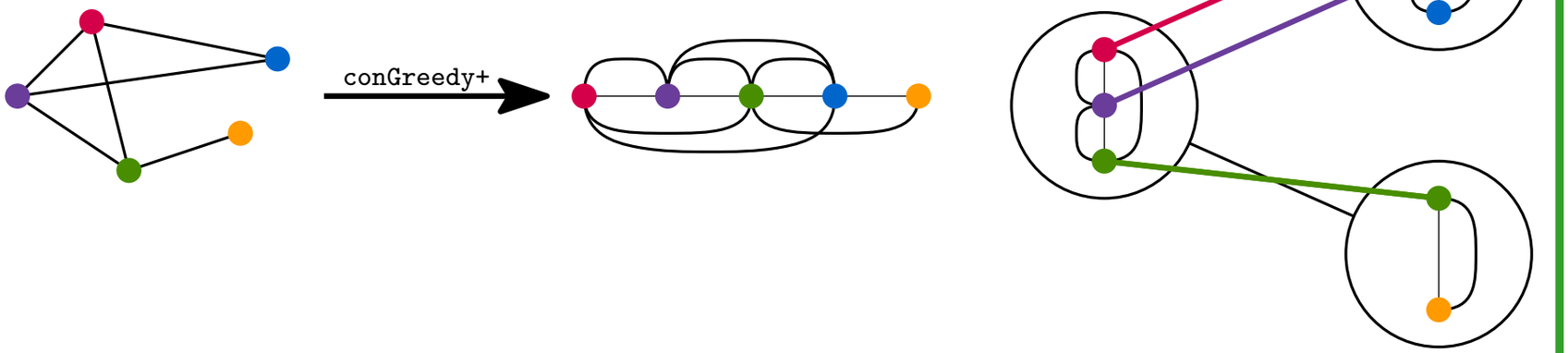

Local conGreedy+

Heuristics for Linear Witness Drawings

Building Block: Book drawing heuristic conGreedy+

[Klawitter, Mchedlidze, & Nöllenburg, GD'17]

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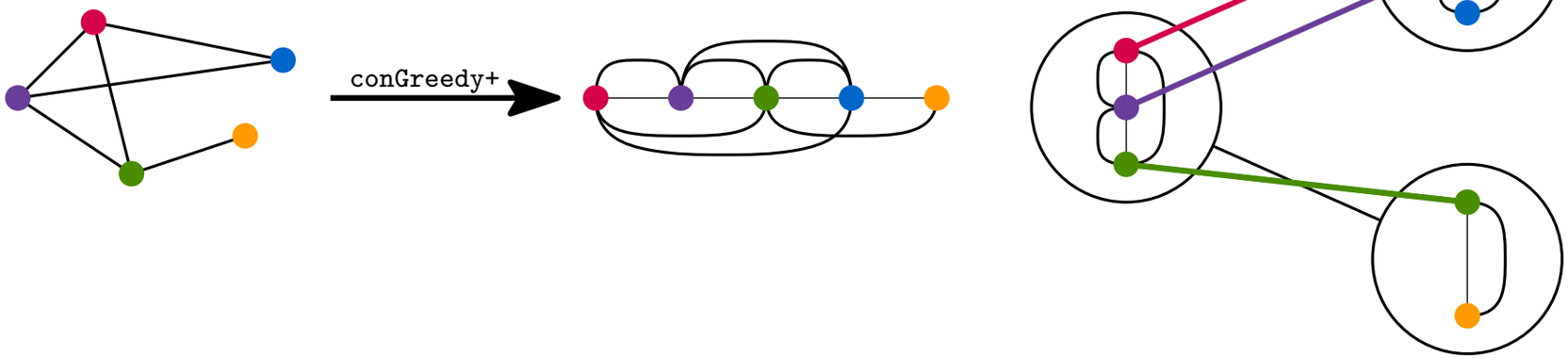
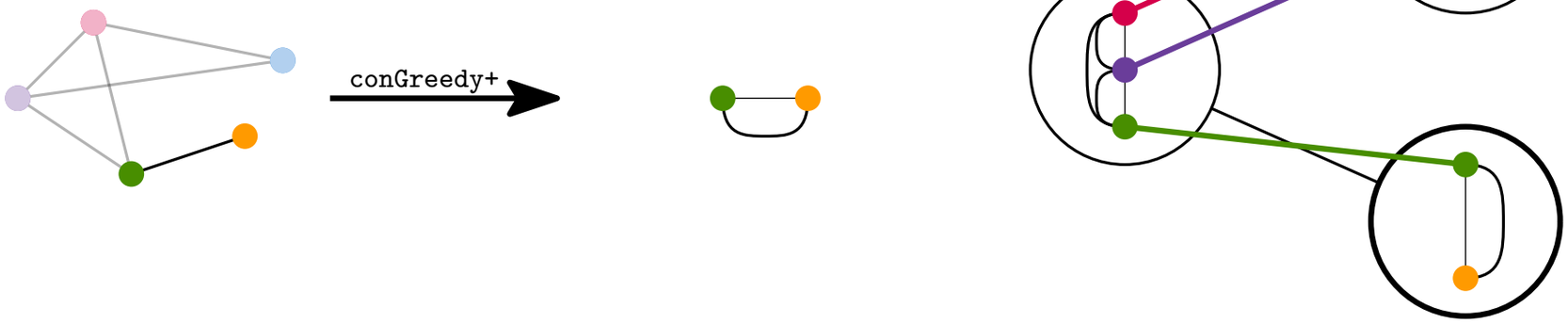


Local conGreedy+



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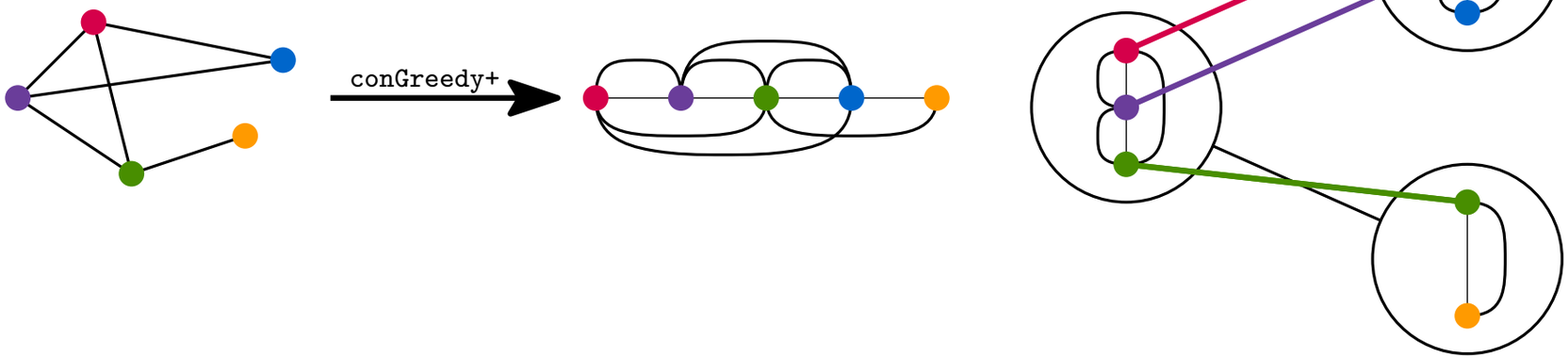


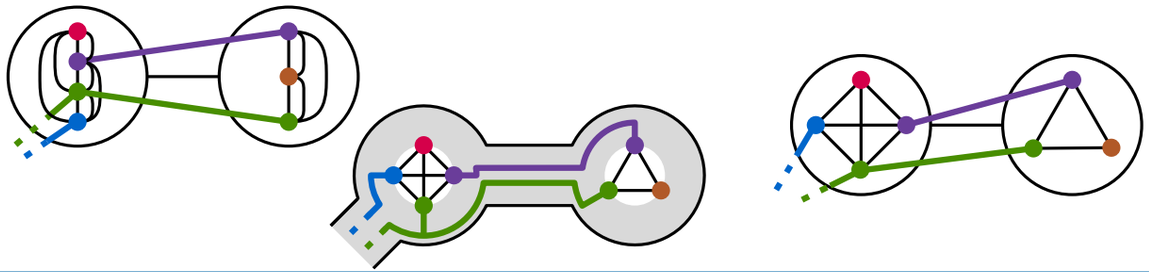
Local conGreedy+

Local Search

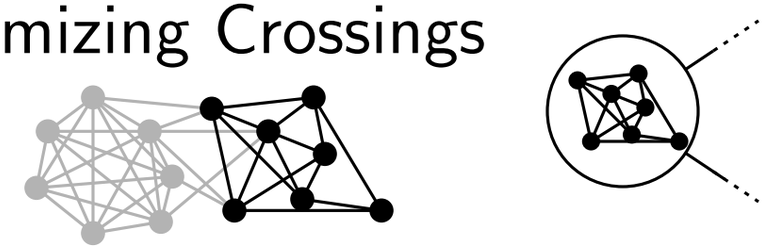
- Vertex-Swap
- Edge-Swap
- Edge-Flip
- Embedding-Flip

Our Contributions

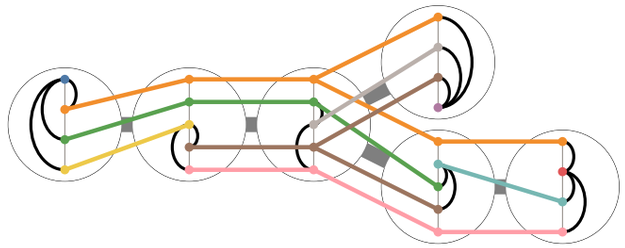
Drawing Paradigms for Treewidth



Complexity and Algorithms for Minimizing Crossings

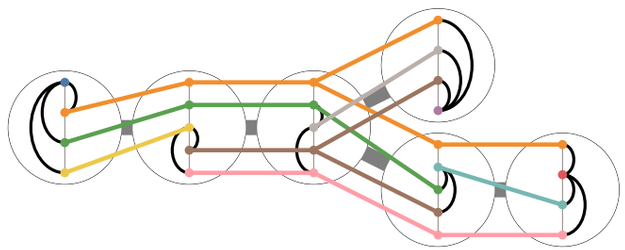


Experimental Evaluation



Our Contributions

Experimental Evaluation



Experimental Evaluation – Running Time

Dataset: 55 graphs from “Sage” with tree decomposition, 15min time limit

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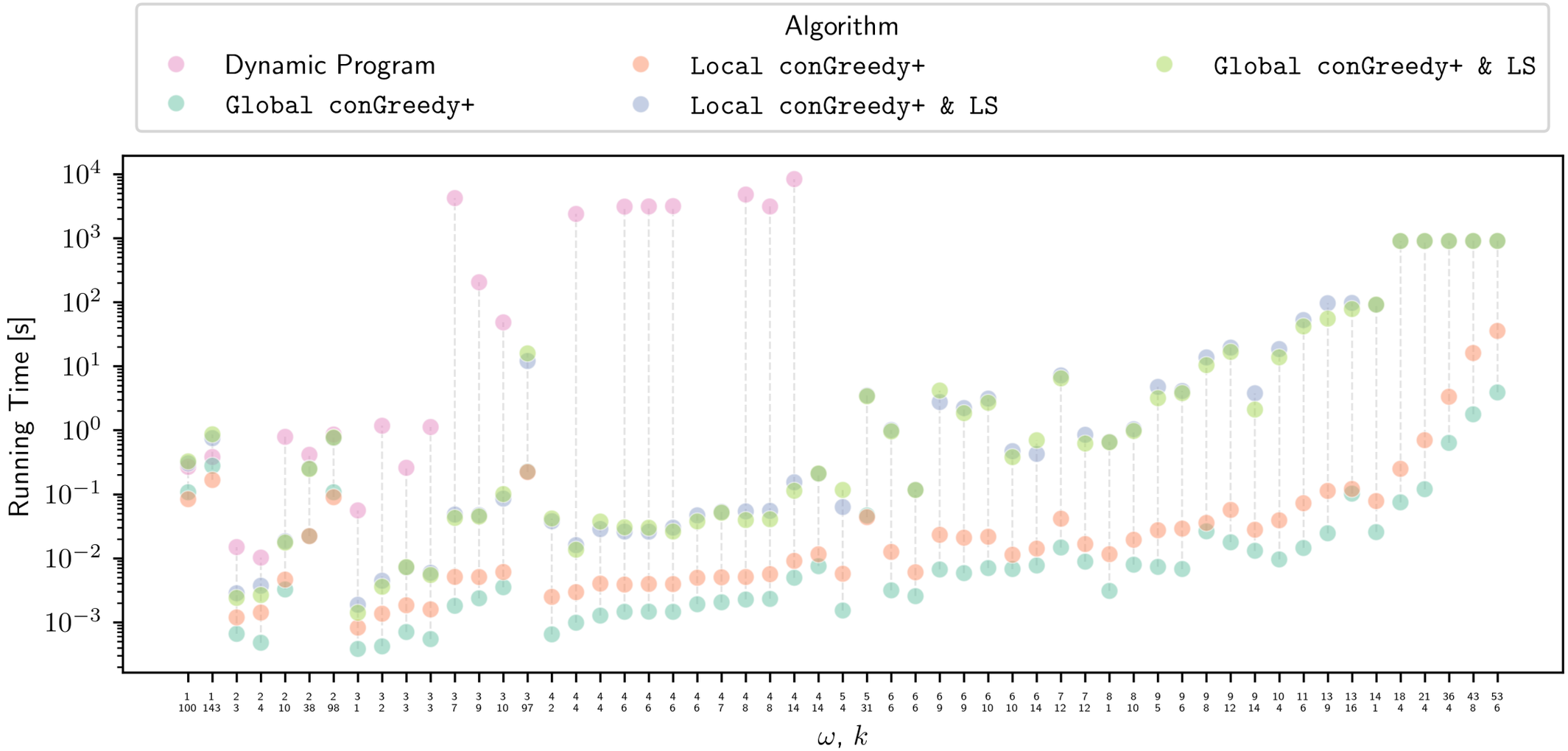

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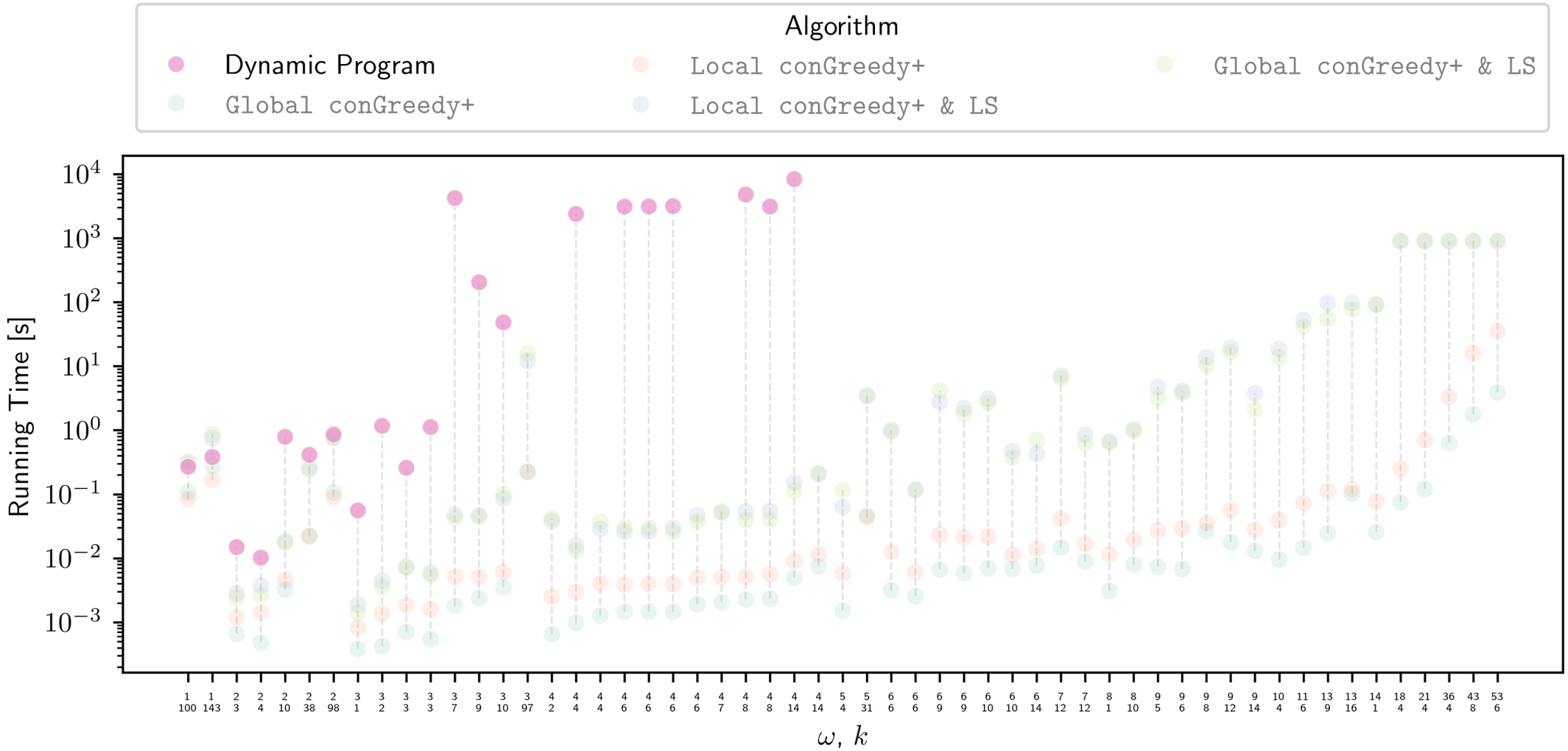

DP only feasible for small decompositions (width ≤ 4)

Experimental Evaluation – Running Time

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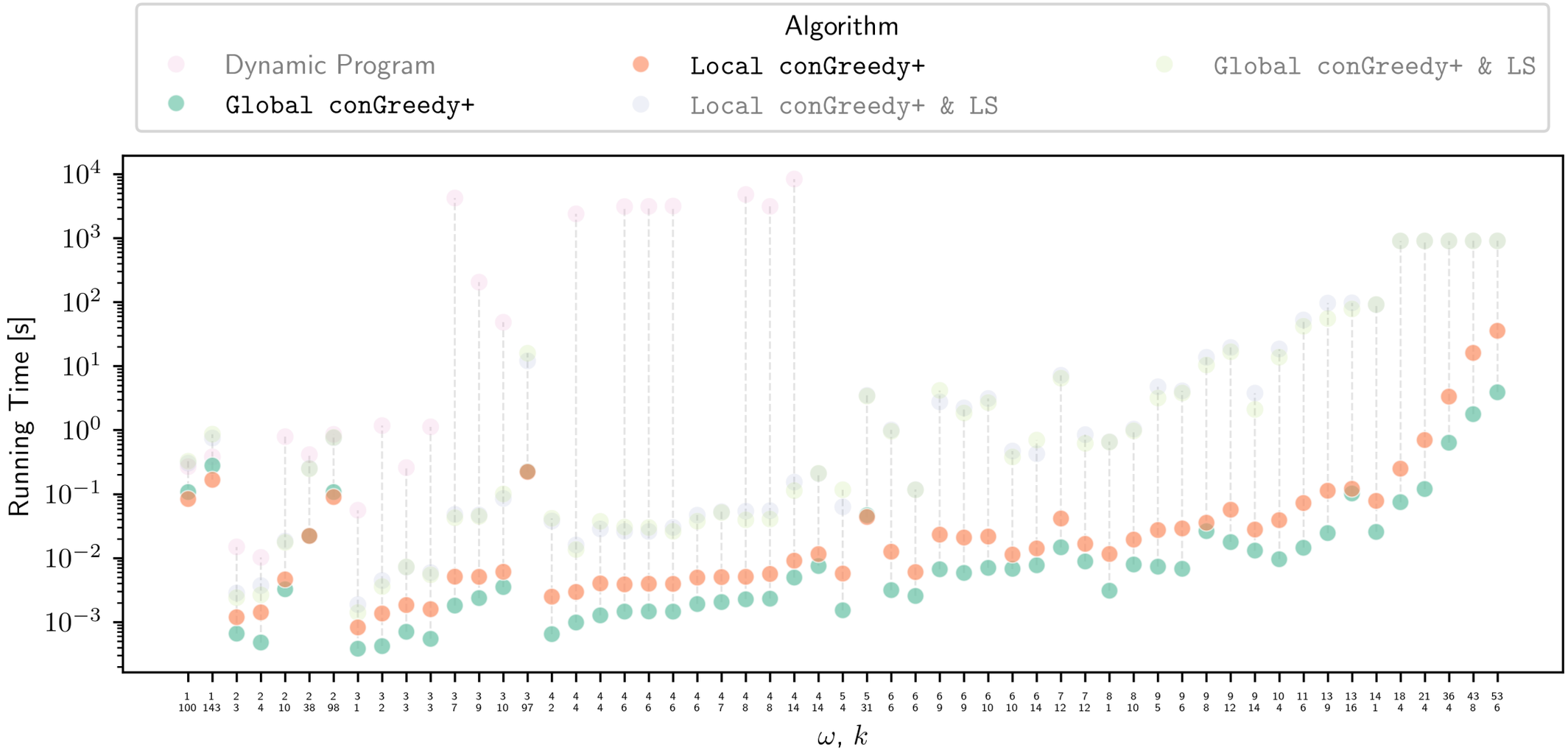
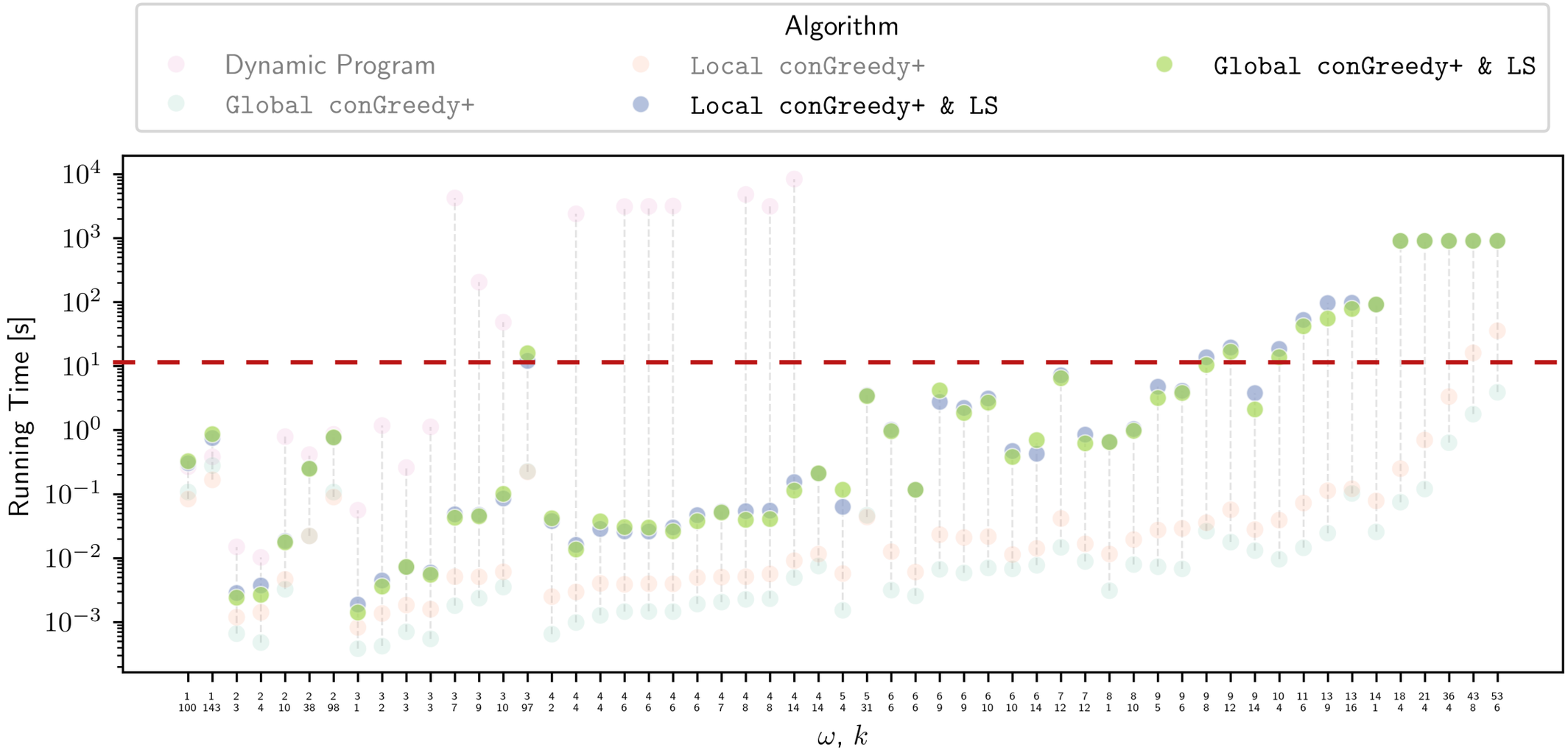


`Global conGreedy+` faster than `Local conGreedy+`

Experimental Evaluation – Running Time

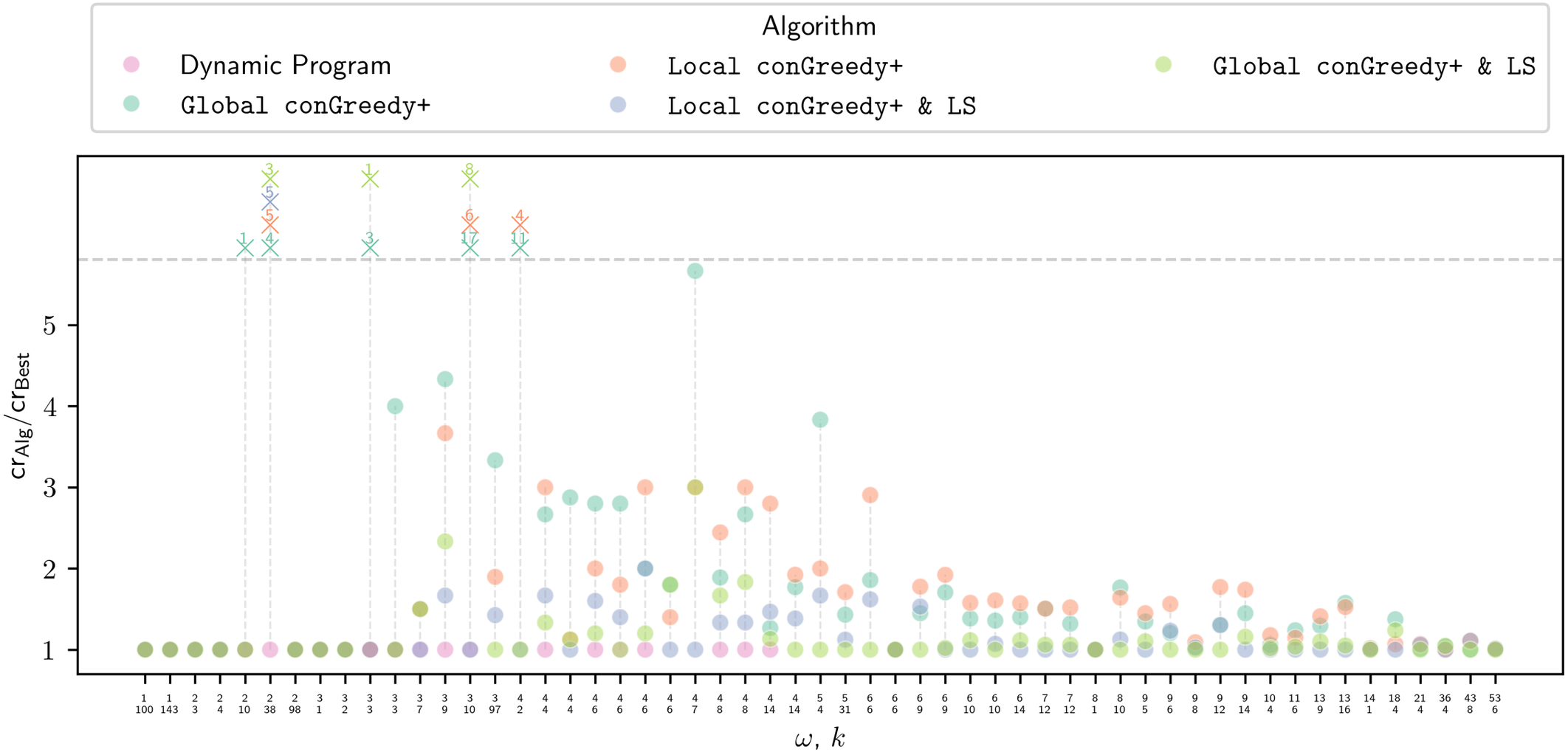
Dataset: 55 graphs from “Sage” with tree decomposition, 15min time limit



Heuristics usually take only few seconds

Experimental Evaluation – Crossings

Dataset: 55 graphs from “Sage” with tree decomposition, 15min time limit



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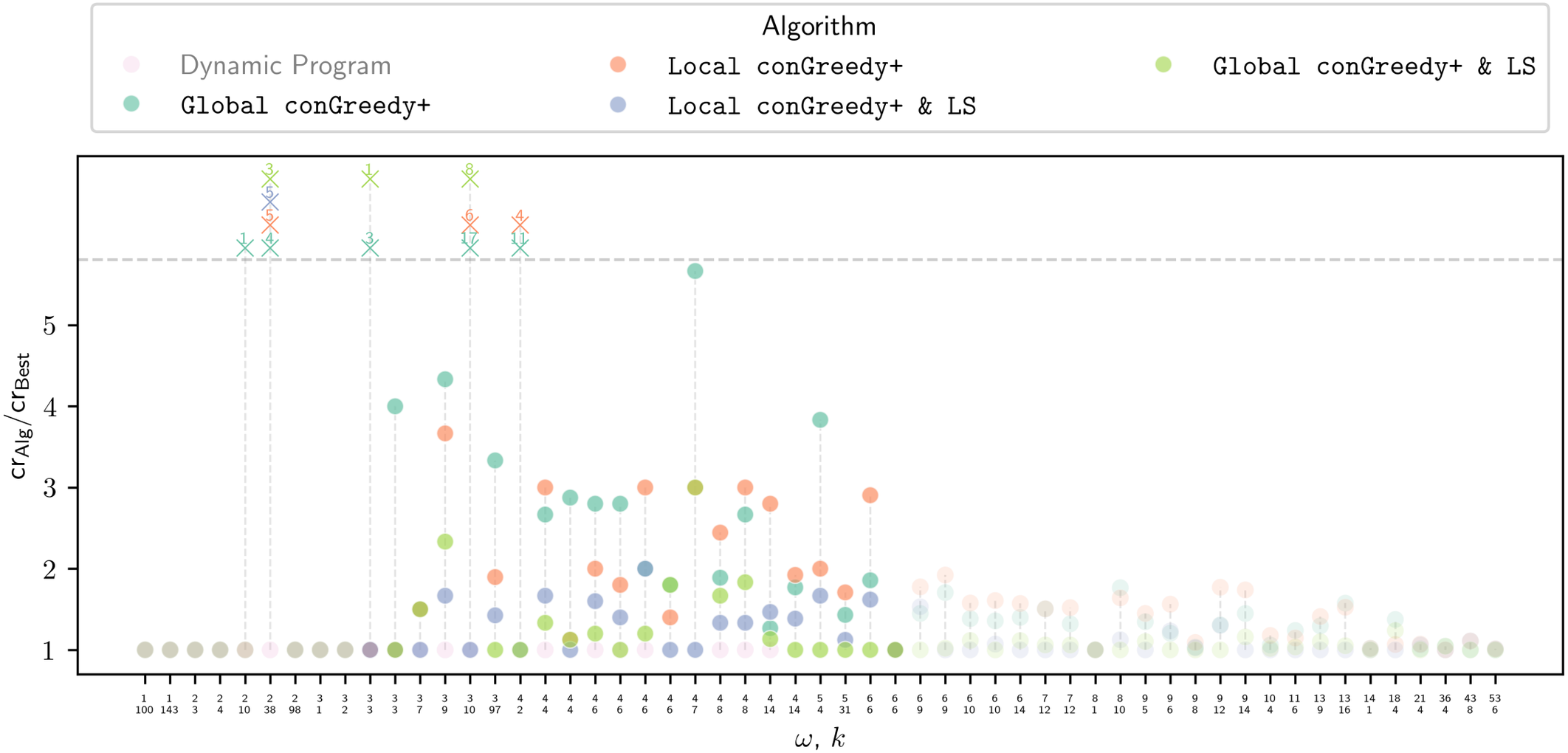
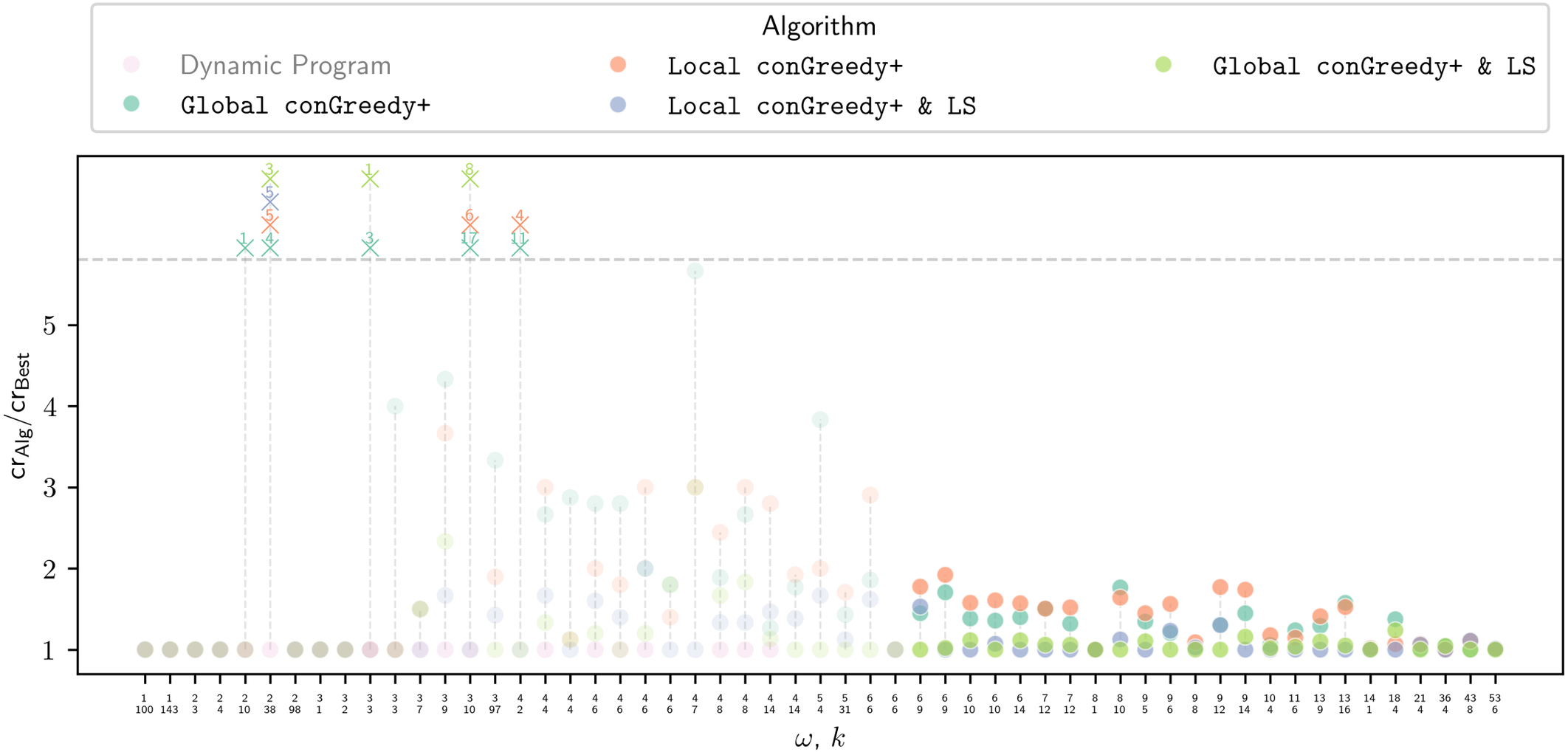


Medium-sized instances: local search helps (a lot)

Experimental Evaluation – Crossings

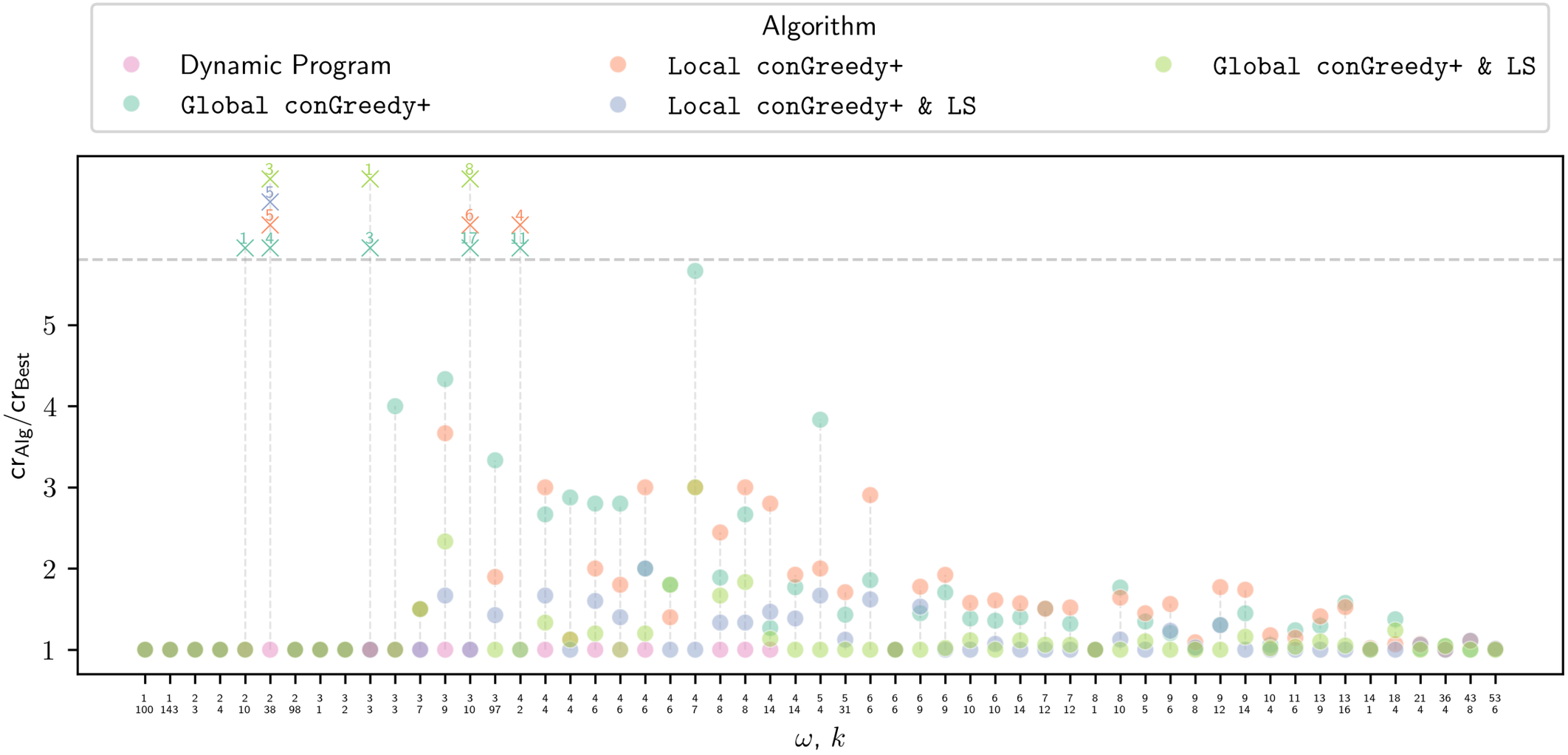
Dataset: 55 graphs from “Sage” with tree decomposition, 15min time limit



Large instances: small/no benefit of local search

Experimental Evaluation – Crossings

Dataset: 55 graphs from “Sage” with tree decomposition, 15min time limit



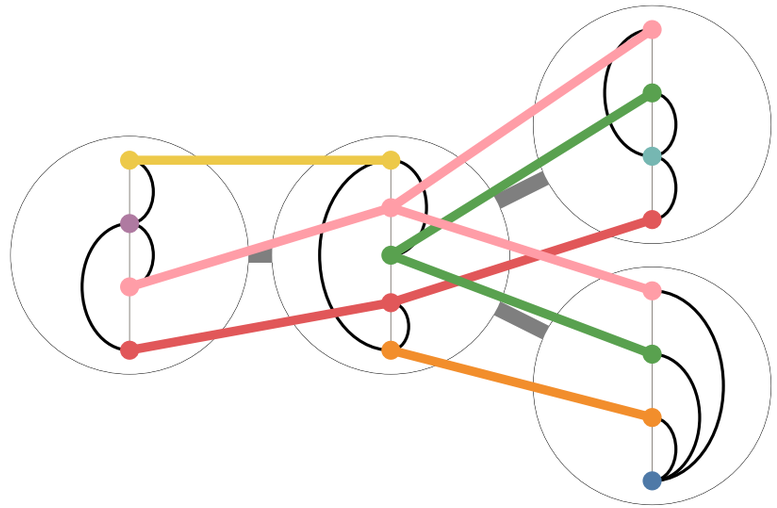
Time for drawings!

Witness Drawings for the Wagner Graph

Witness Drawings for the Wagner Graph

Witness Drawings for the Wagner Graph

`Global conGreedy+`:
8 crossings
1ms



Witness Drawings for the Wagner Graph

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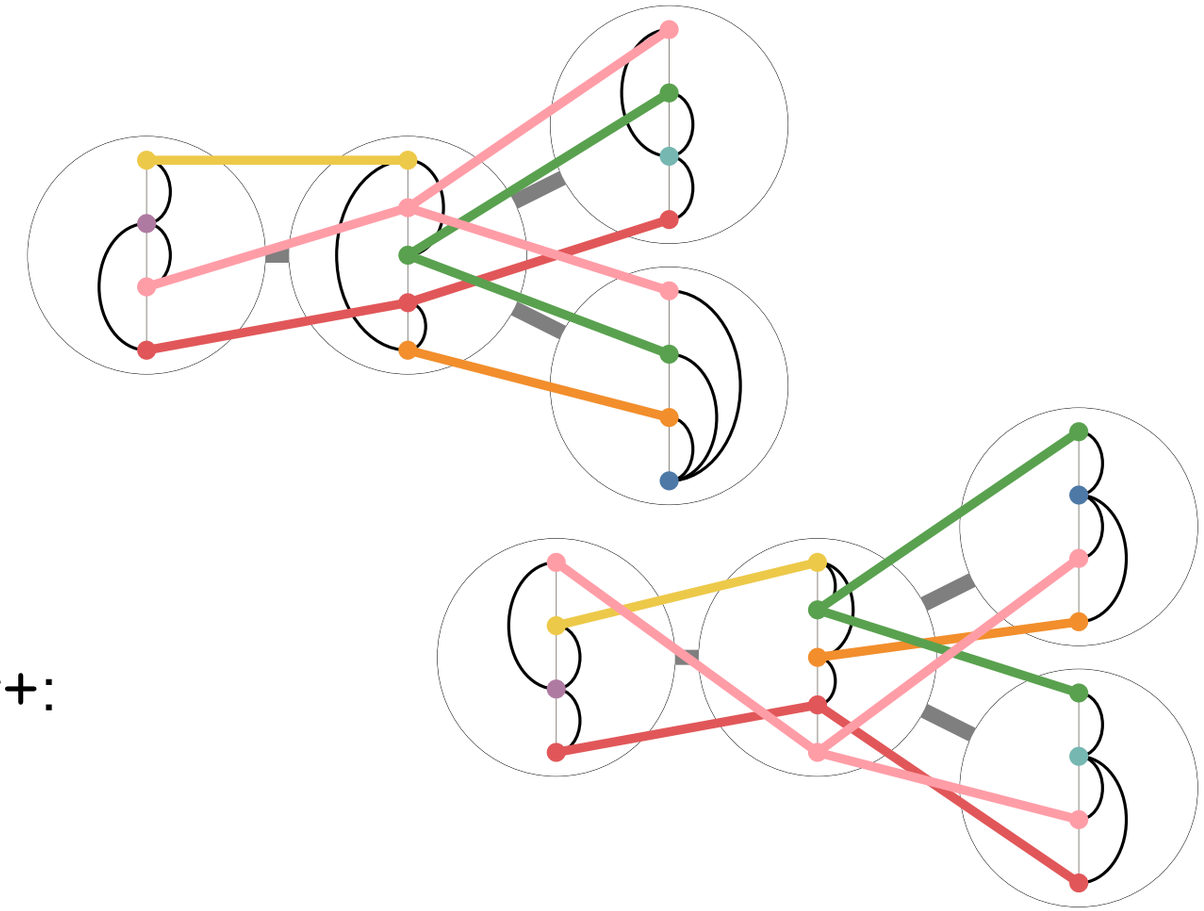


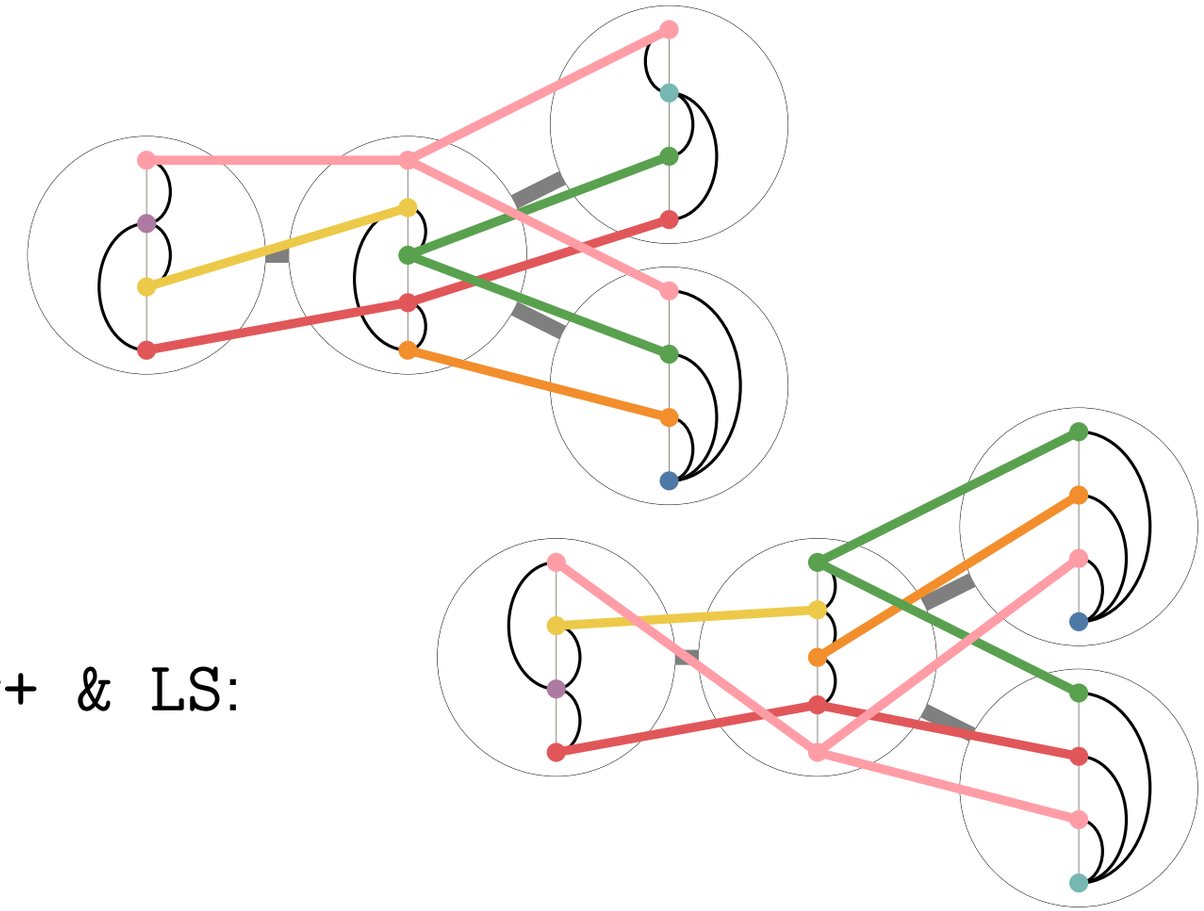
`Local conGreedy+`:
9 crossings
3ms

Witness Drawings for the Wagner Graph

`Global conGreedy+ & LS:`

~~8~~ $\rightarrow$ 4 crossings

~~1~~ $\rightarrow$ 135ms

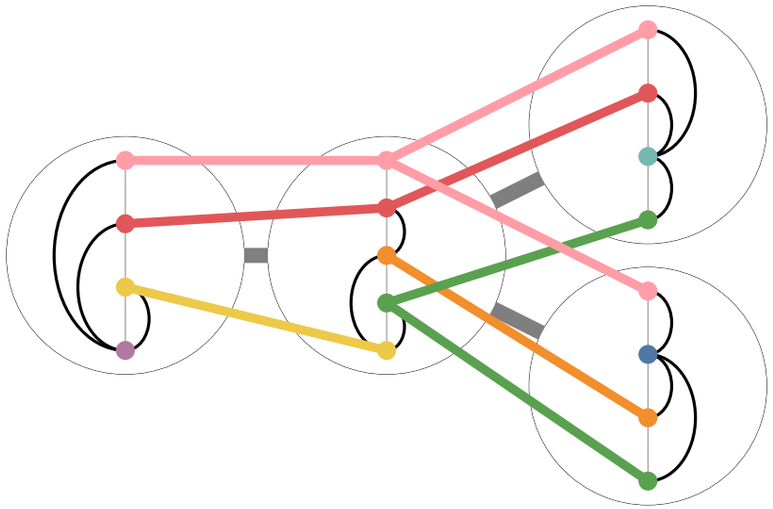


`Local conGreedy+ & LS:`

~~9~~ $\rightarrow$ 5 crossings

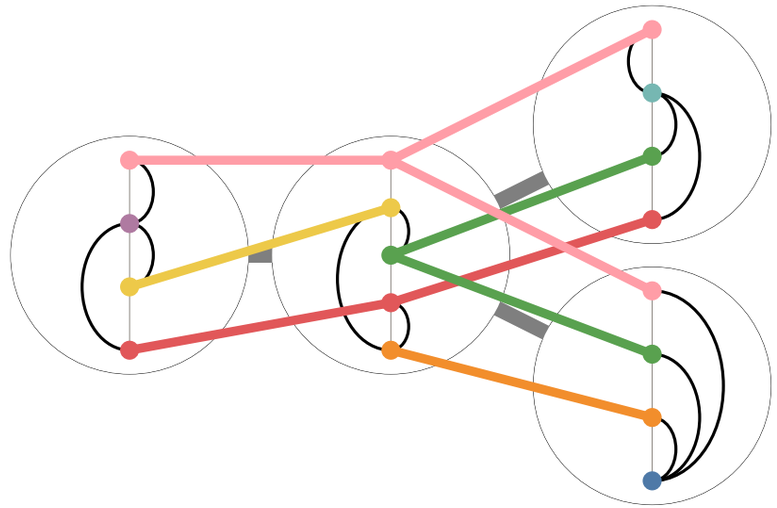
~~3~~ $\rightarrow$ 160ms

Witness Drawings for the Wagner Graph

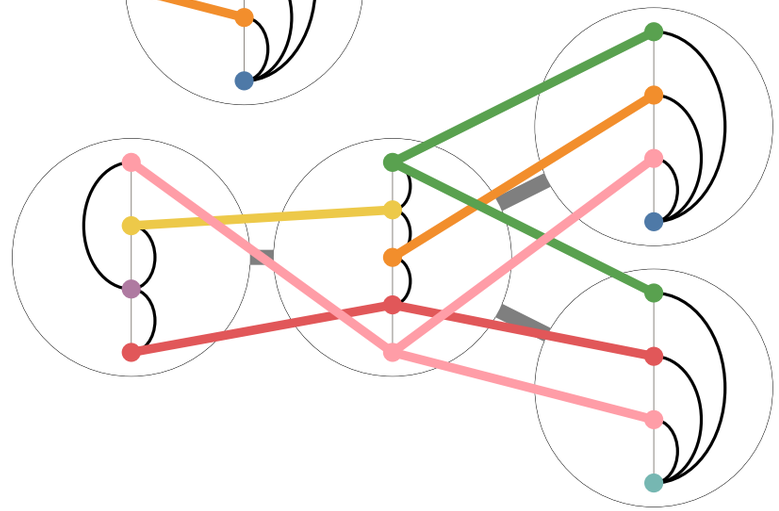


DP:
3 crossings
40min

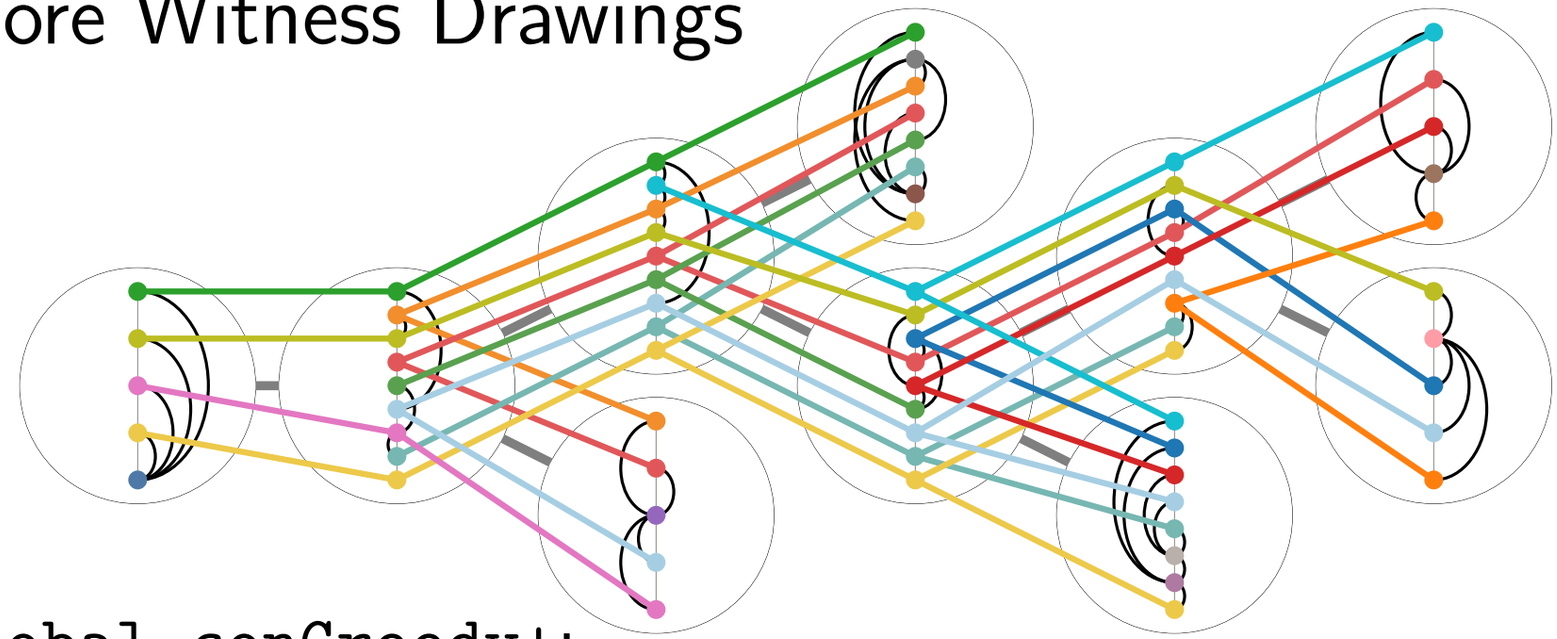
Global conGreedy+ & LS:
~~8~~ → 4 crossings
~~1~~ → 135ms

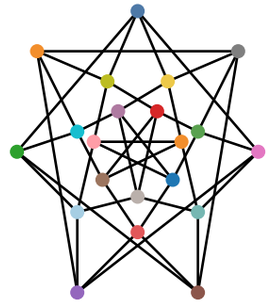


Local conGreedy+ & LS:
~~9~~ → 5 crossings
~~3~~ → 160ms

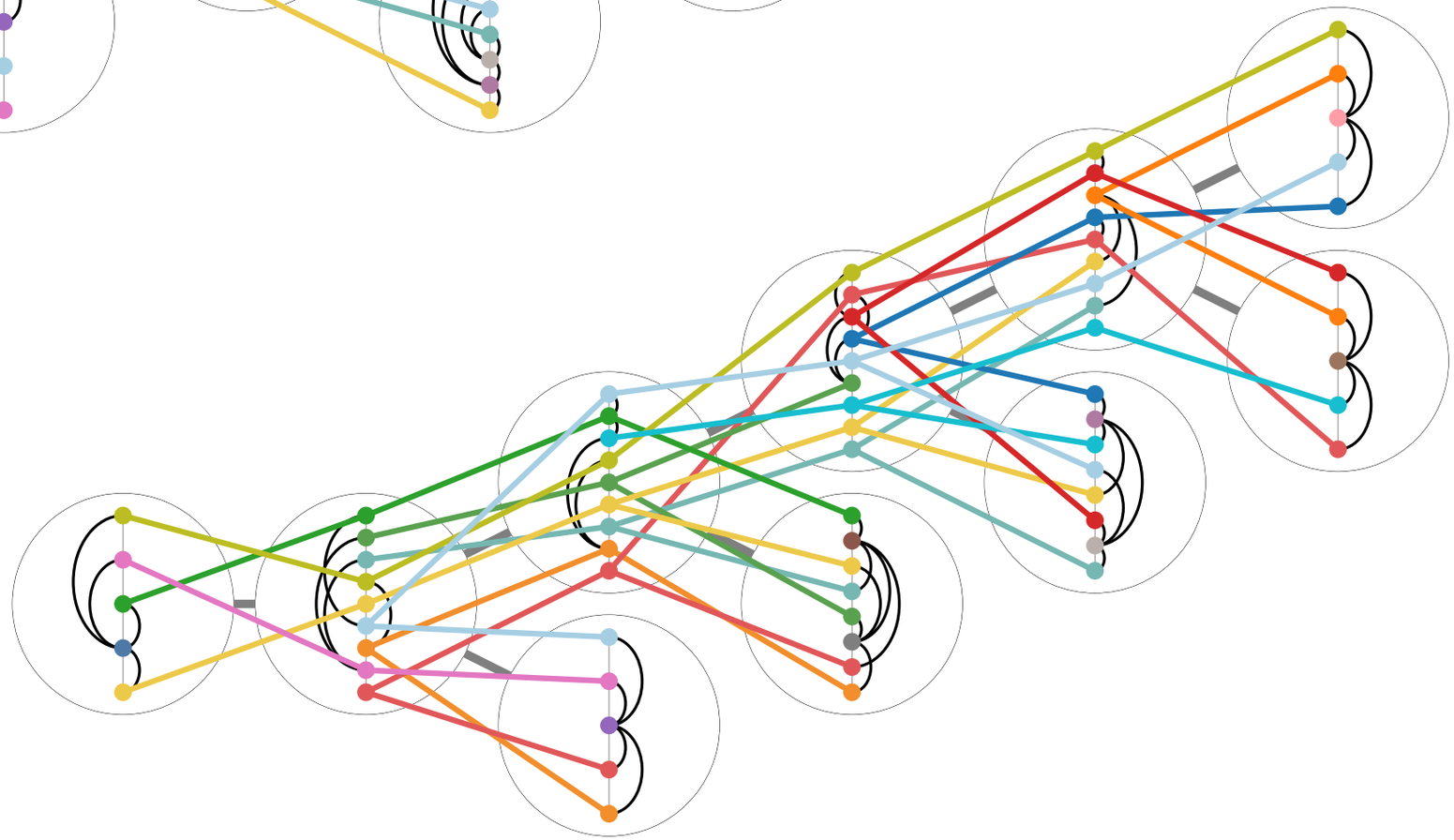


More Witness Drawings



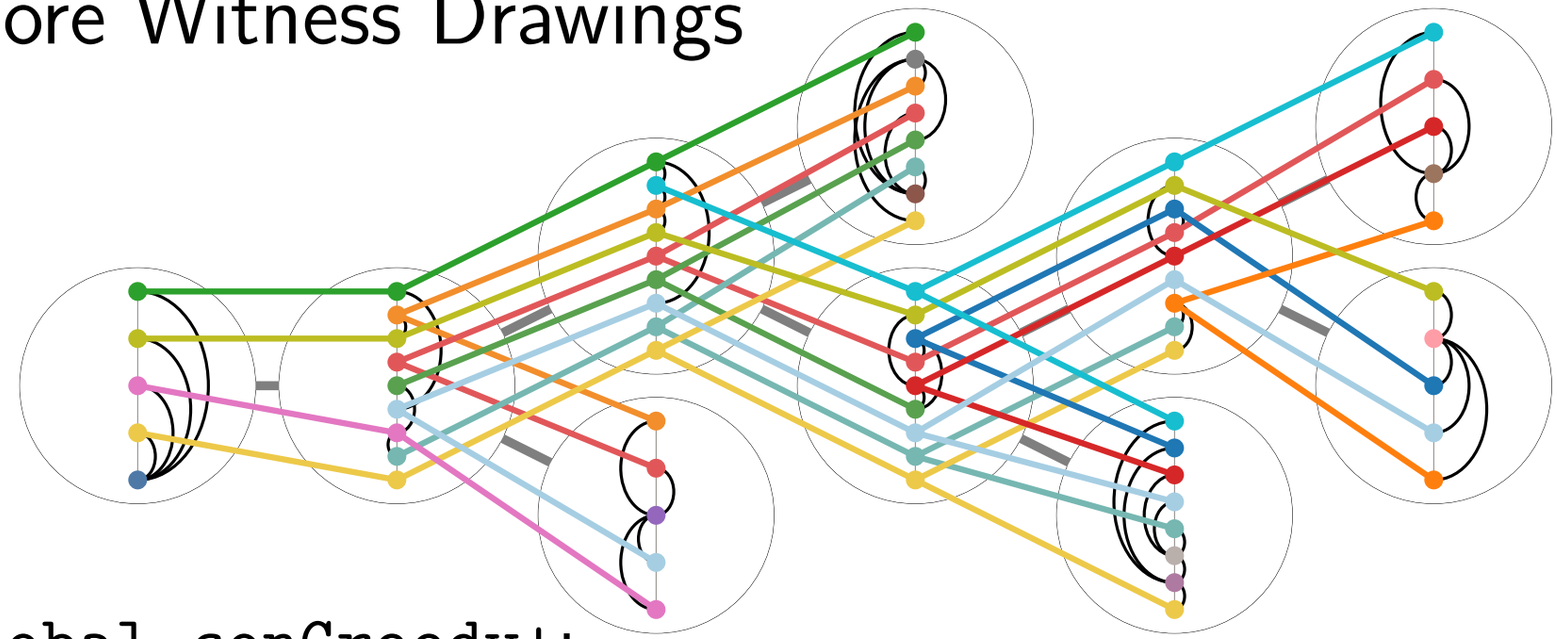


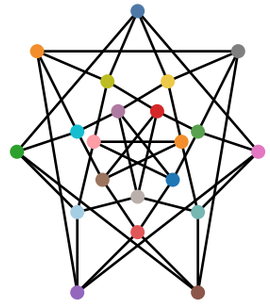
`Global conGreedy+`:
113 crossings
8ms



`Local conGreedy+`:
105 crossings
19ms

More Witness Drawings





More readable?

`Global conGreedy+`:
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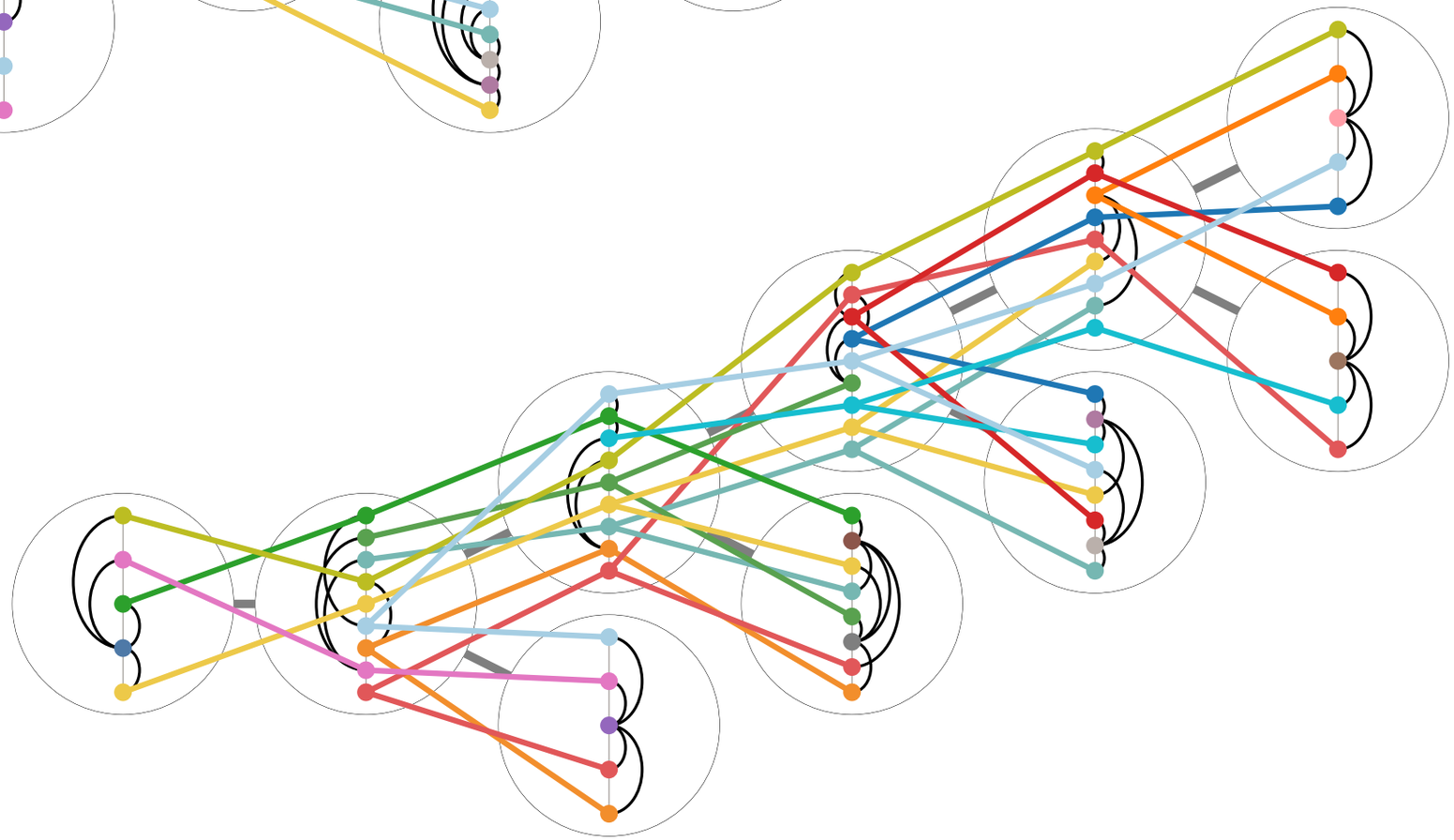

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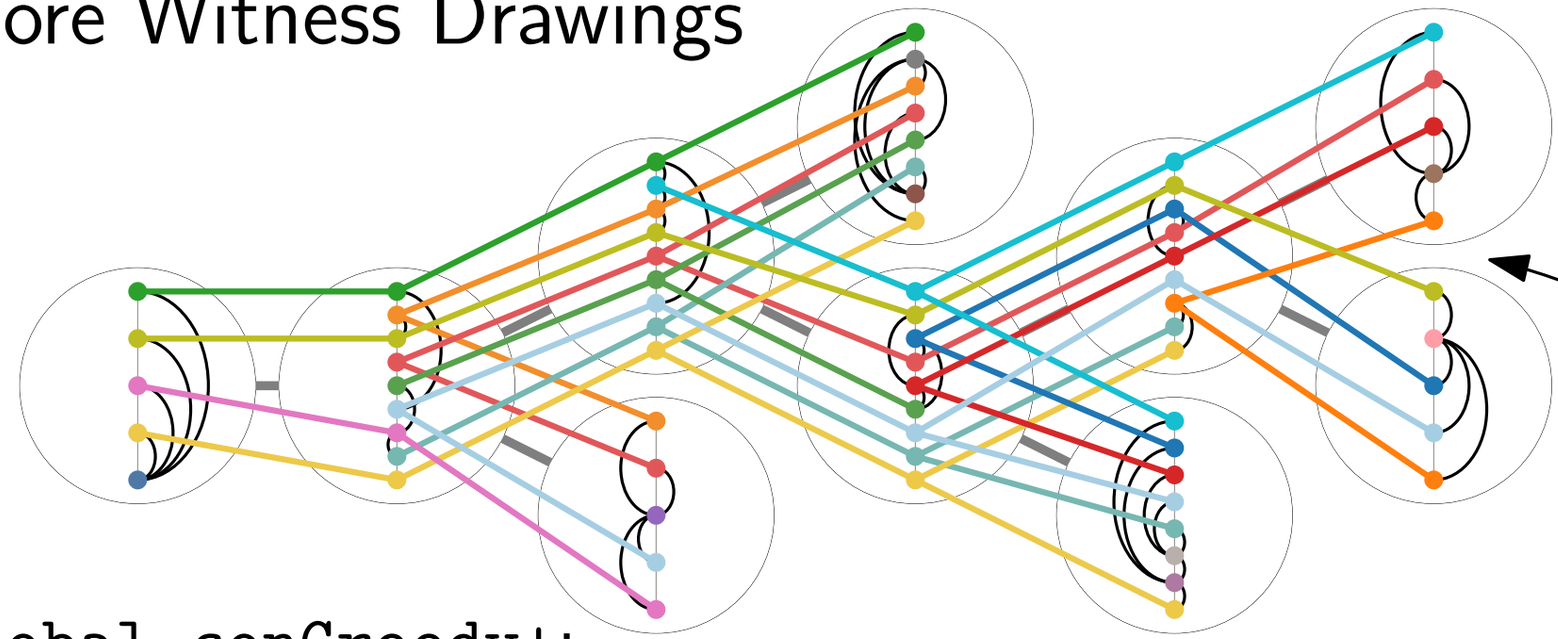
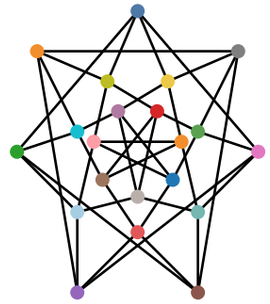
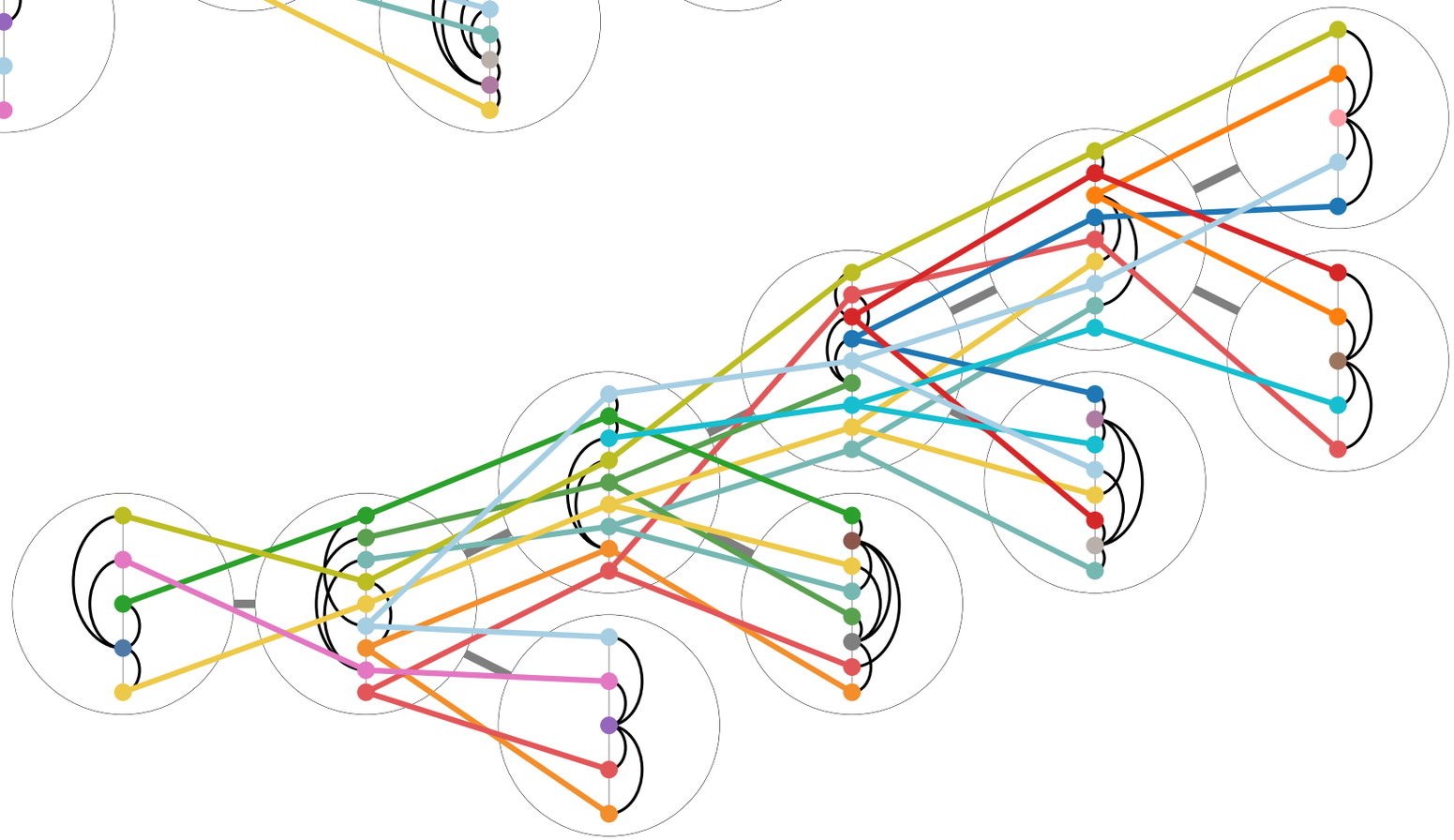
More Witness Drawings





More readable?

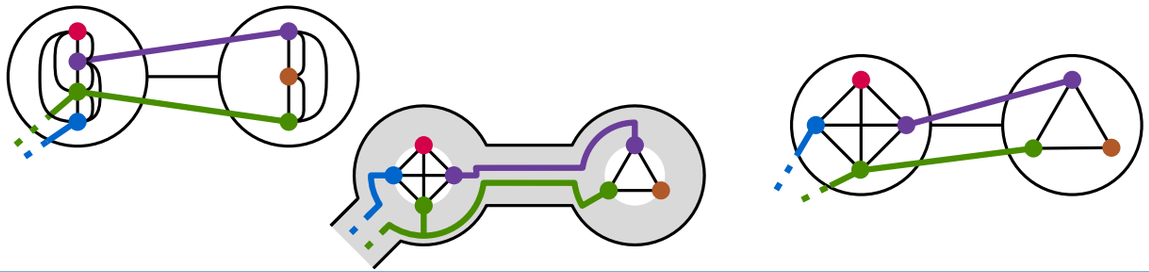
`Global conGreedy+`:
113 crossings
8ms



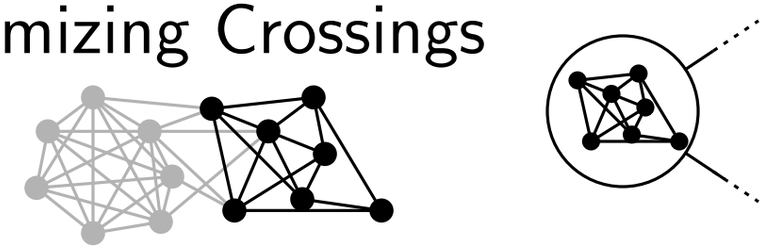
`Local conGreedy+`:
105 crossings
19ms

Conclusion

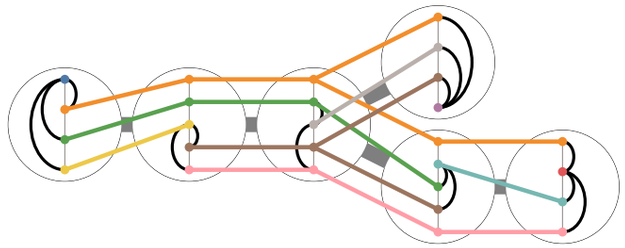
Drawing Paradigms for Treewidth



Complexity and Algorithms for Minimizing Crossings

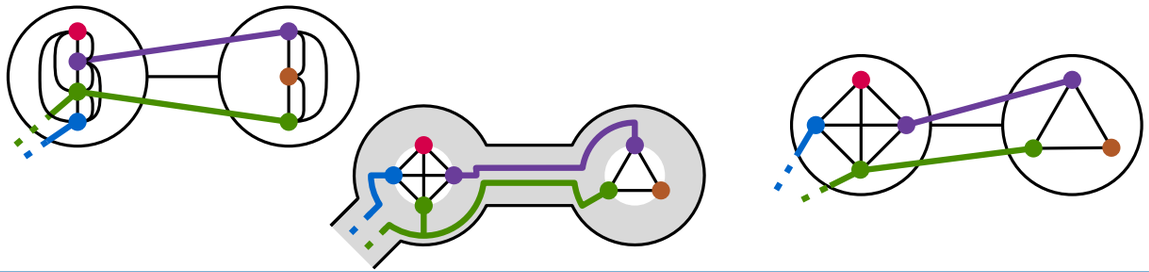


Experimental Evaluation

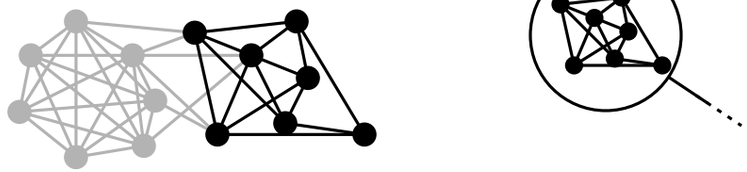


Conclusion

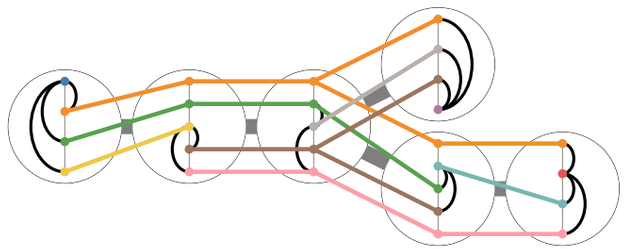
Drawing Paradigms for Treewidth



Complexity and Algorithms for Minimizing Crossings



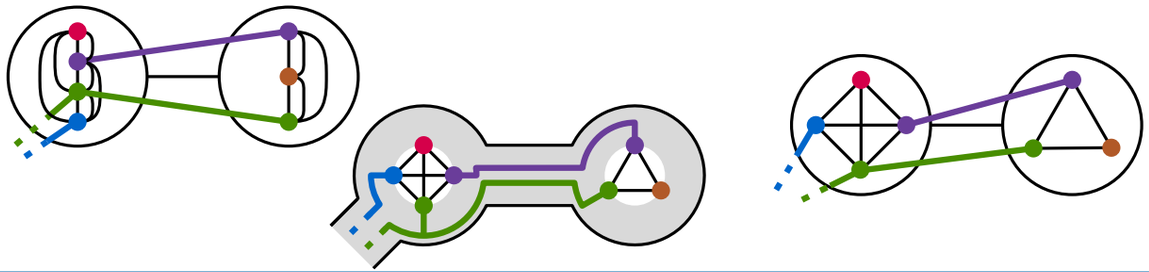
Experimental Evaluation



Prototype

Conclusion

Drawing Paradigms for Treewidth



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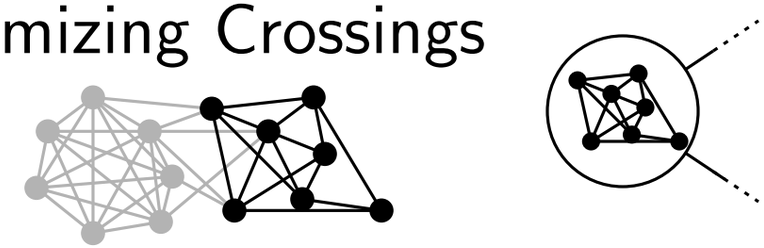
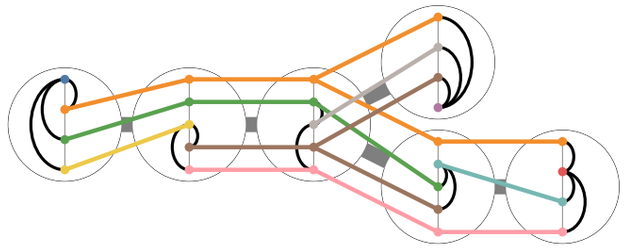


Experimental Evaluation

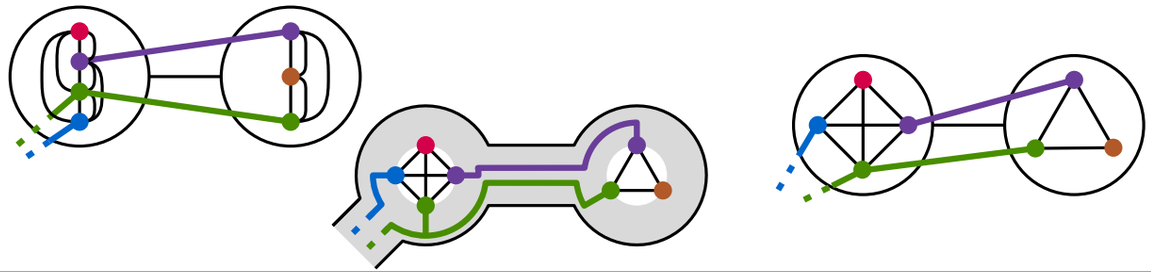


Prototype

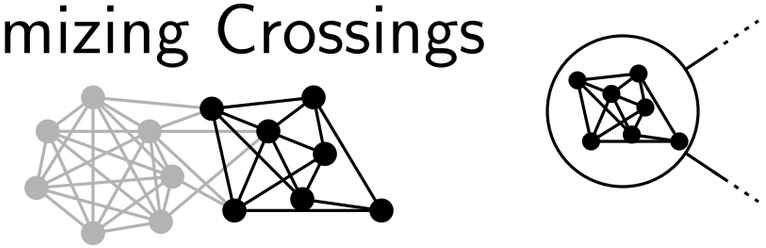
Future Work

Conclusion

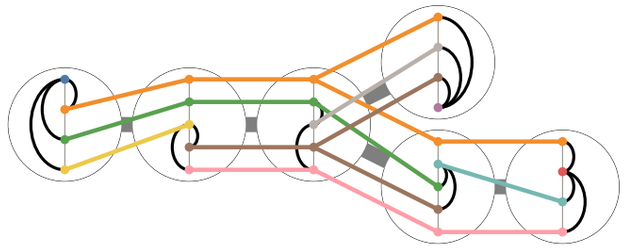
Drawing Paradigms for Treewidth



Complexity and Algorithms for Minimizing Crossings



Experimental Evaluation



Prototype

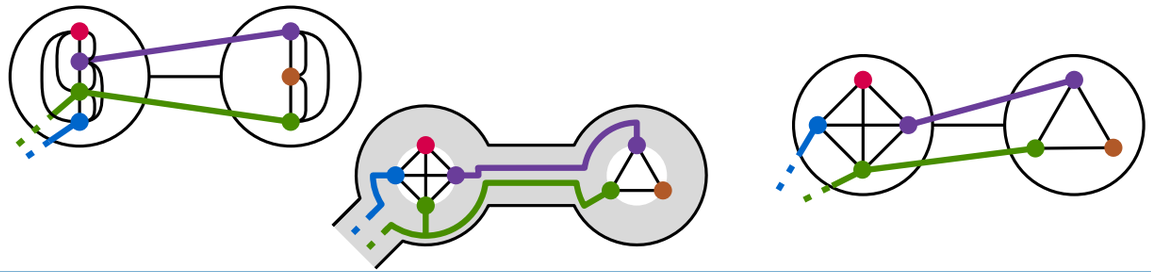
Future Work

Further Explore Design Space

Other Optimization Criteria

Conclusion

Drawing Paradigms for Treewidth



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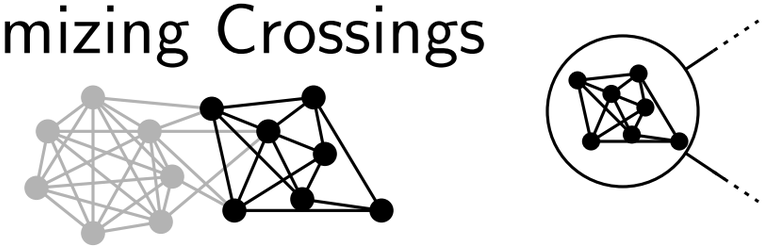
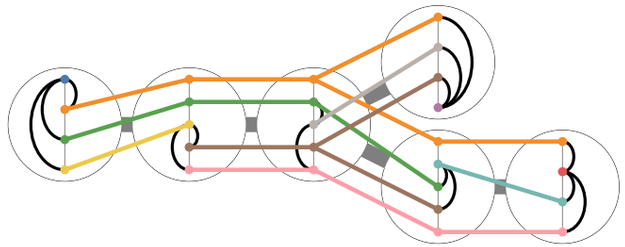


Experimental Evaluation



Prototype

Future Work

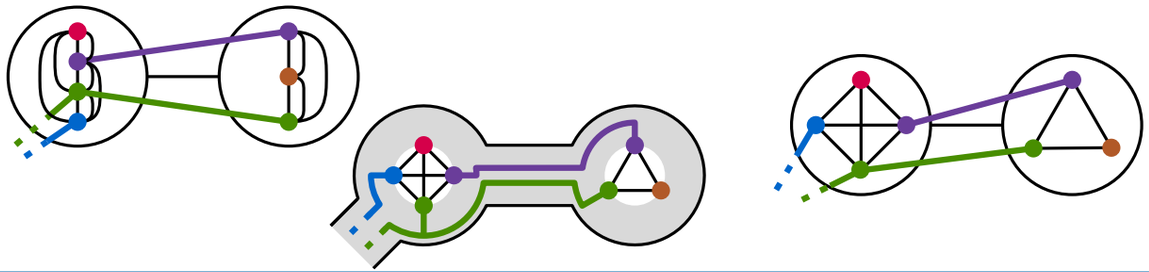
Further Explore Design Space

Other Optimization Criteria

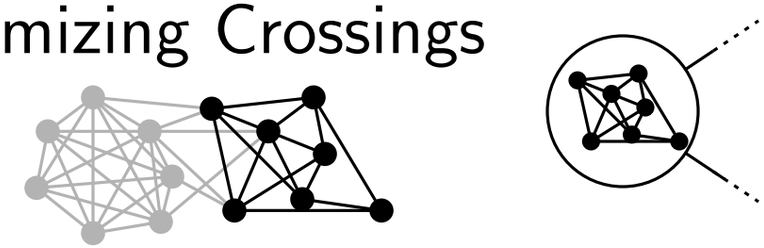
Trees of Larger Degree

Conclusion

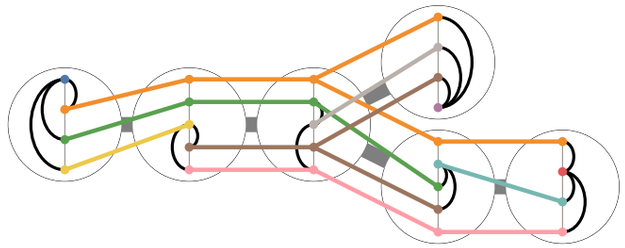
Drawing Paradigms for Treewidth



Complexity and Algorithms for Minimizing Crossings



Experimental Evaluation



Prototype

Future Work

Further Explore Design Space

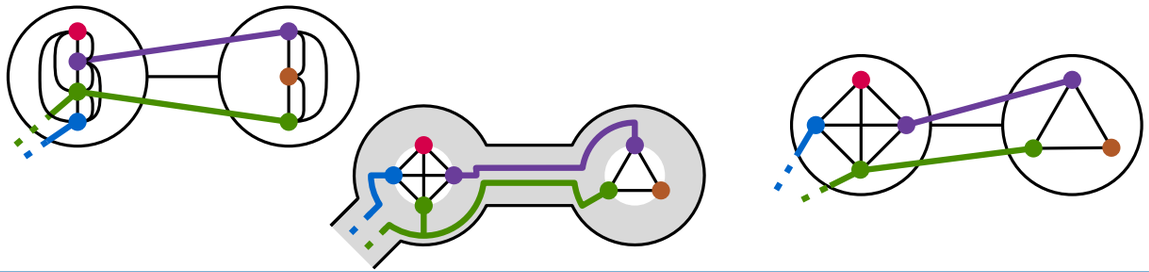
Other Optimization Criteria

Trees of Larger Degree

Witness Drawings for other Graph Parameters

Conclusion

Drawing Paradigms for Treewidth



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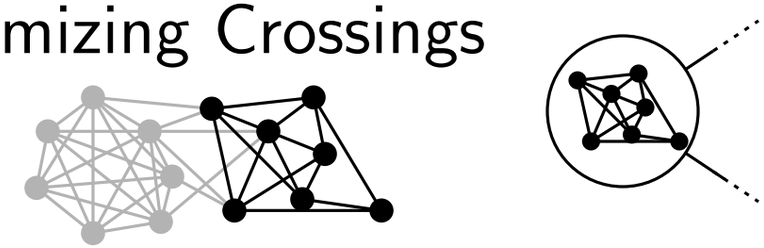
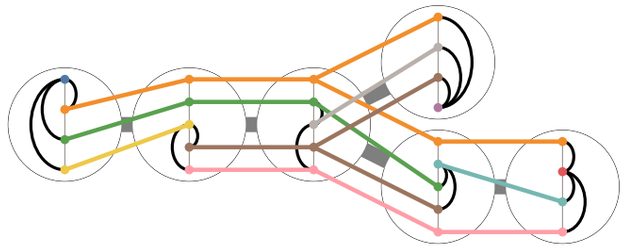


Experimental Evaluation



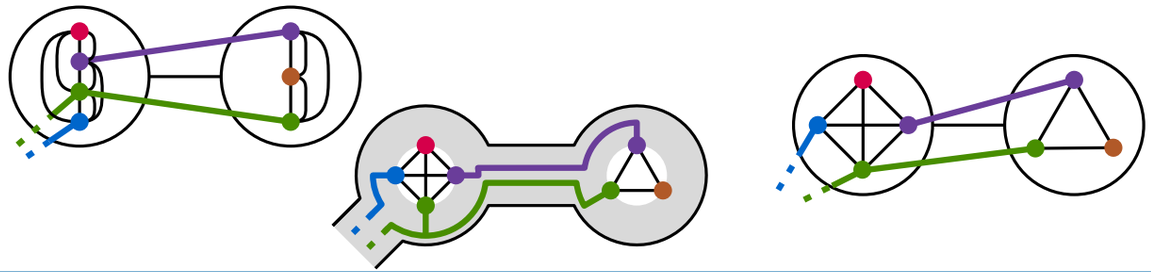
Prototype

Future Work

- Further Explore Design Space
- Other Optimization Criteria
- Trees of Larger Degree
- Witness Drawings for other Graph Parameters
- User Study

Conclusion

Drawing Paradigms for Treewidth



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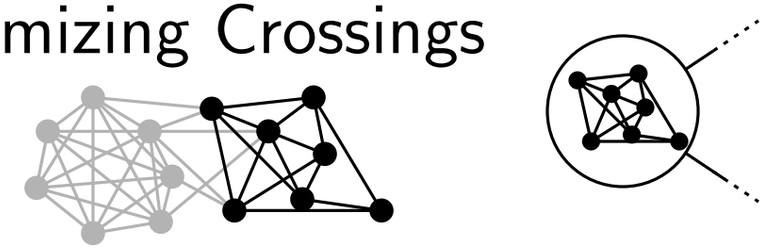
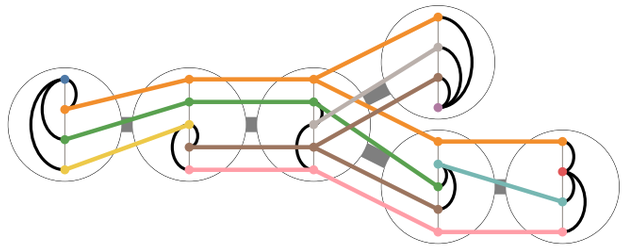


Experimental Evaluation



Prototype

Future Work

- Further Explore Design Space
- Other Optimization Criteria
- Trees of Larger Degree
- Witness Drawings for other Graph Parameters
- User Study

Thank you for your attention!

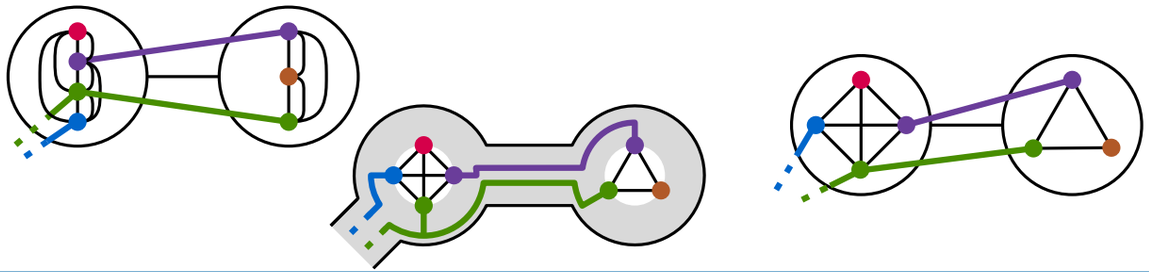
Invitation:

Participate in a **User Study** on Treewidth Visualizations

https://url.tuwien.at/yqgbg

Conclusion

Drawing Paradigms for Treewidth



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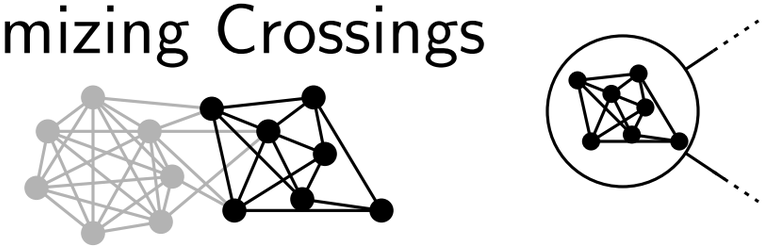

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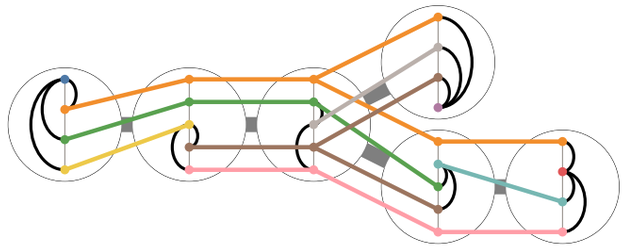


Prototype

Future Work

Further Explore Design Space

Other Optimization Criteria

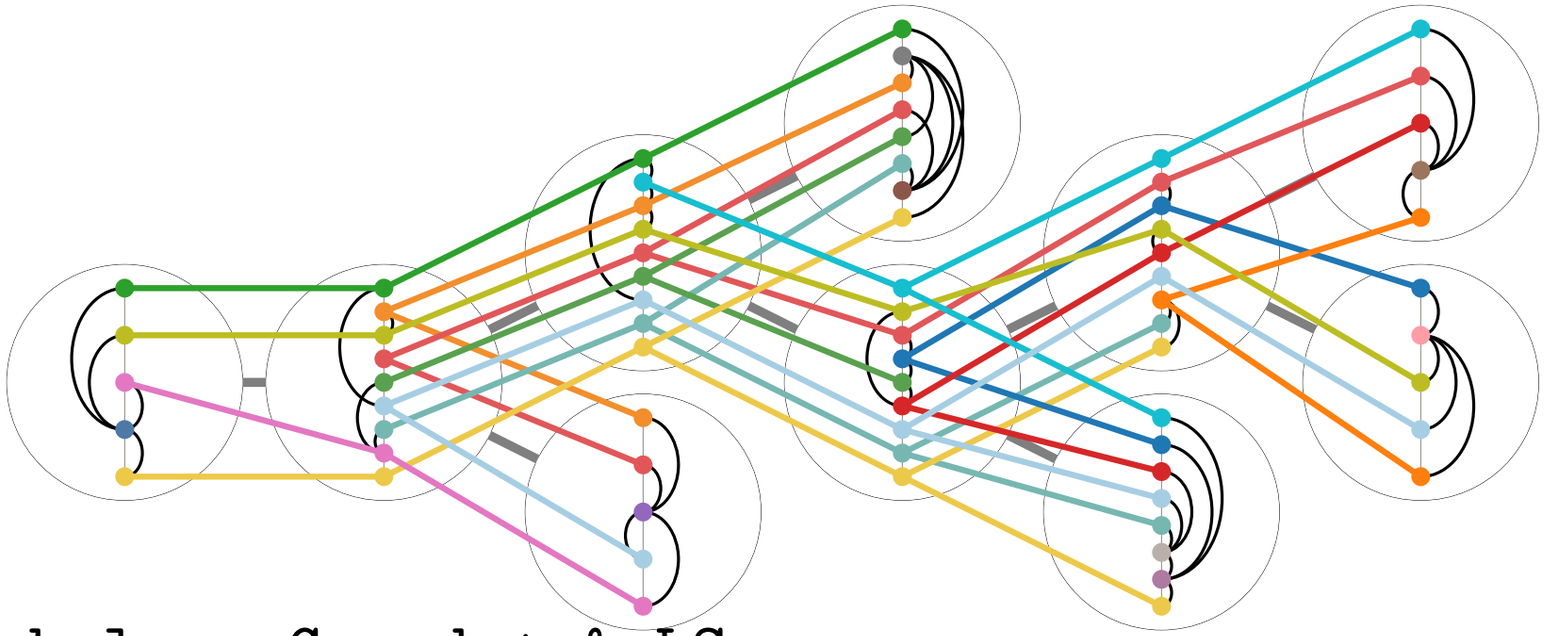
Trees of Larger Degree

Witness Drawings for other Graph Parameters

User Study →

Thank you for your attention!

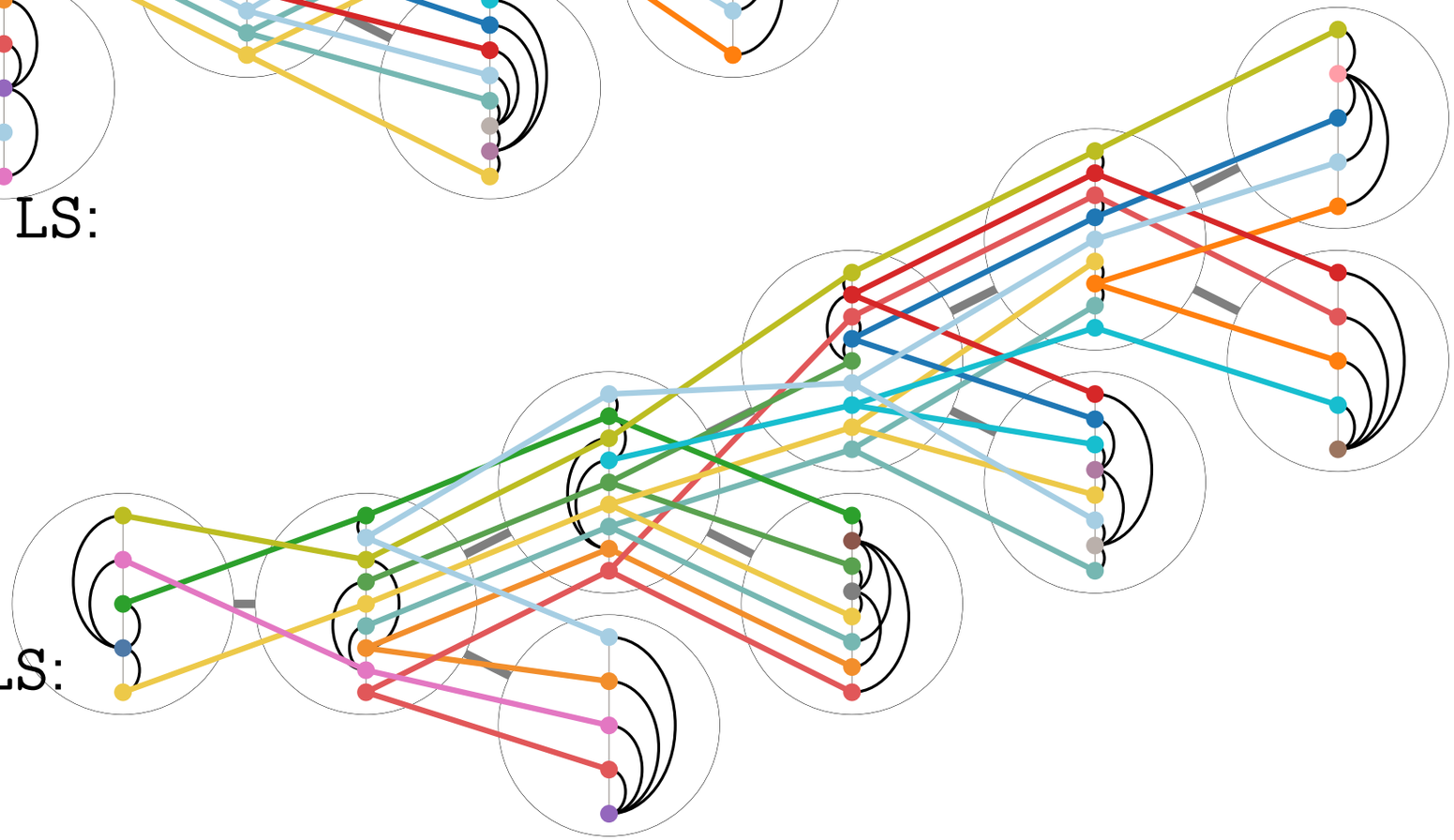
More Witness Drawings (LS)



`Global conGreedy+ & LS:`

64 crossings

970ms



`Local conGreedy+ & LS:`

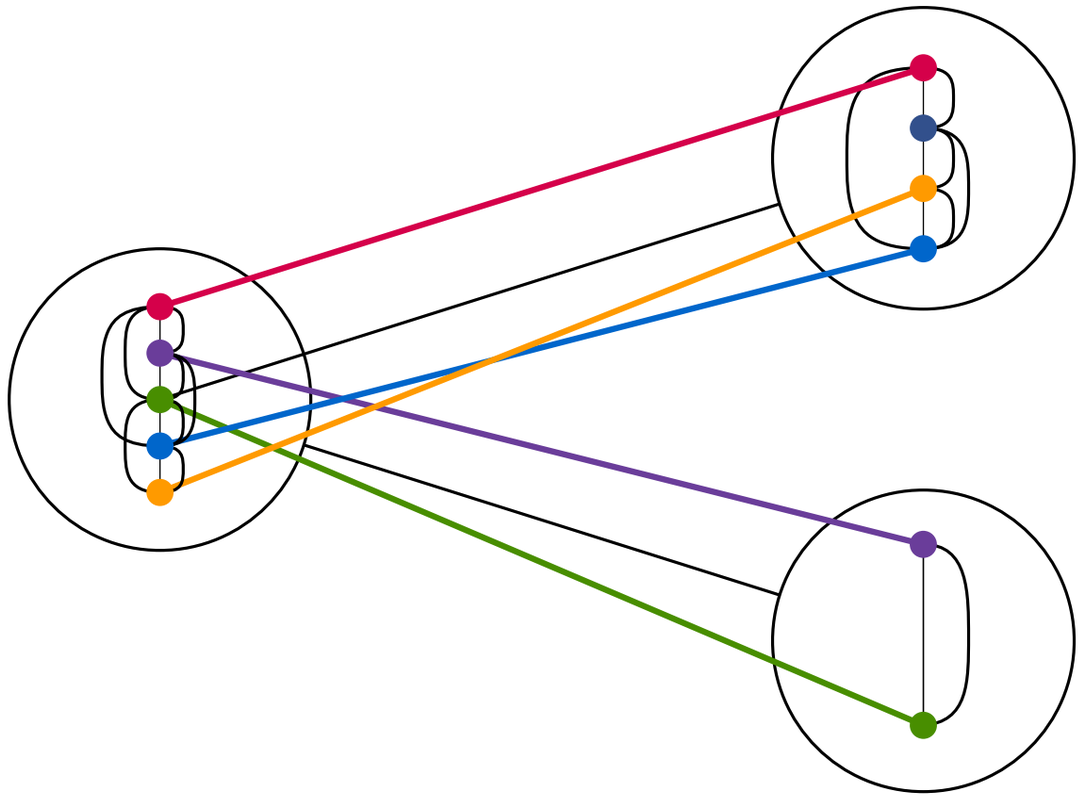
72 crossings

1s

Crossing Minimization for Linear Drawings

Goal: Minimize Σ (t/t, t/e, e/e-crossings)

Theorem:
A crossing-minimal linear witness drawing of a tree decomposition of width ω can be computed in $\mathcal{O}(f(\omega) \cdot n)$ time.



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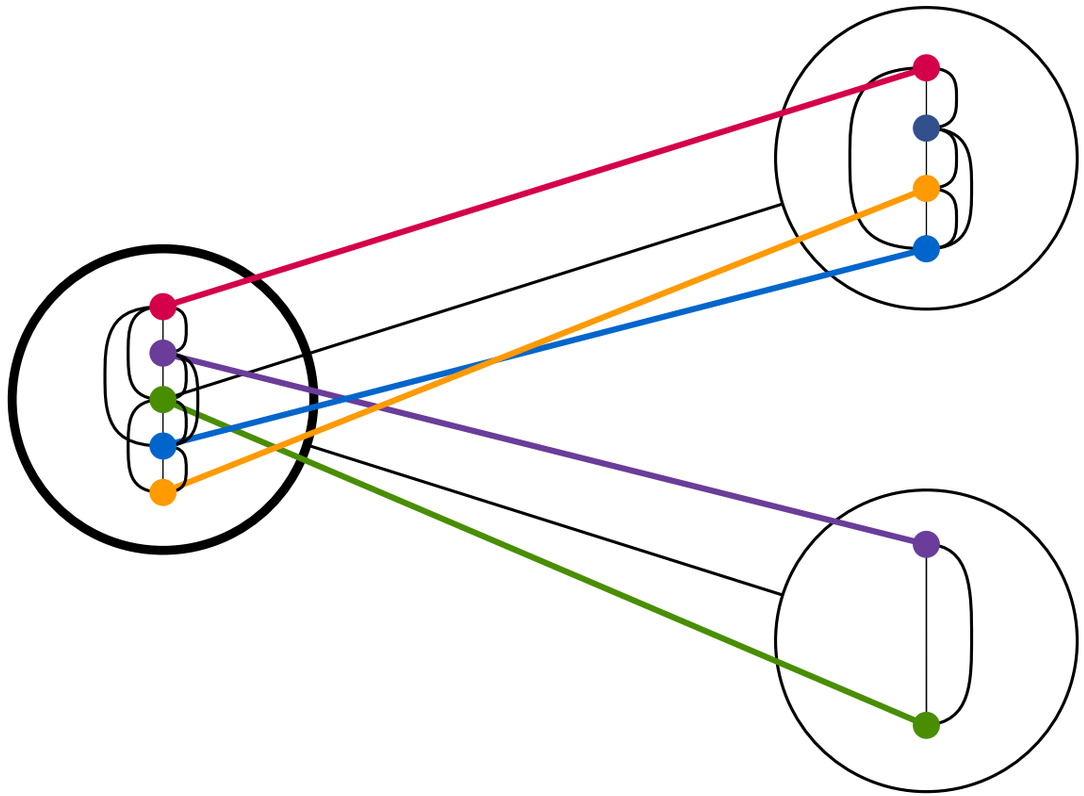

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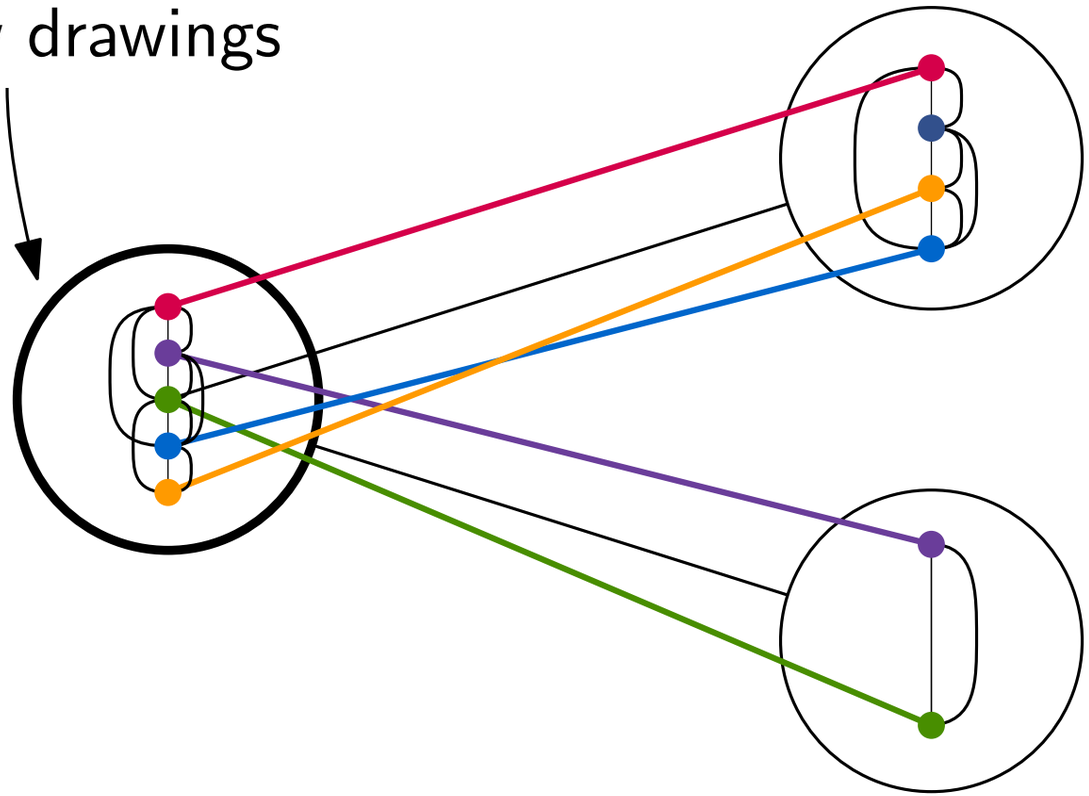
Few drawings



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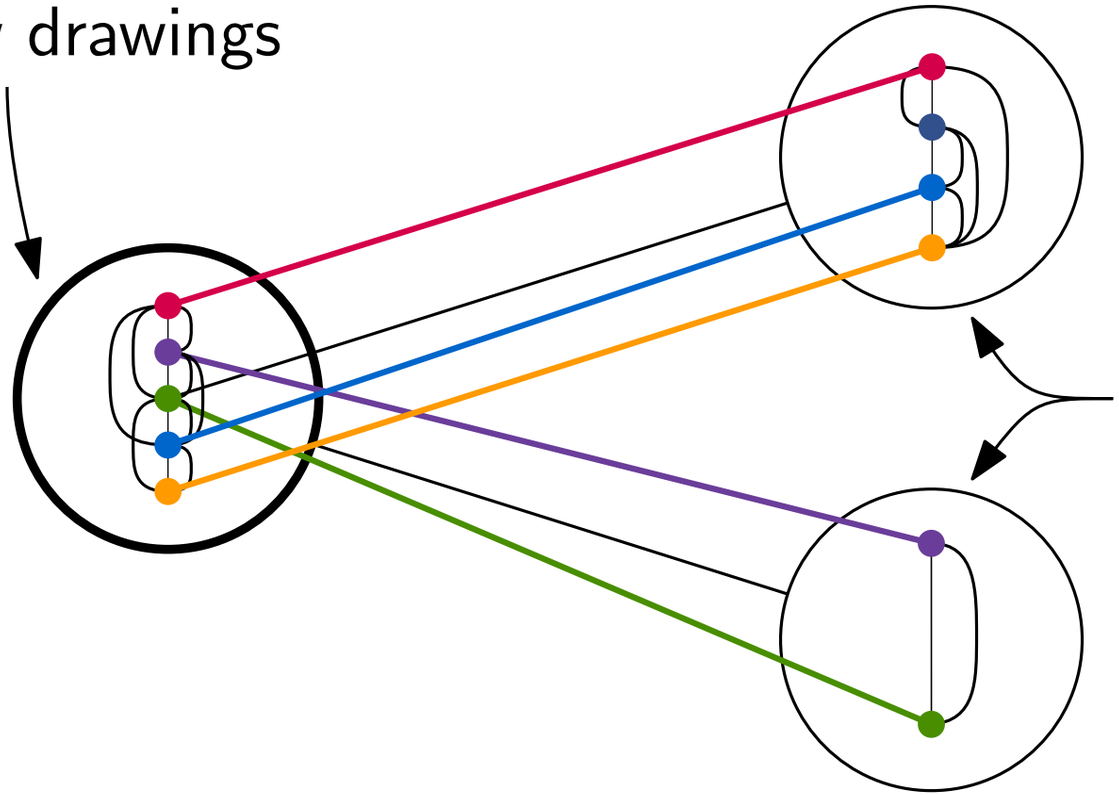

Matters: Drawing

Crossing Minimization for Linear Drawings

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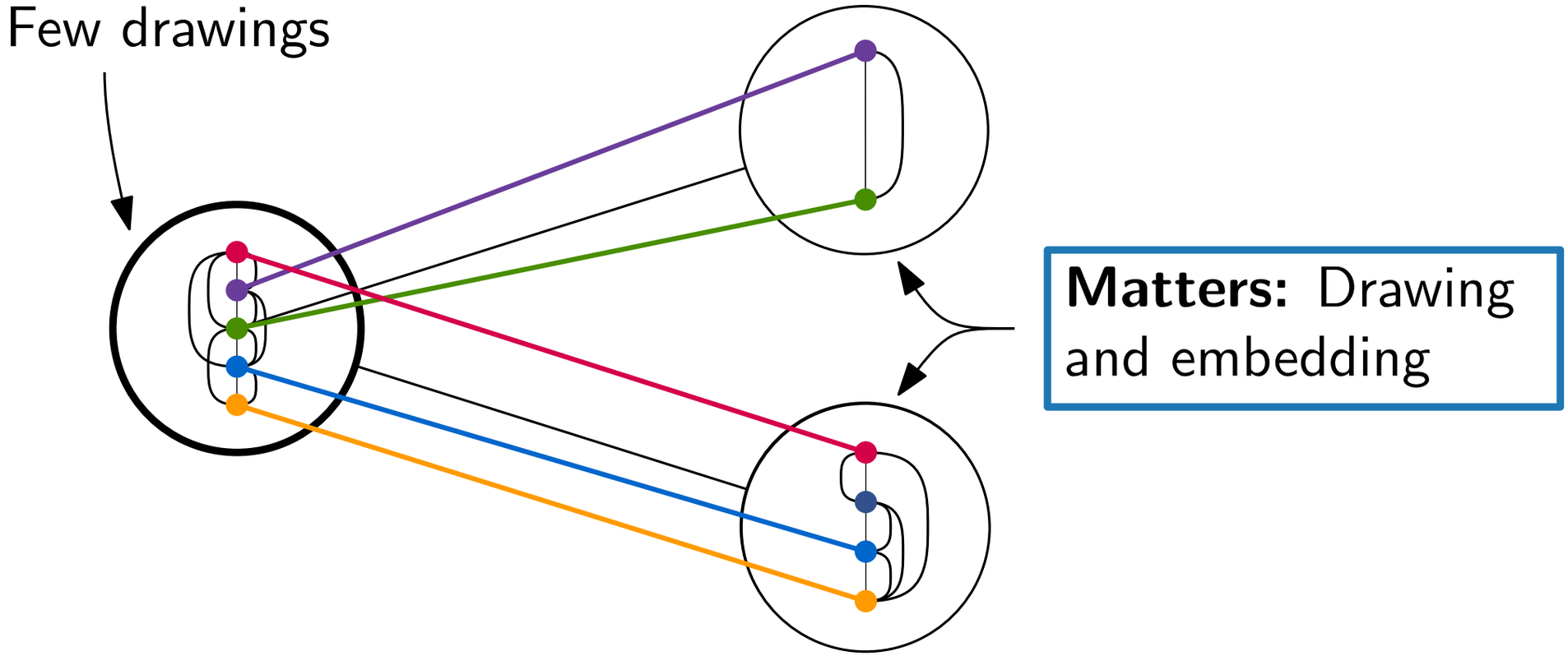

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