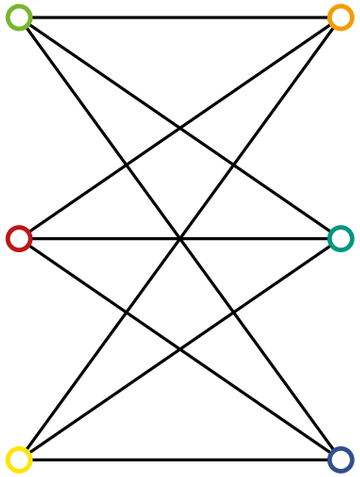
Geometry Matters in Planar Storyplans

Alexander Dobler, Maximilian Holzmüller, Martin Nöllenburg

September 24, 2025 · GD 2025

supported by WWTF

Planar Storyplan



Planar Storyplan

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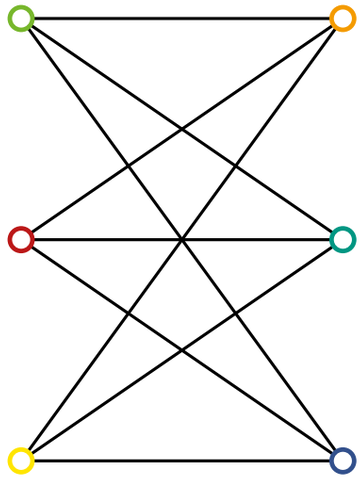

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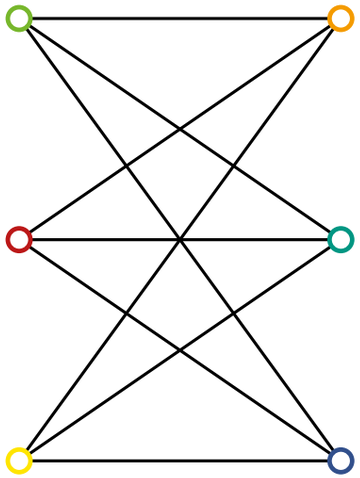

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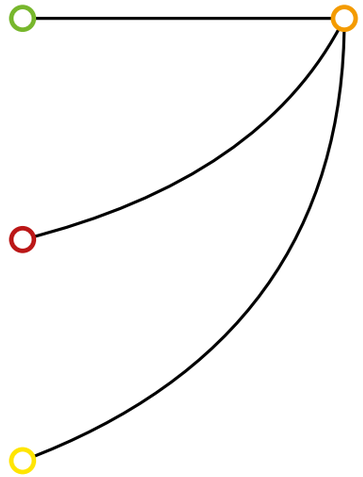
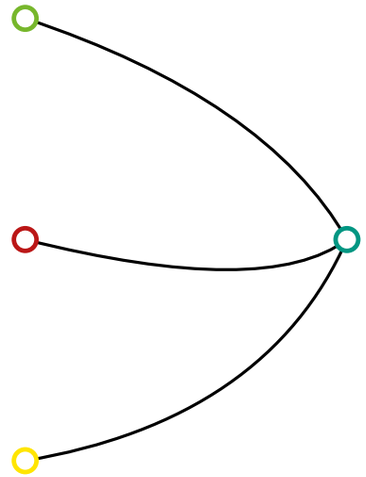
$\Rightarrow$ Split it into a **sequence of frames**

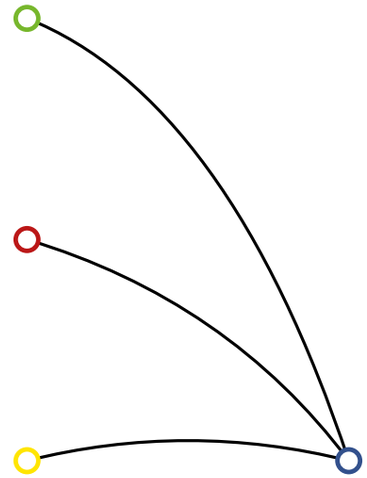
Γ_1



Γ_2



Γ_3



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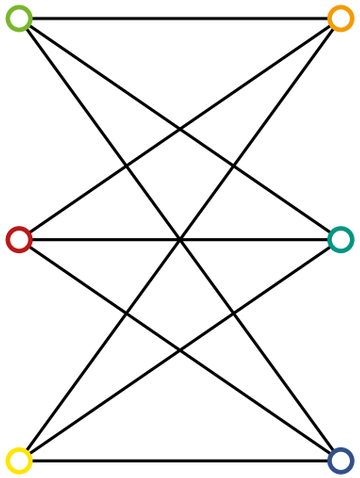

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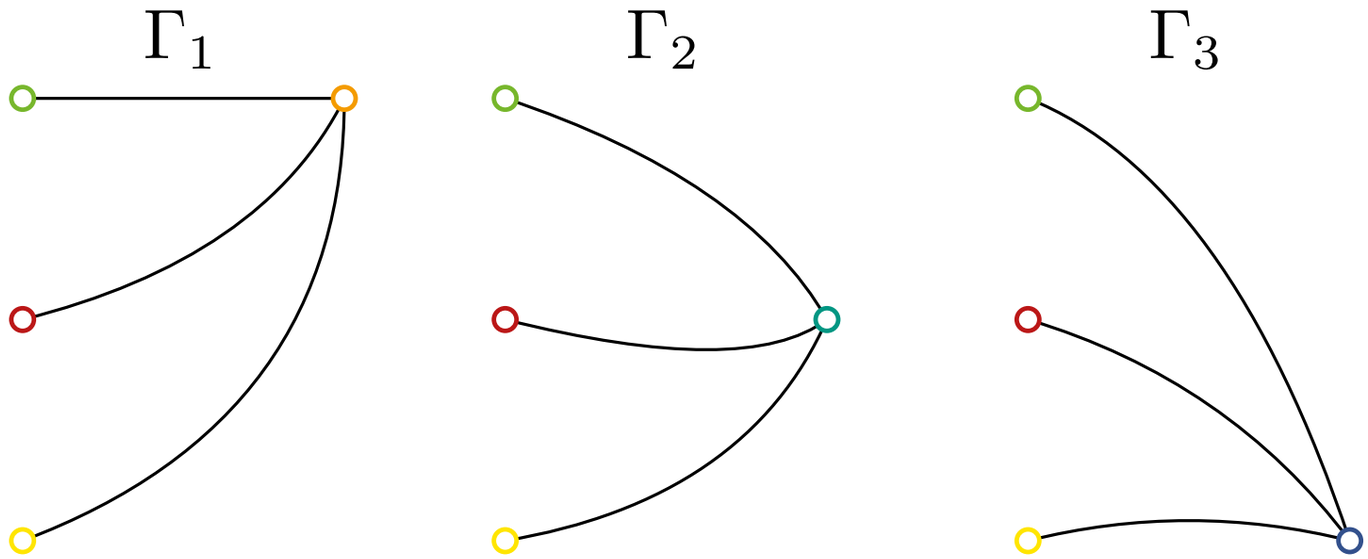

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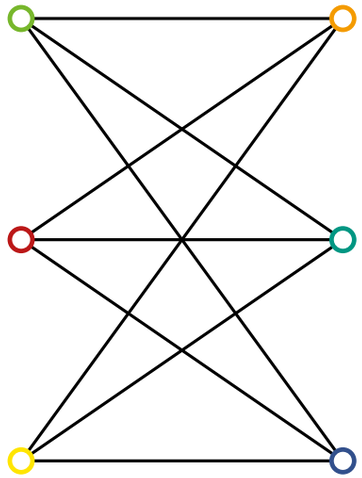

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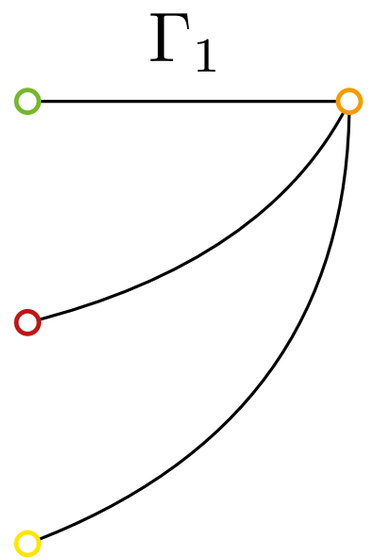

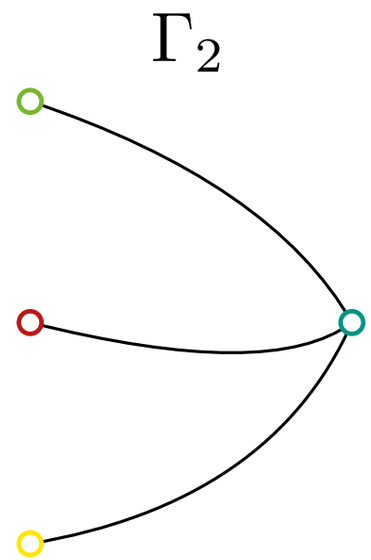

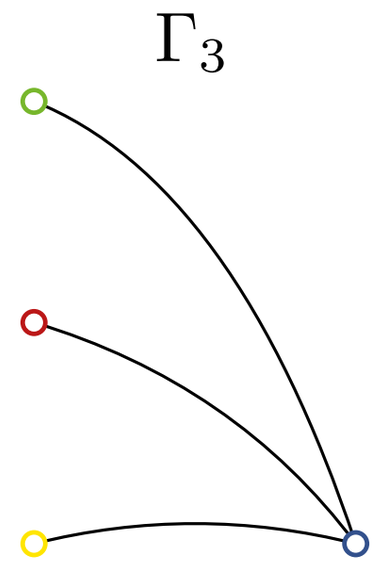

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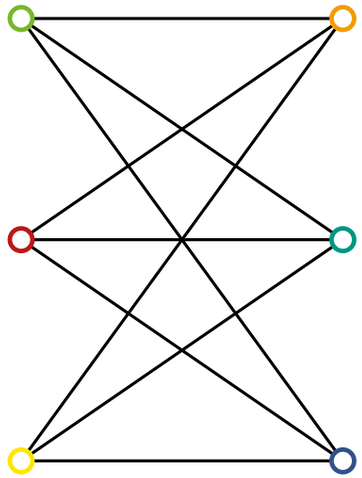

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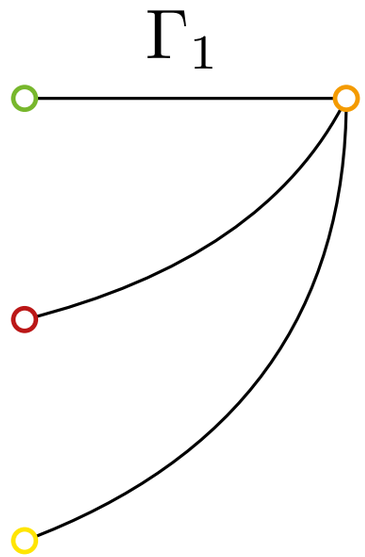

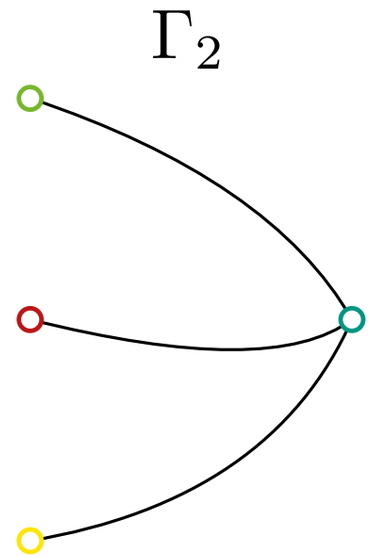

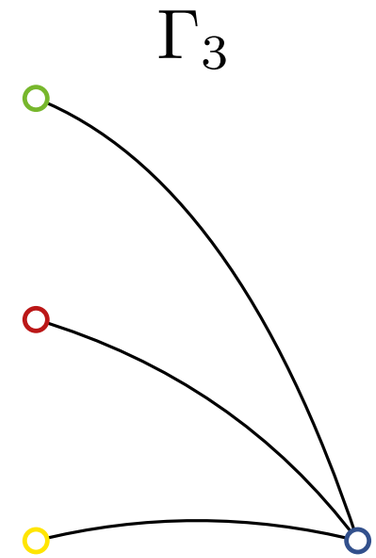

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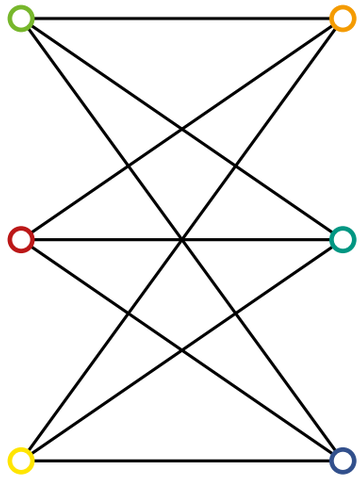

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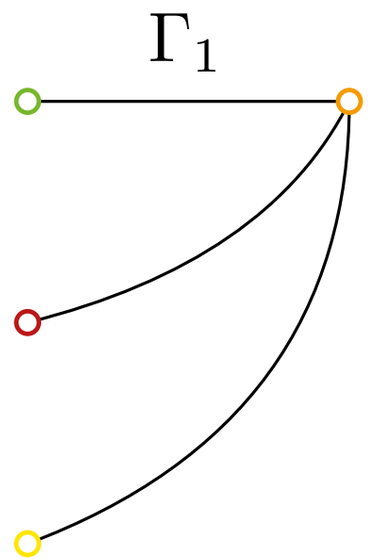

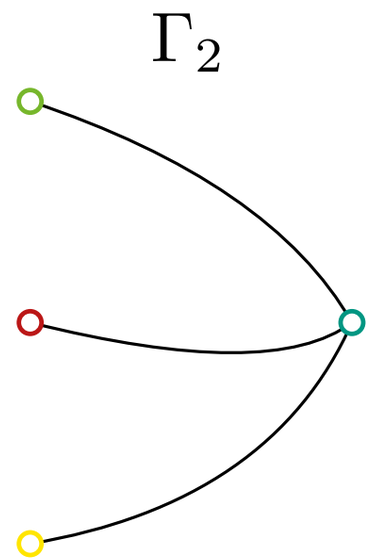

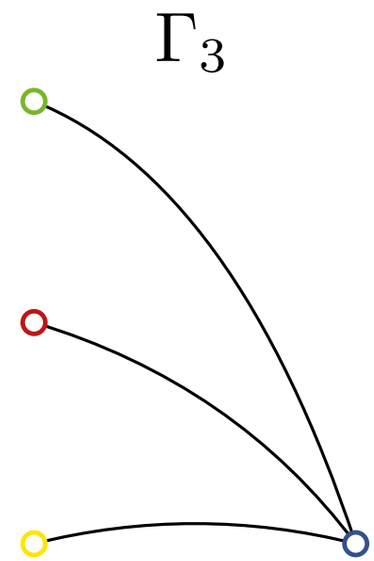

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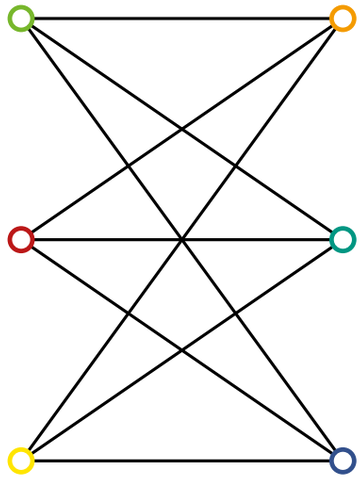

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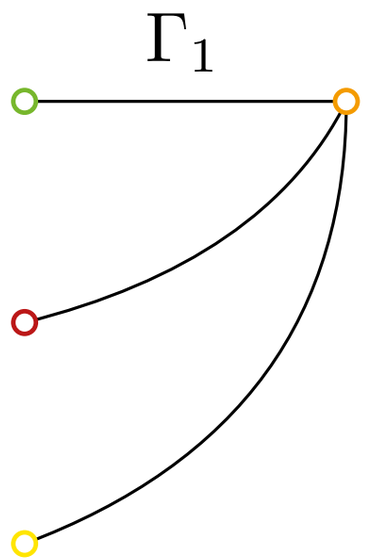

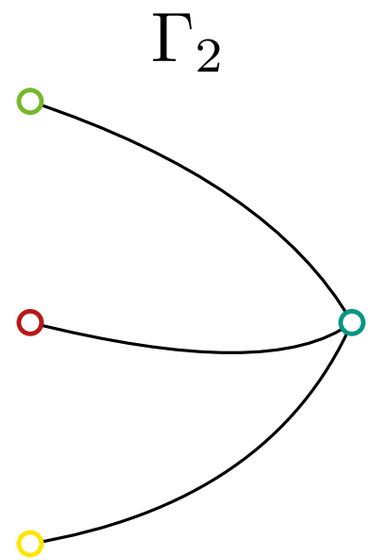

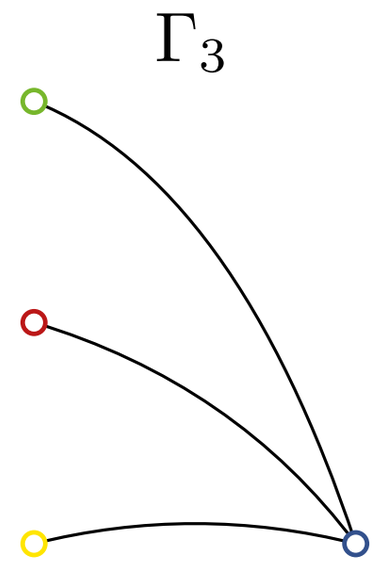

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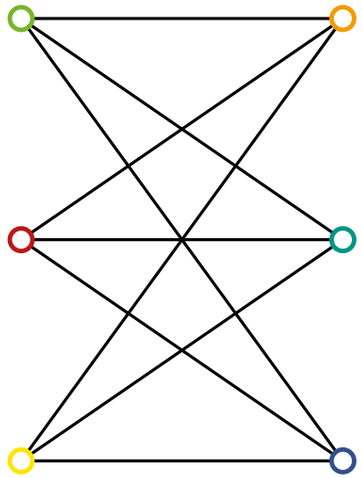

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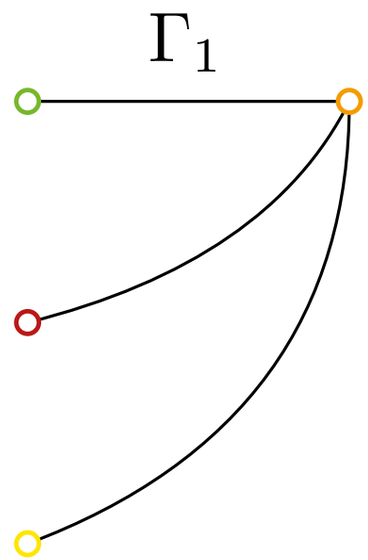

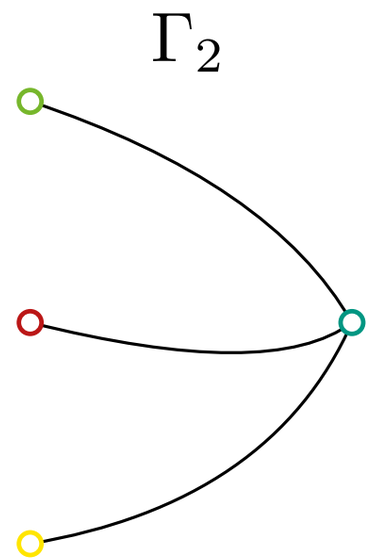

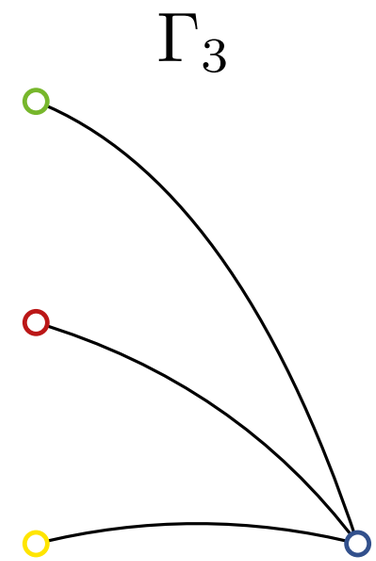

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planar storyplan $\Rightarrow$ each frame is (topological) planar

Known Results and Our Contribution

Planar Storyplan Problem: Given G, does G have a planar storyplan?

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[Binucci et al., GD22]:

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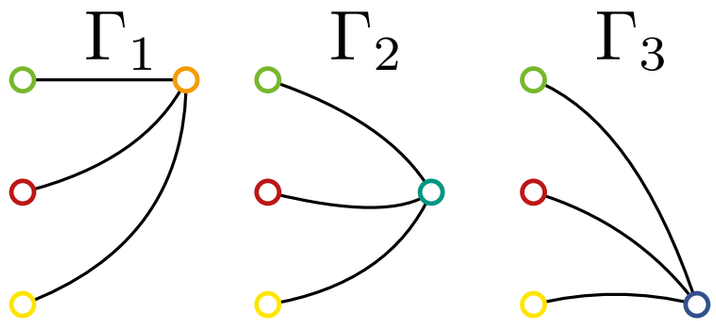

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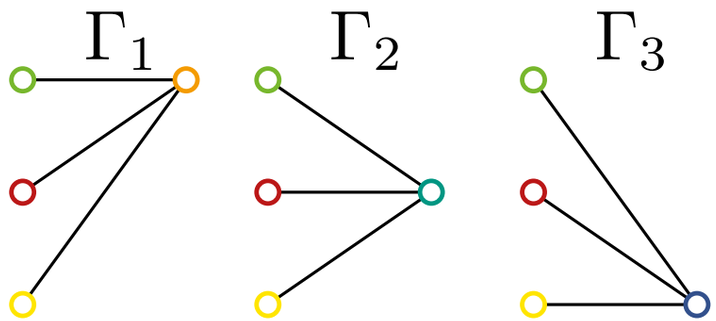

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k-planar $\not\approx$ geometric k-planar

thickness $\not\approx$ geometric thickness

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Theorem 1. There is a graph G which has a planar but no planar geometric storyplan.

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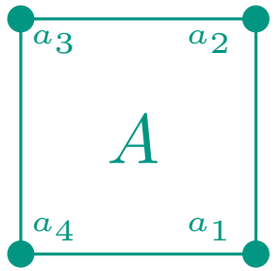

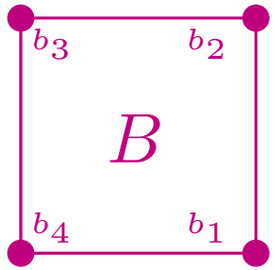

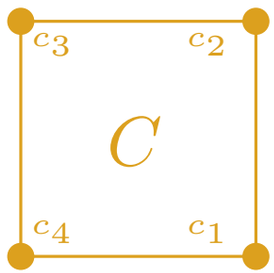

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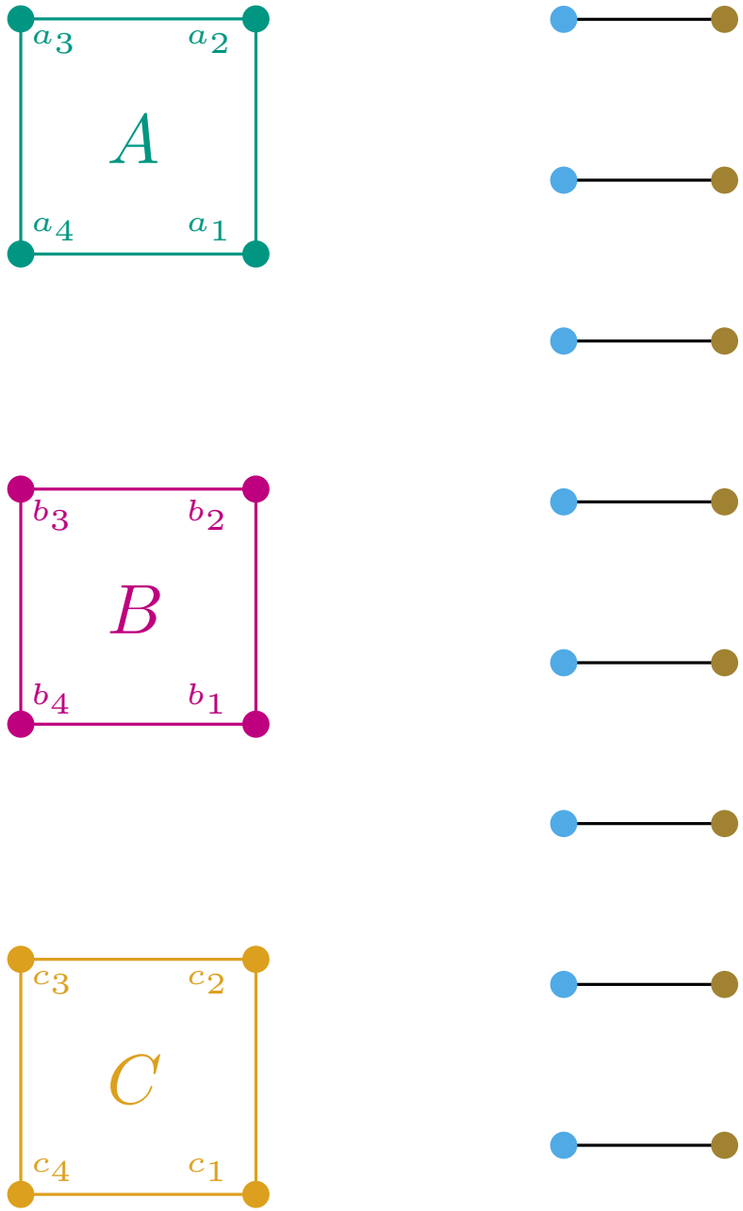

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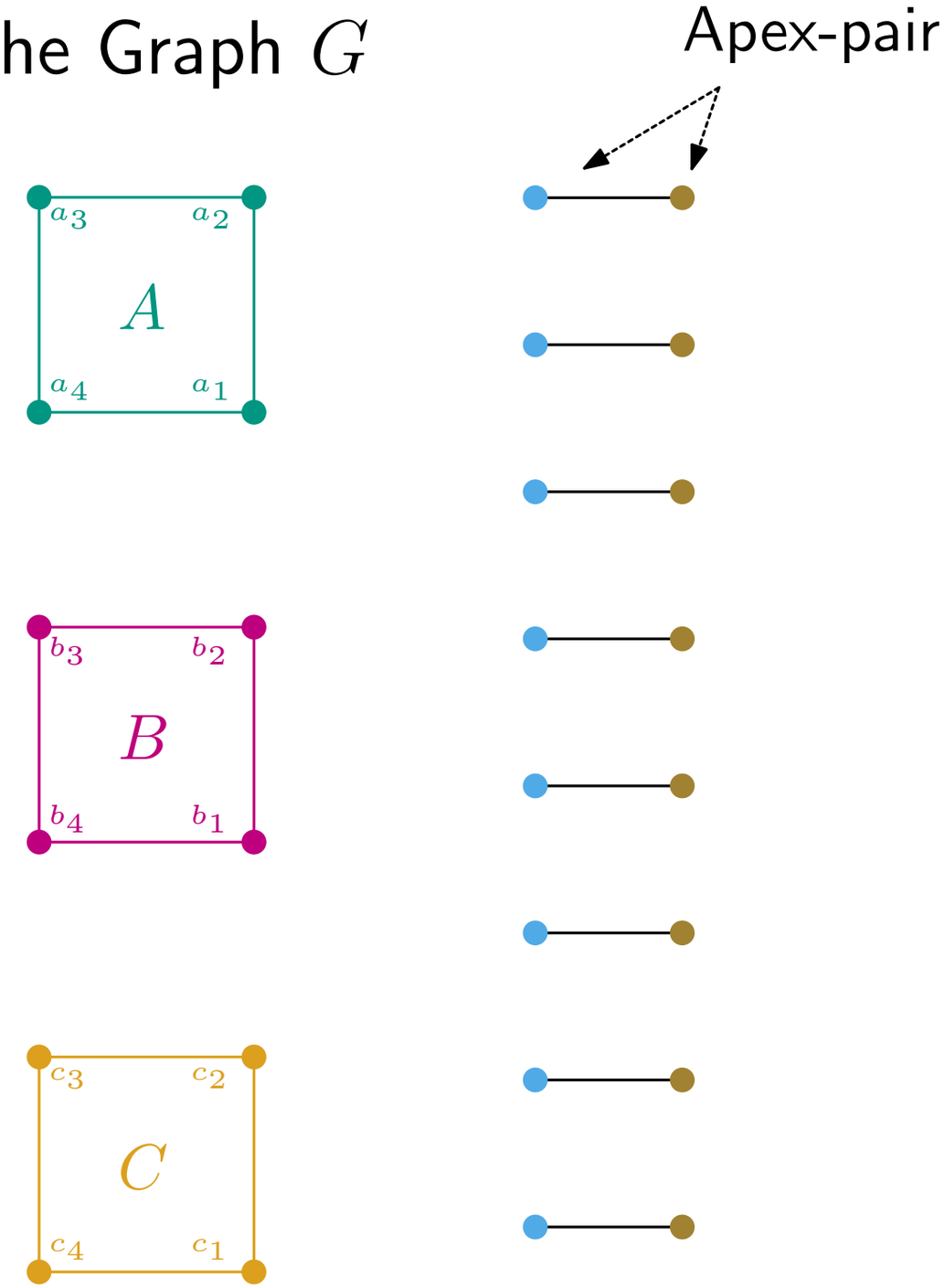

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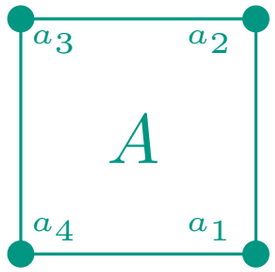
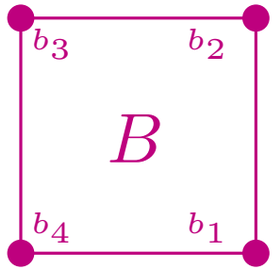
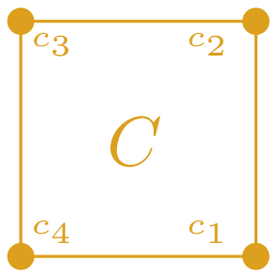
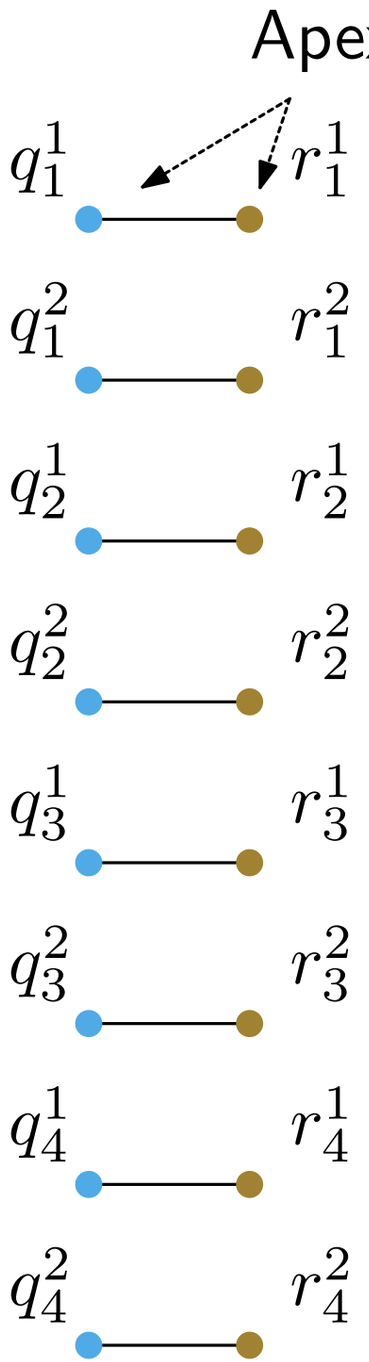

a_3
a_2
A
a_4
a_1


b_3
b_2
B
b_4
b_1


c_3
c_2
C
c_4
c_1


Apex-pair
q_1^1
r_1^1
q_1^2
r_1^2
q_2^1
r_2^1
q_2^2
r_2^2
q_3^1
r_3^1
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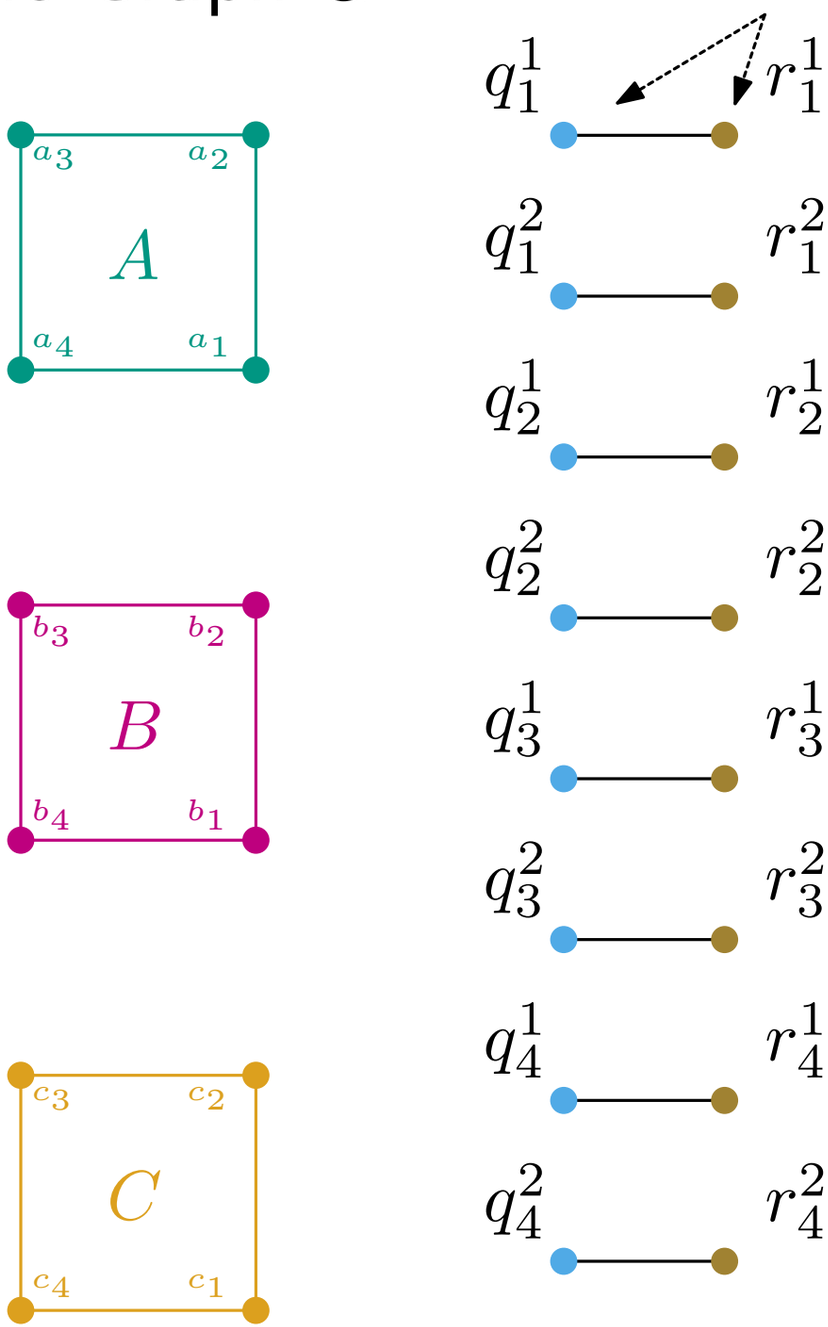

Each q_i^j connected with all four-cycle vertices.

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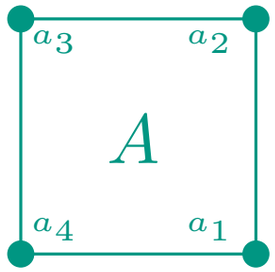

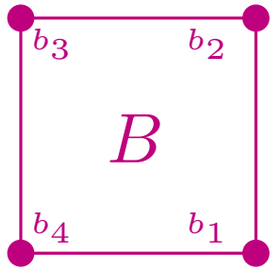

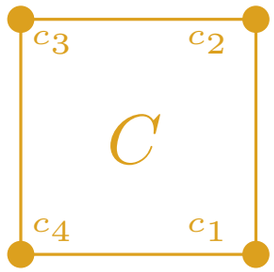

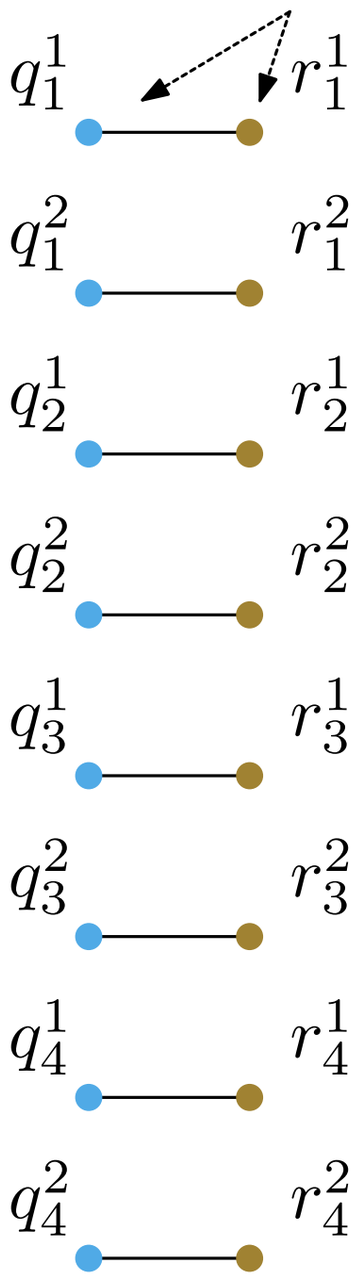

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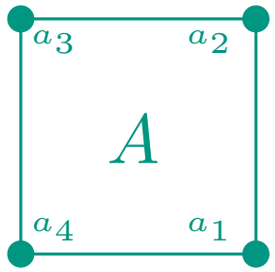

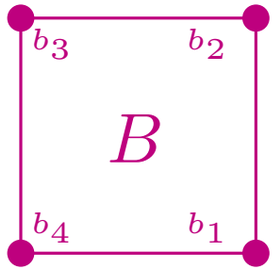

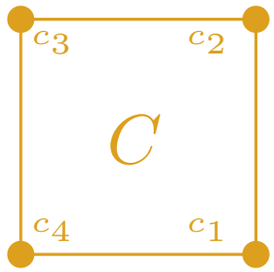

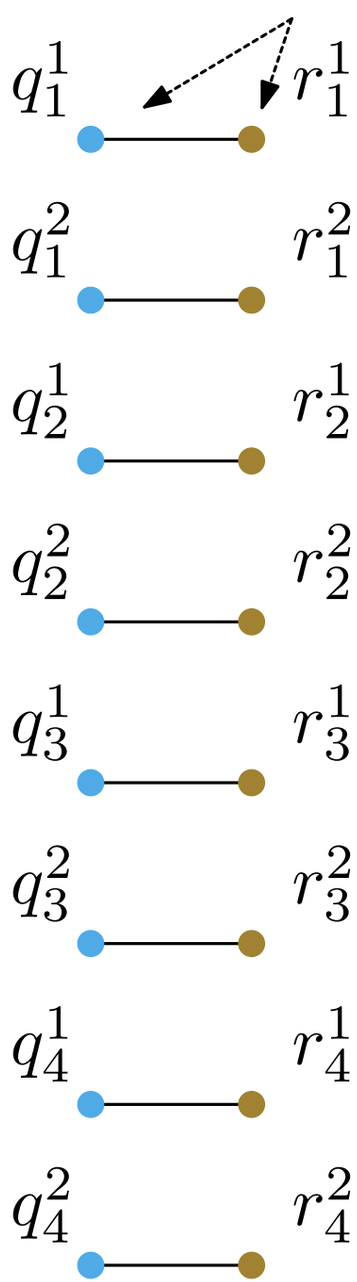

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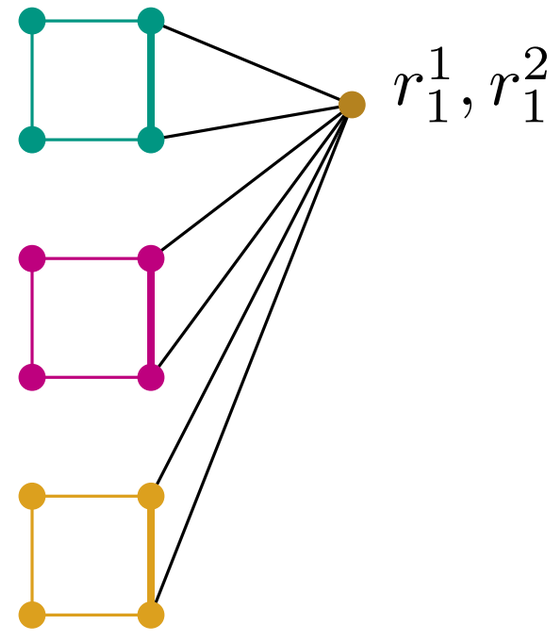

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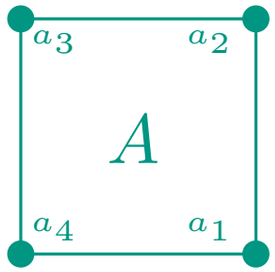

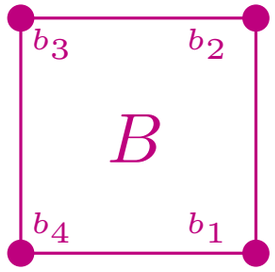

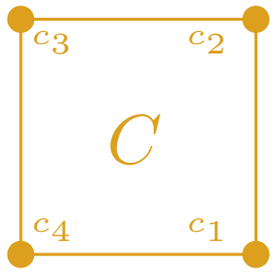

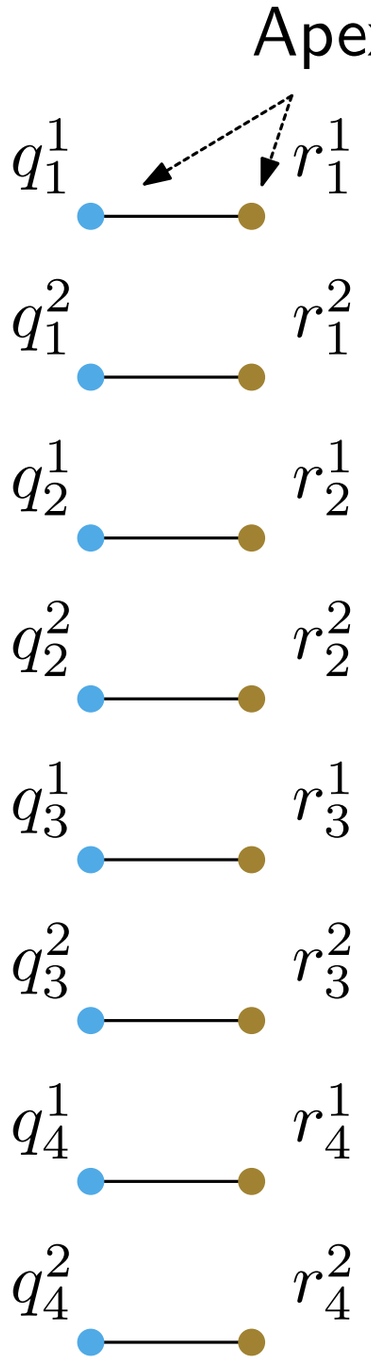

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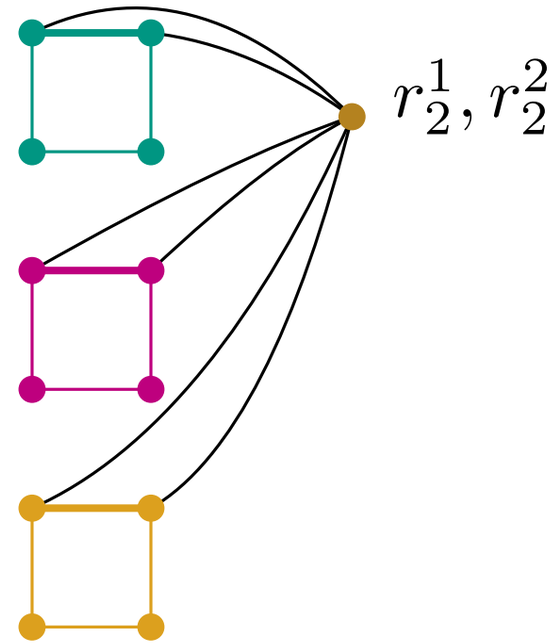

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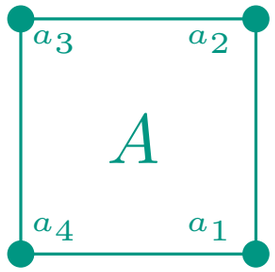

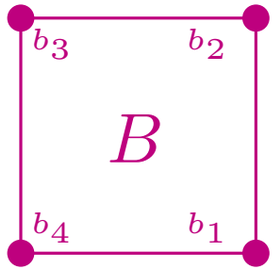

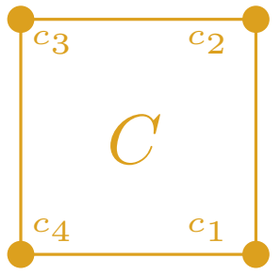

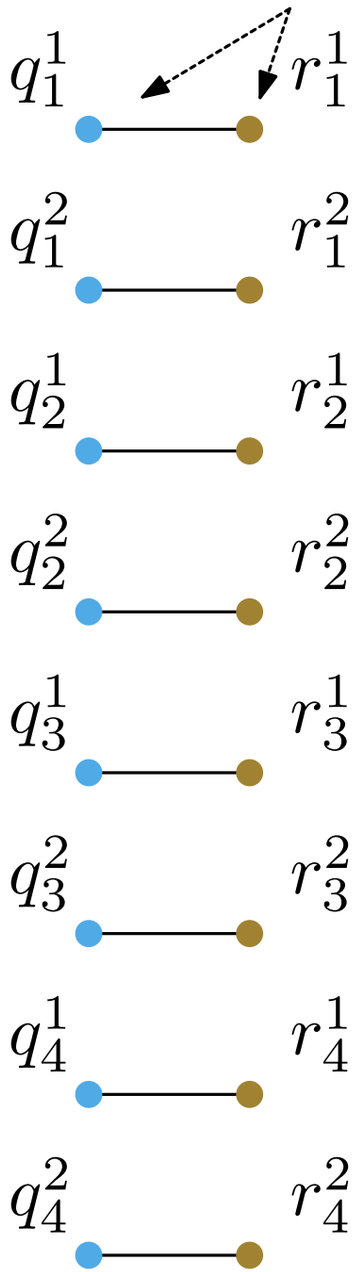

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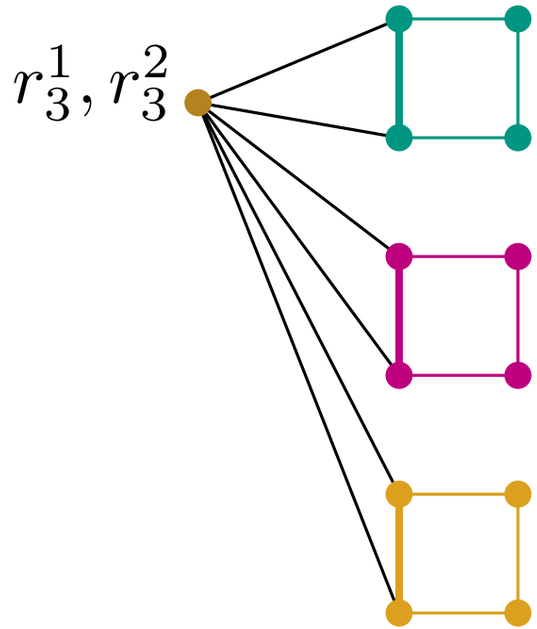

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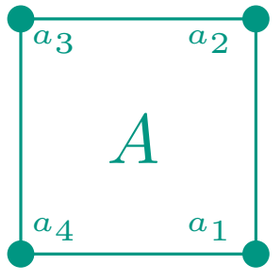

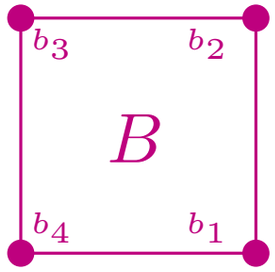

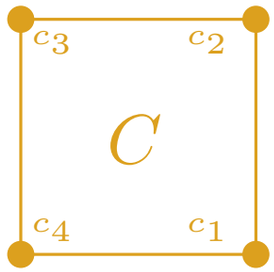

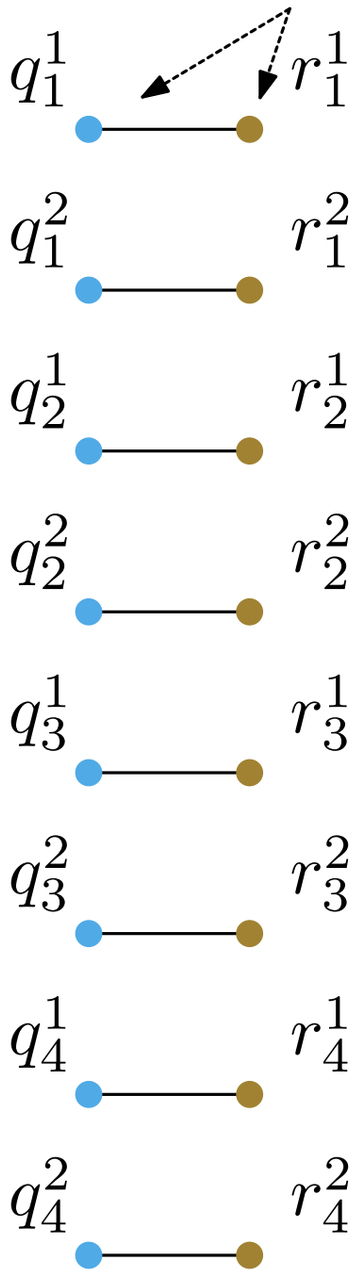

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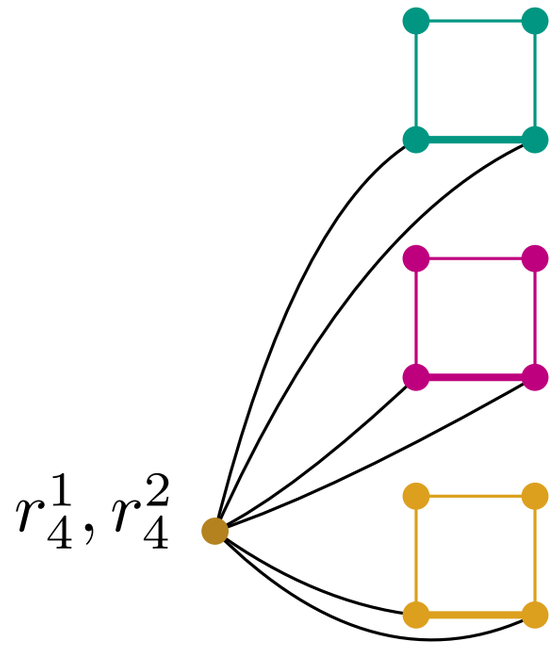
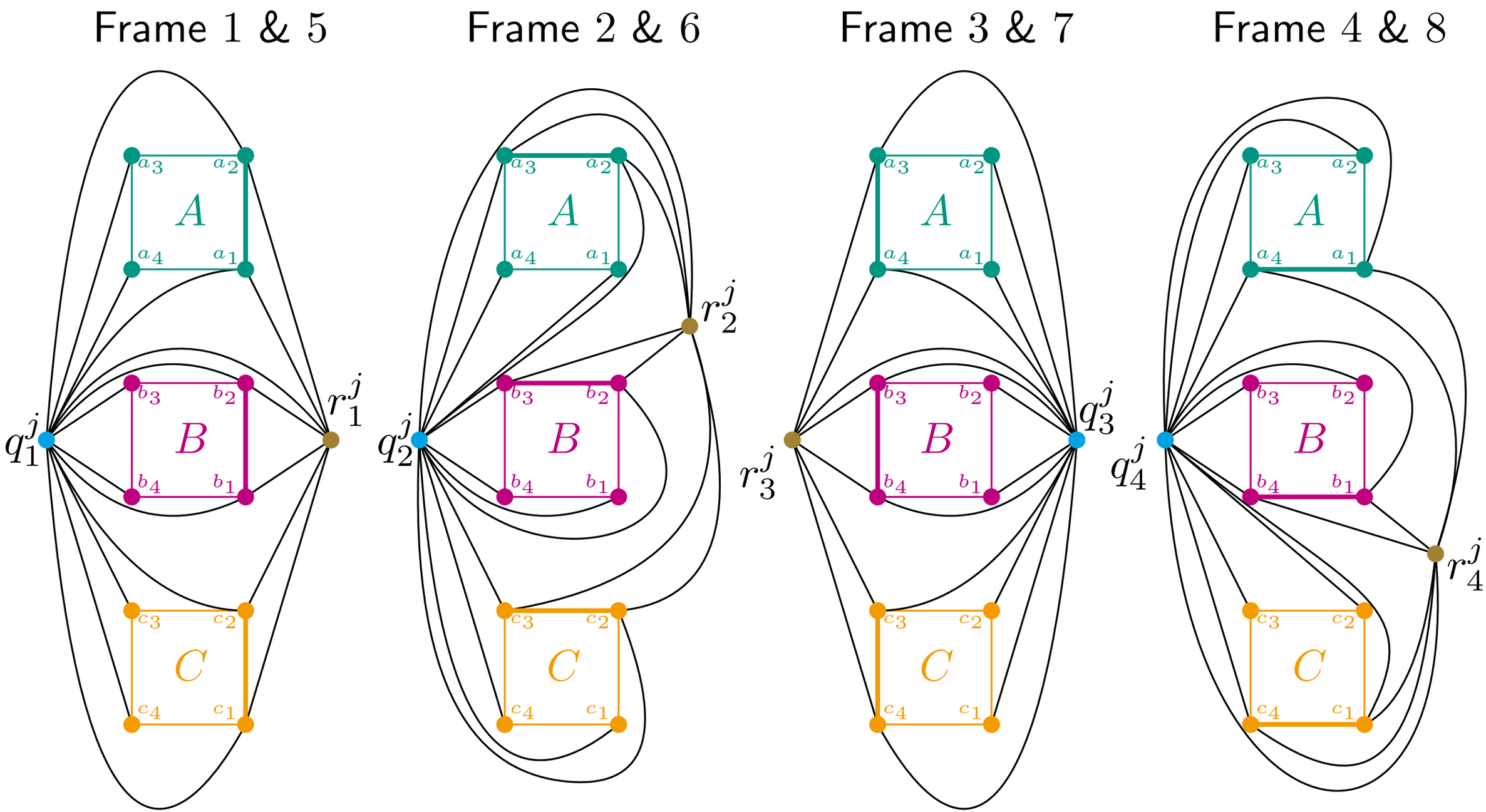


G Has a Planar Topological Storyplan



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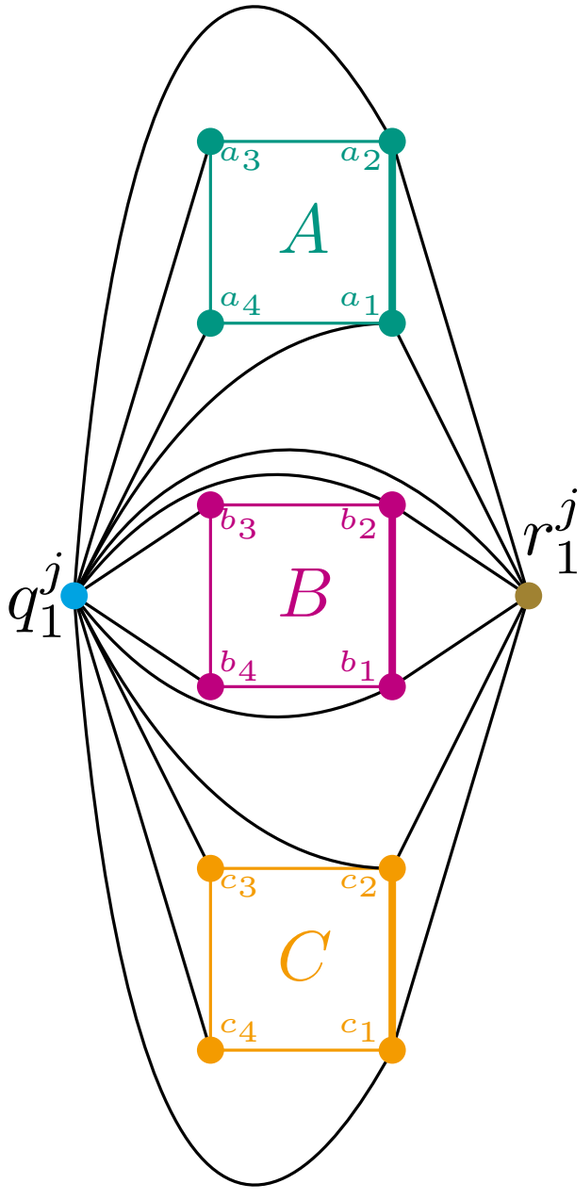
Frame 1 & 5



Frame 2 & 6

Frame 3 & 7

Frame 4 & 8

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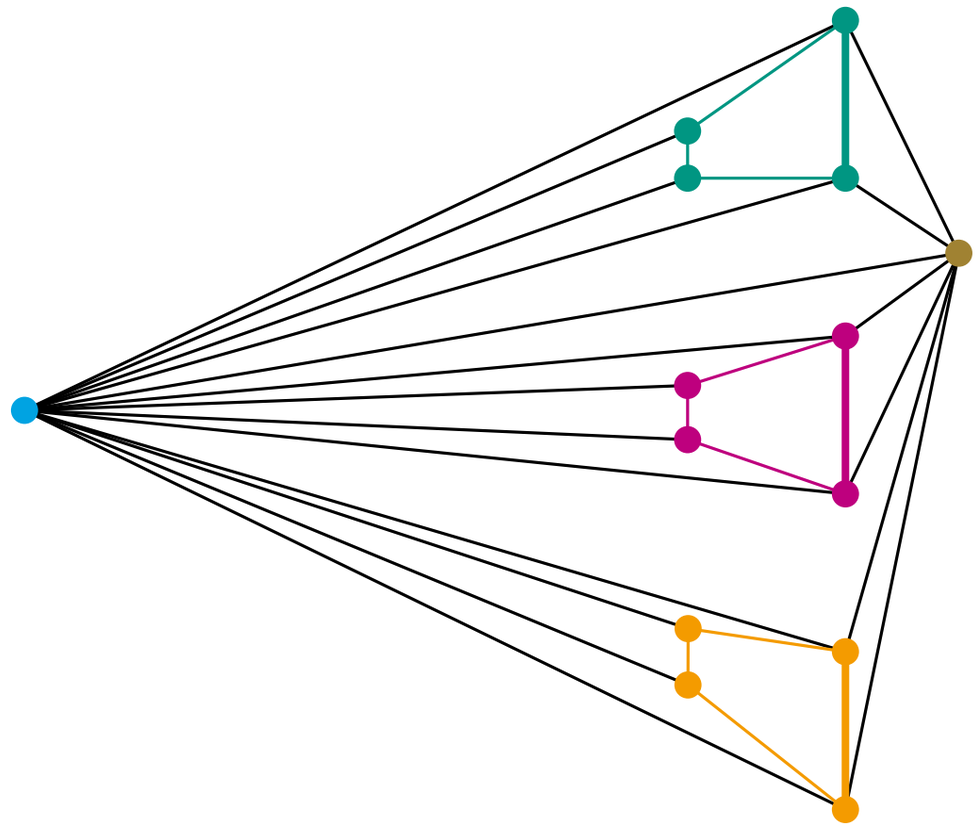

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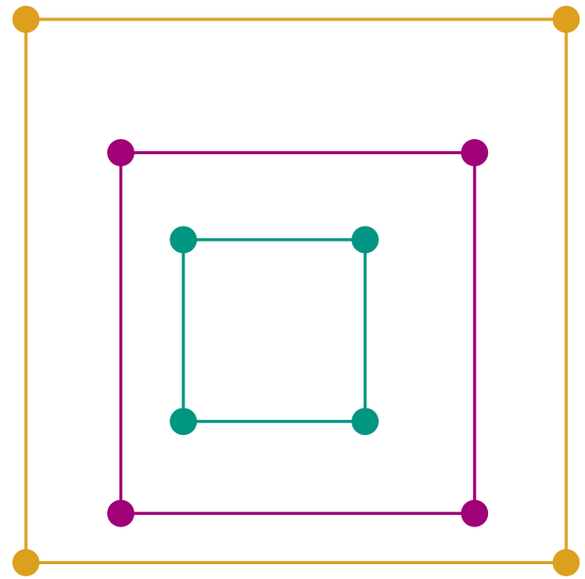

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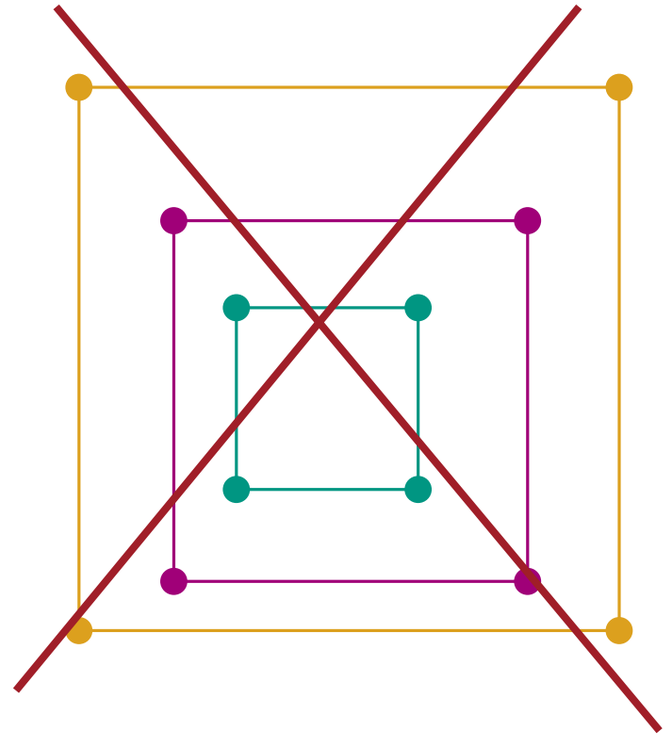

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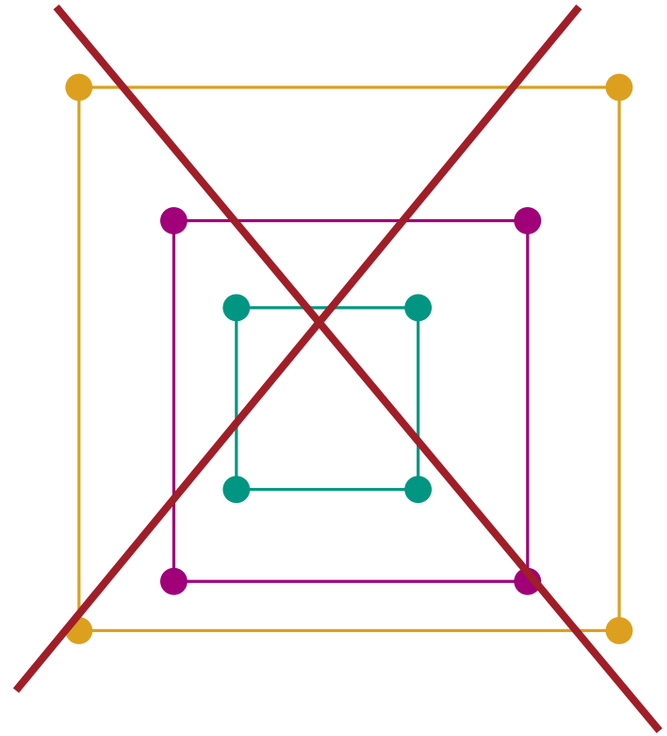

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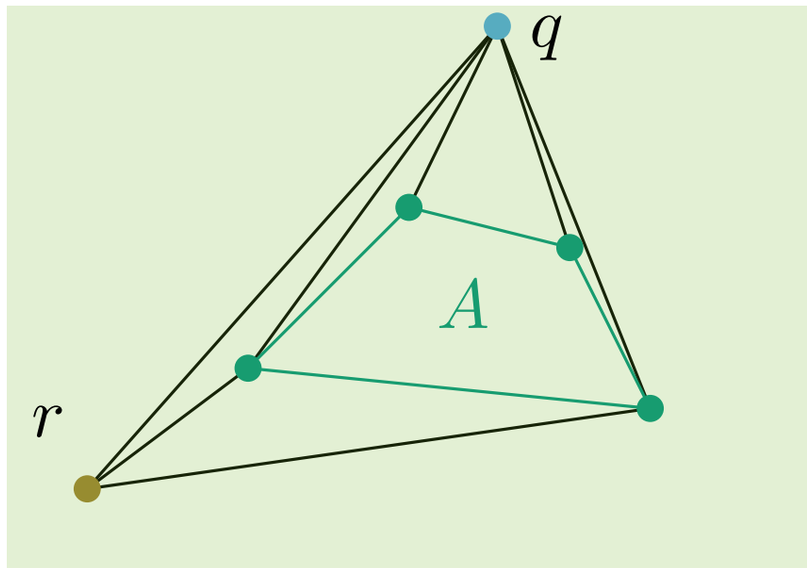

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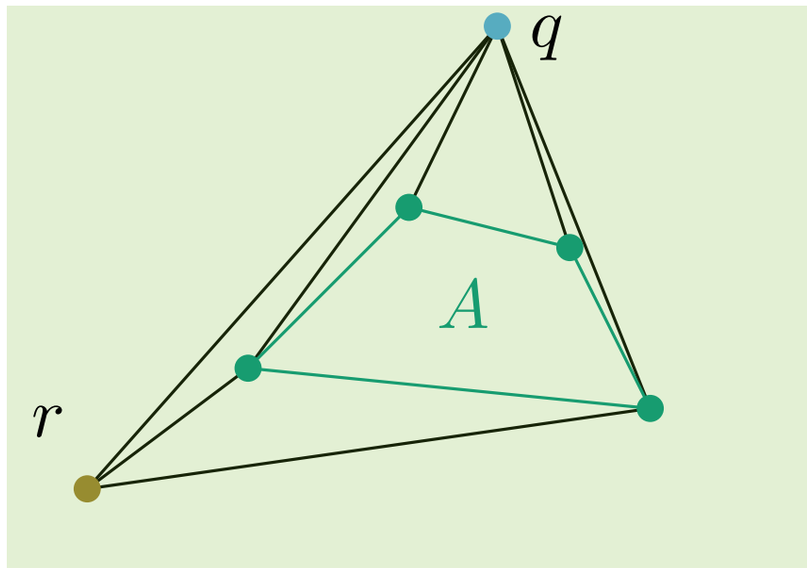

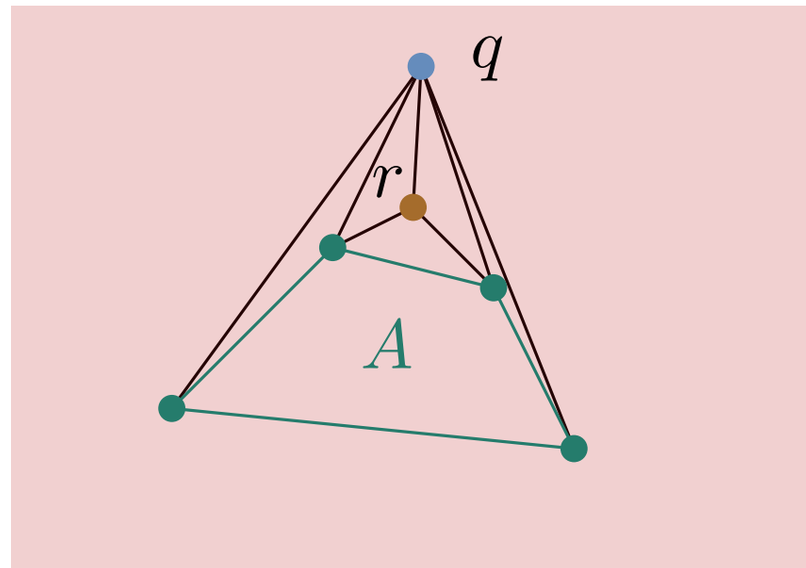

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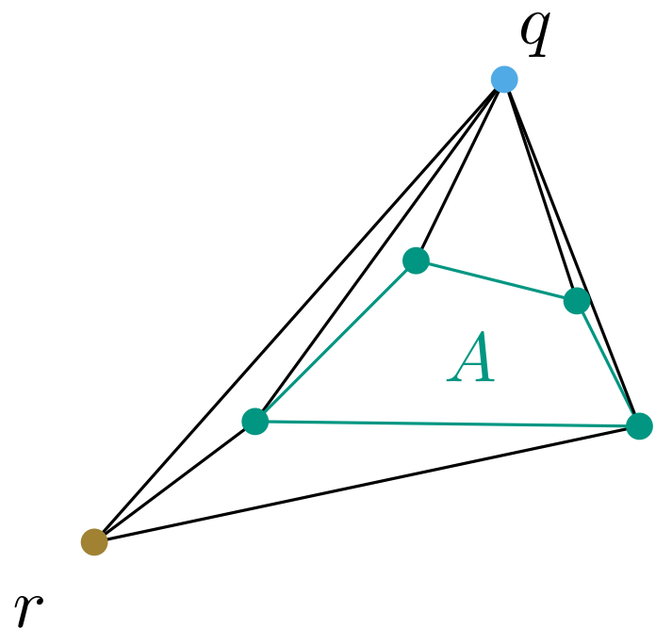

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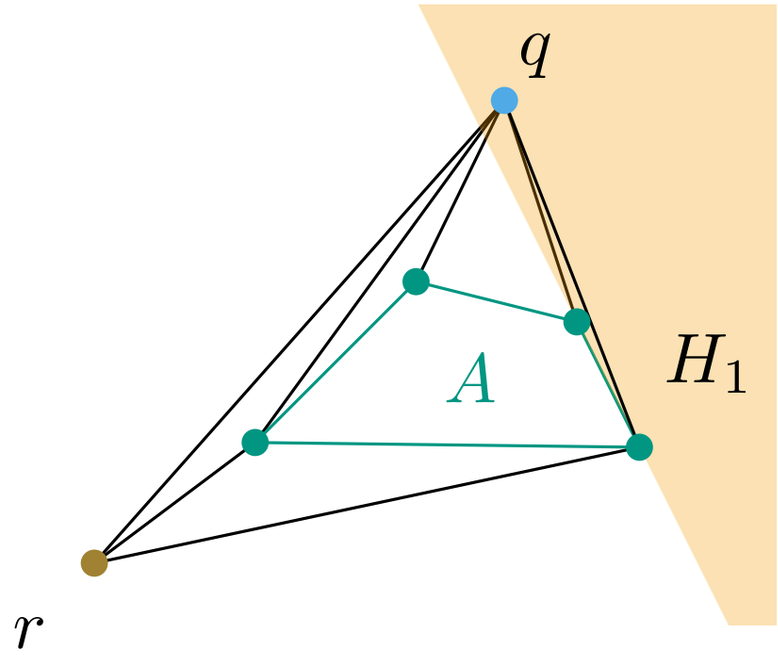

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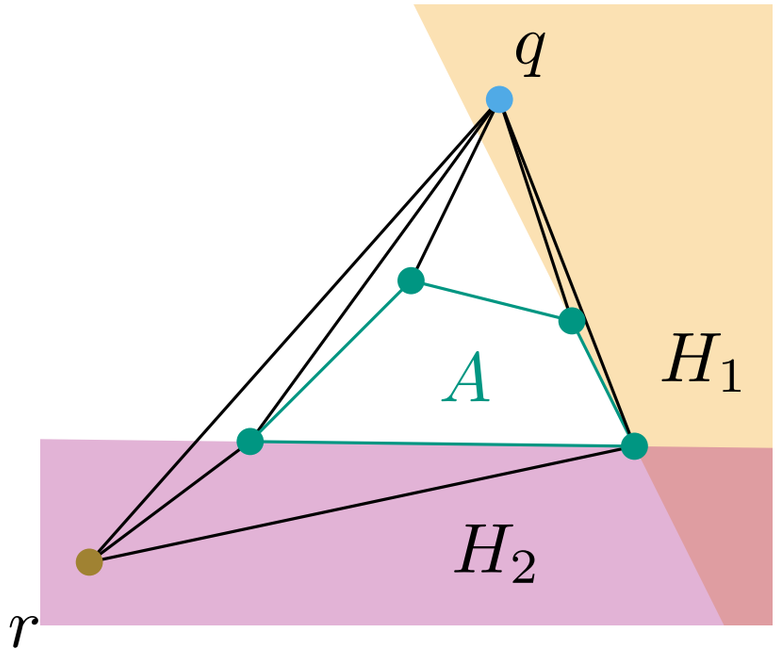

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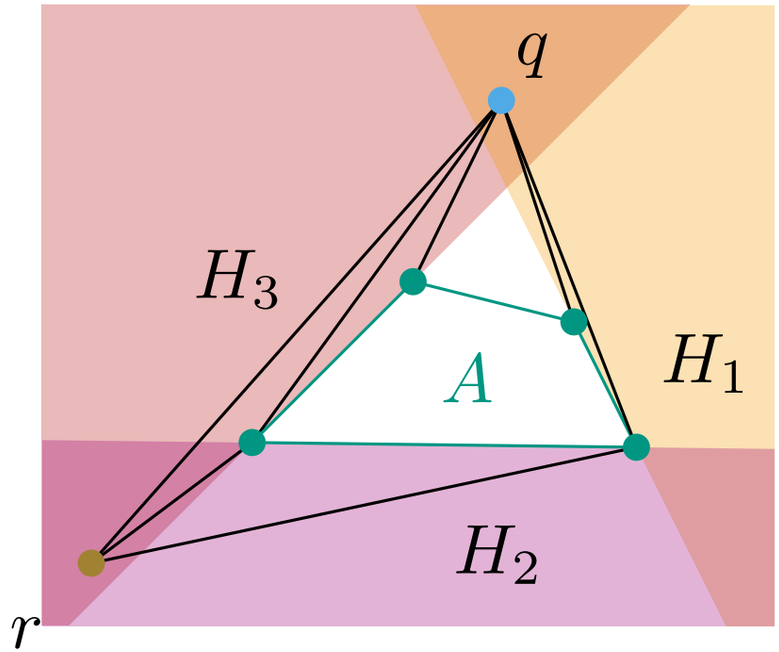

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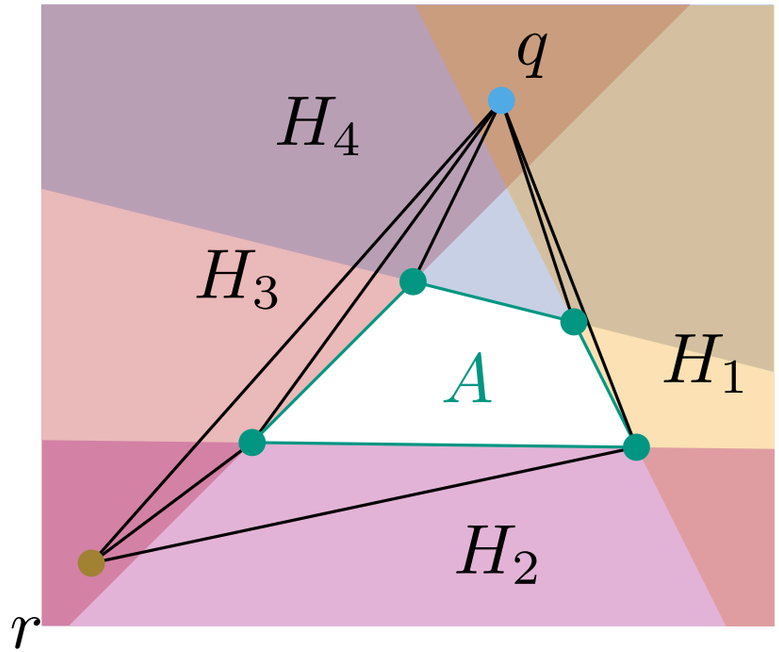

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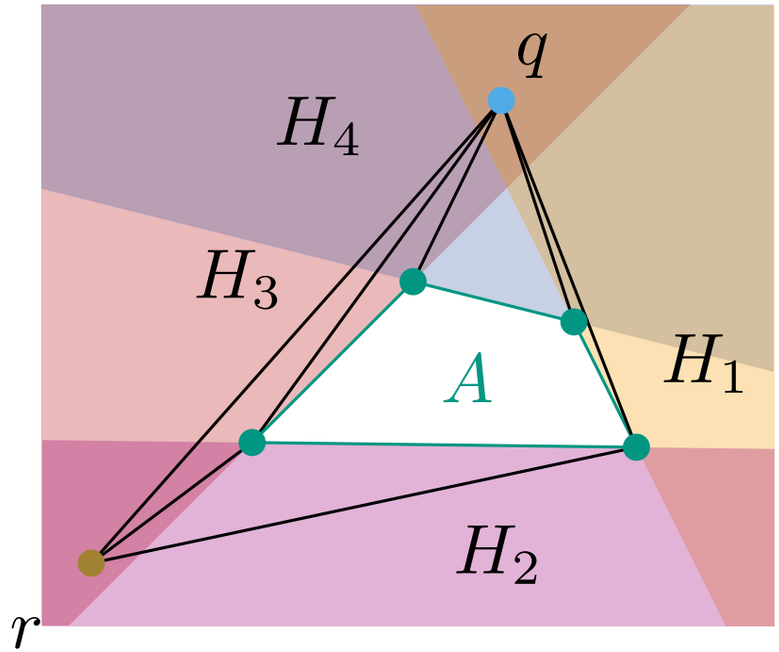

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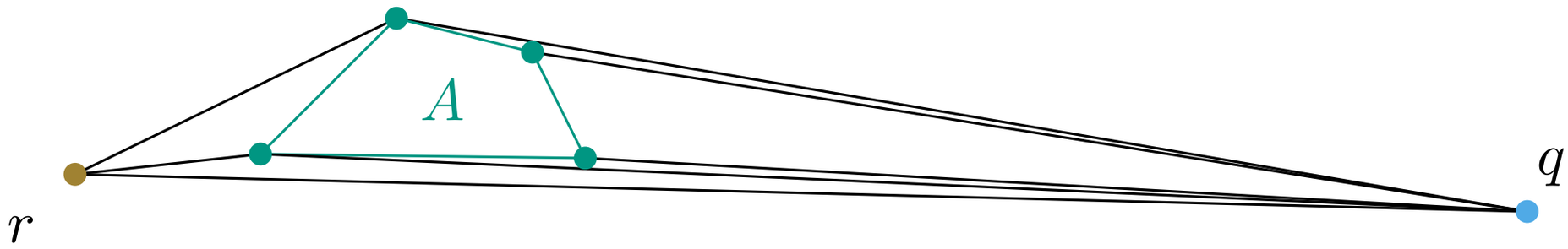

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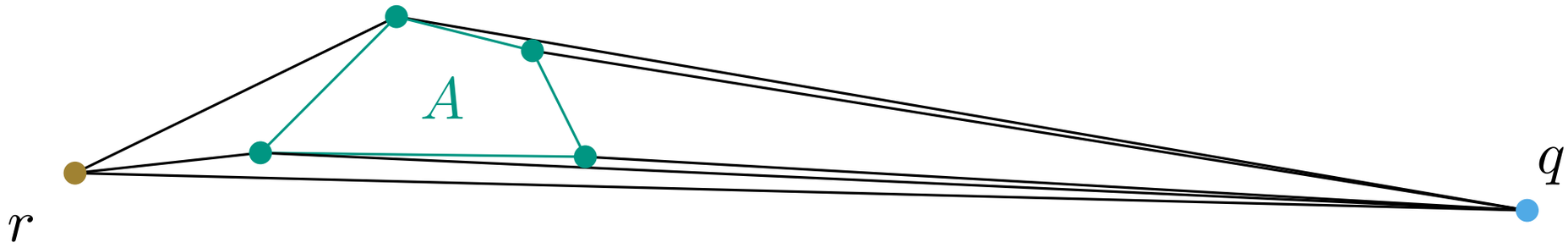

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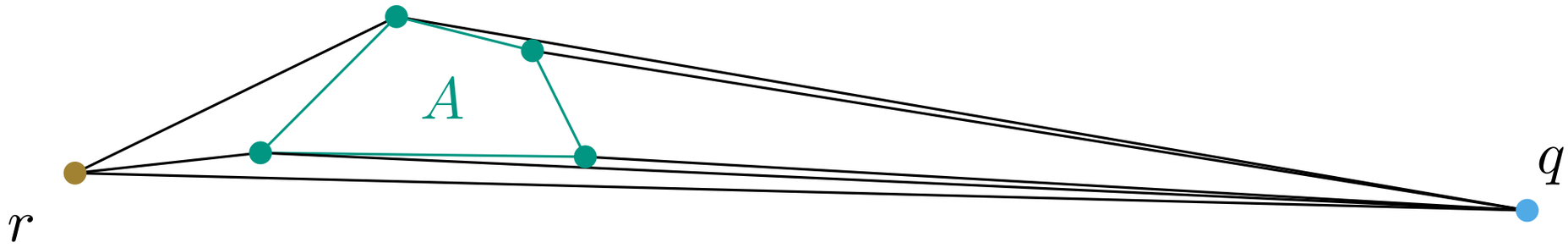

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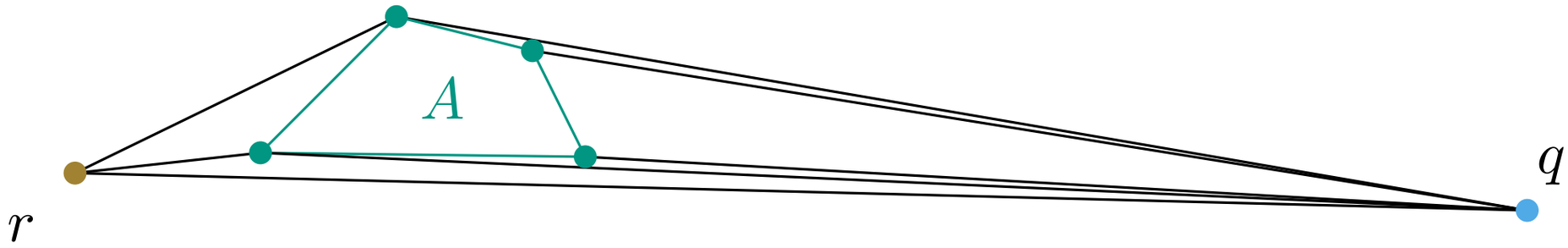

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Contradiction!

Outlook

$\exists$planar storyplan$(G) \rightarrow \exists$planar geometric storyplan(G)?

NO!

Computational complexity?

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