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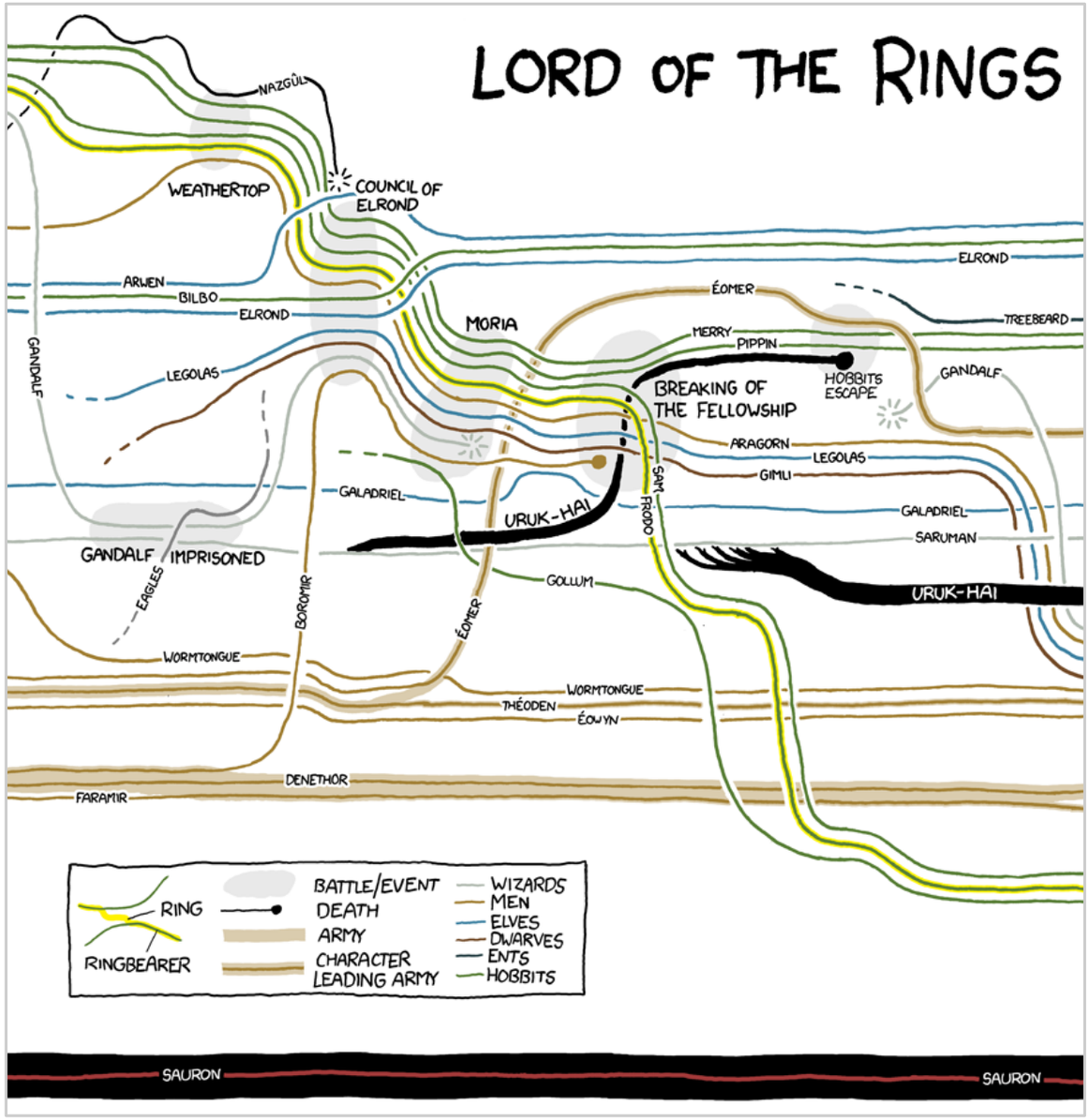
WÜRZBURG

Optimizing Wiggle in Storylines

Alexander Dobler **Tim Hegemann**
Martin Nöllenburg Alexander Wolff

The 33rd International Symposium on Graph Drawing and Network Visualization

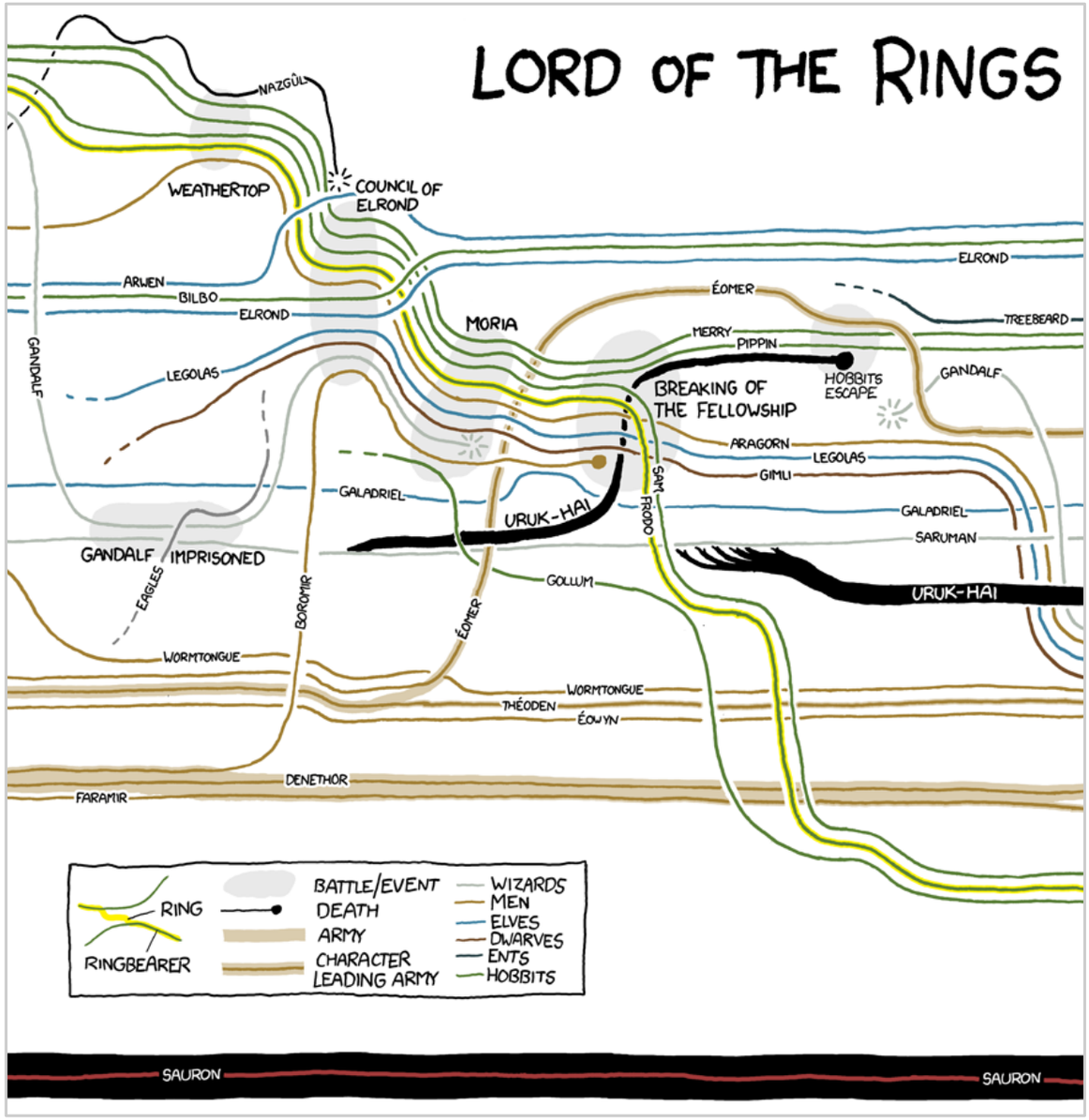
Storylines



[xkcd 657. Munroe. 2009 (clipping)]

Storylines

Time



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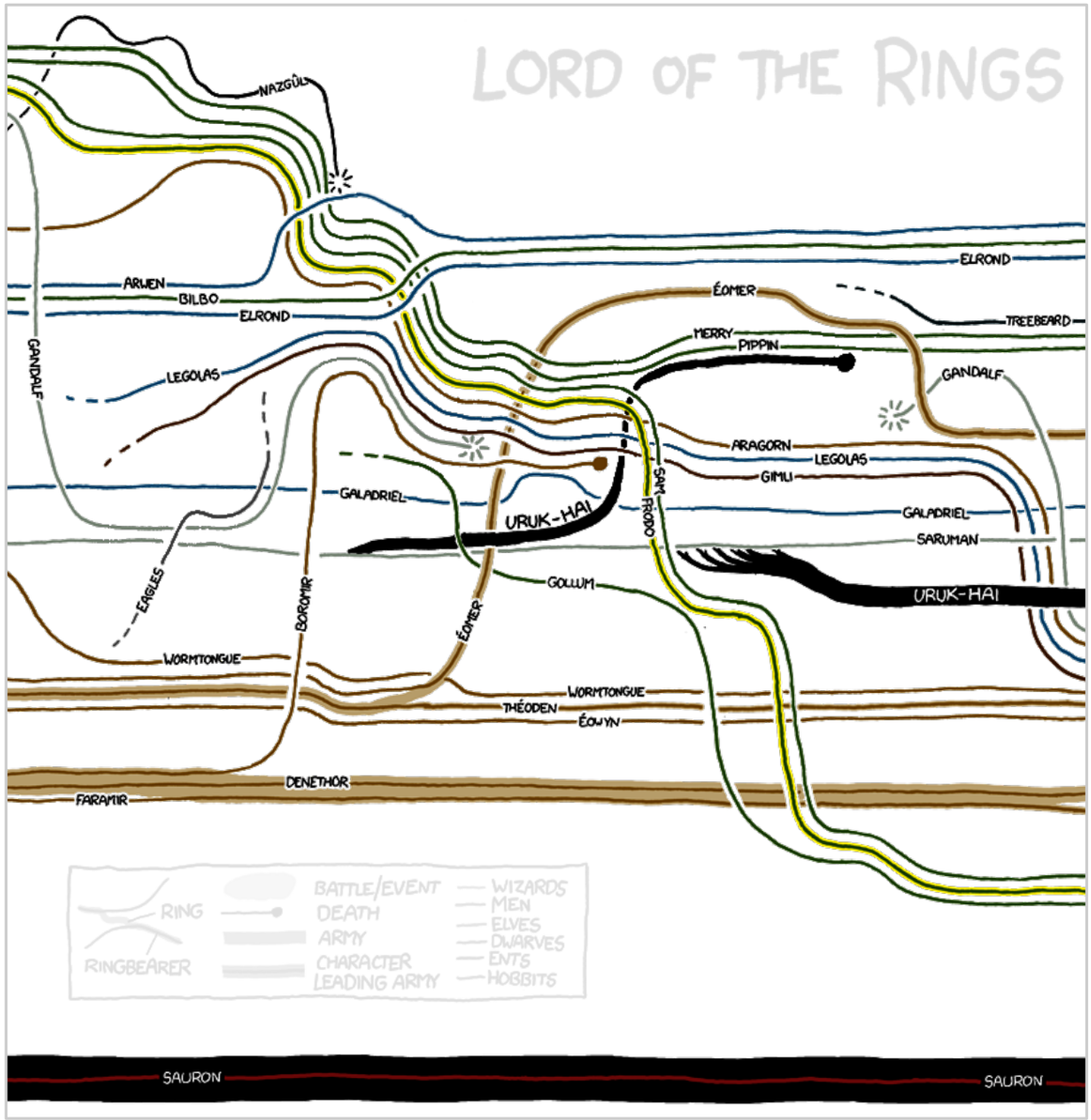
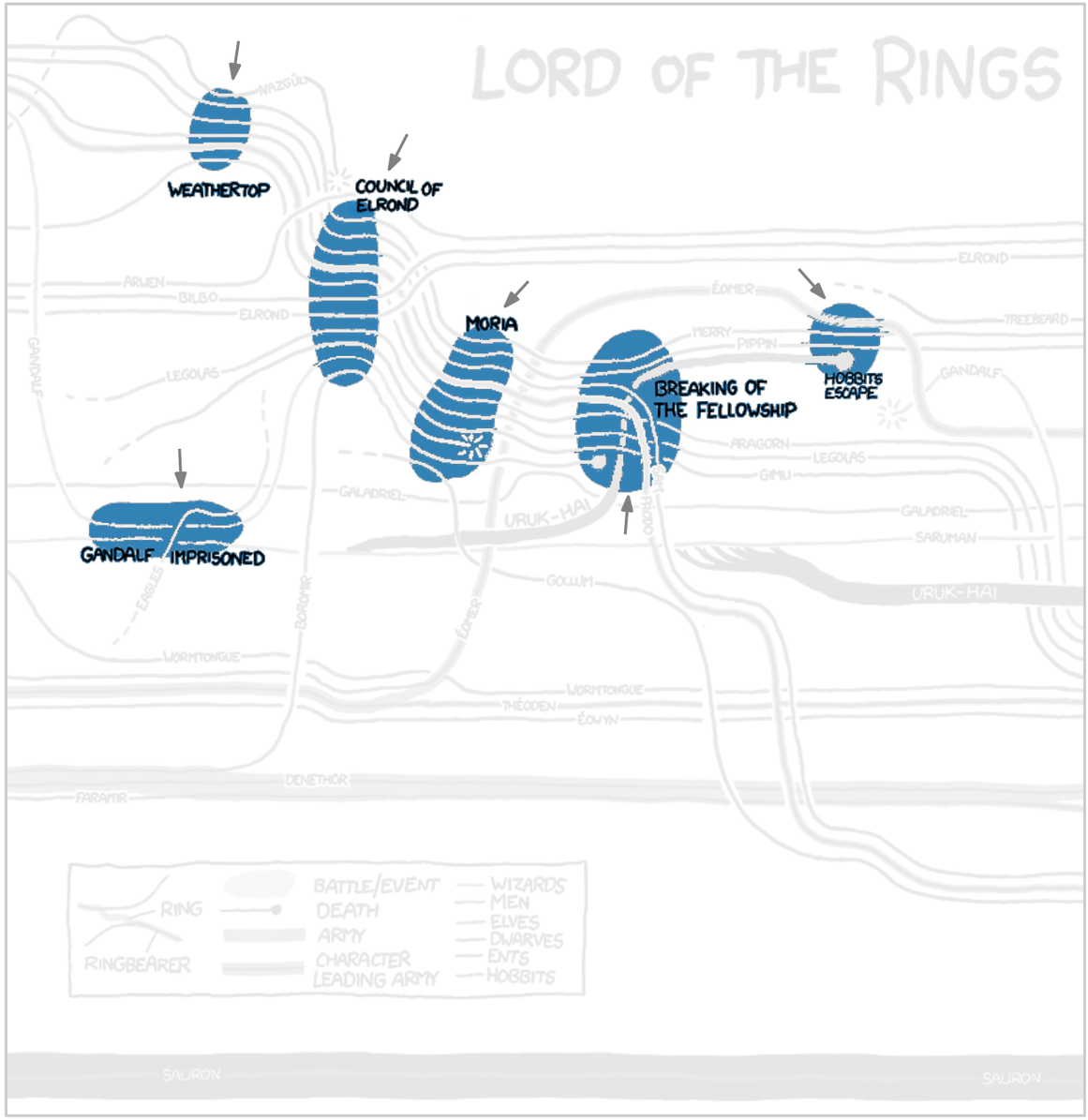


Characters $\mathcal{C}$

[xkcd 657. Munroe. 2009 (clipping)]

Storylines

Time



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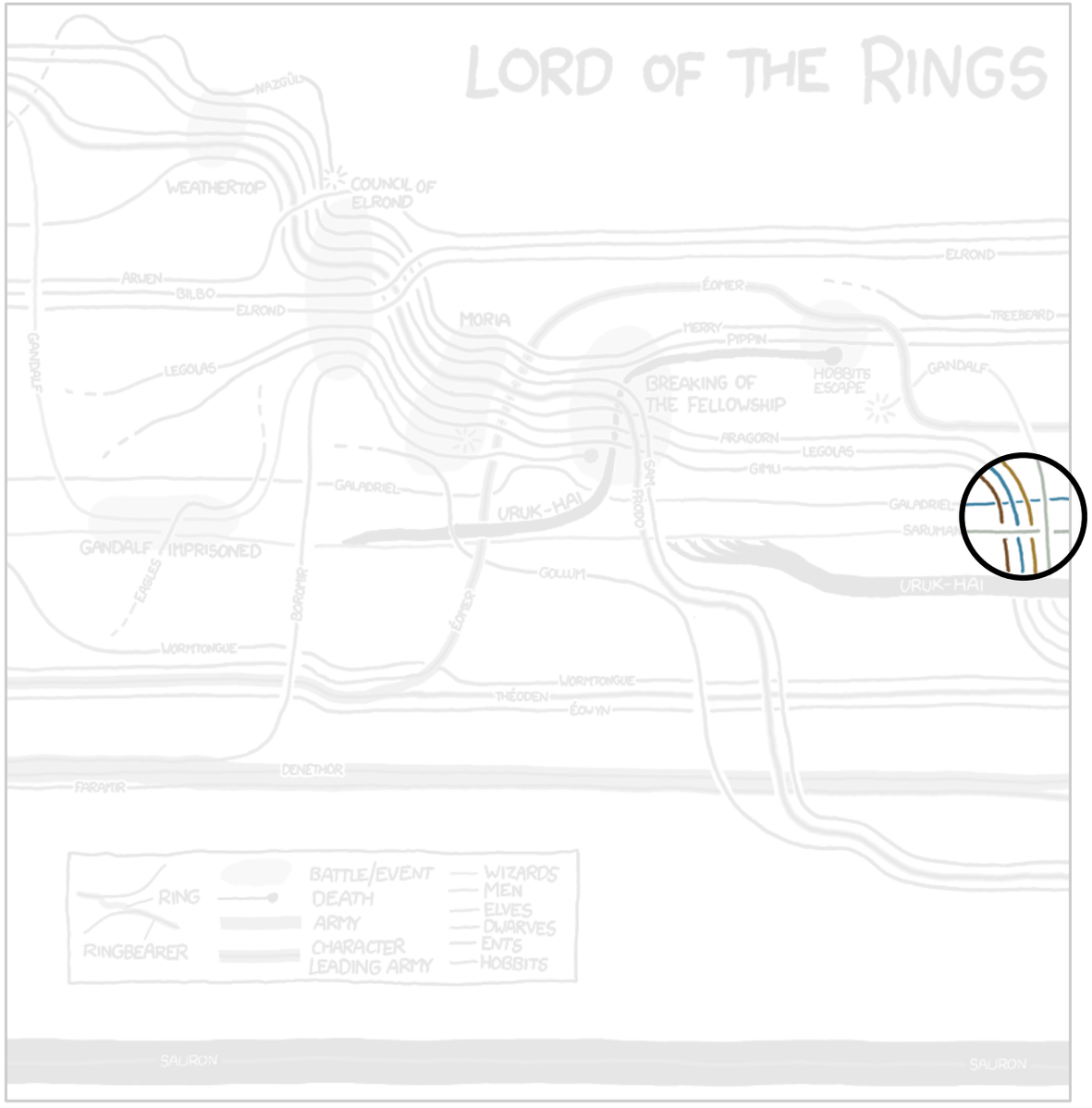
Meetings $\mathcal{M}$

Storylines

Time



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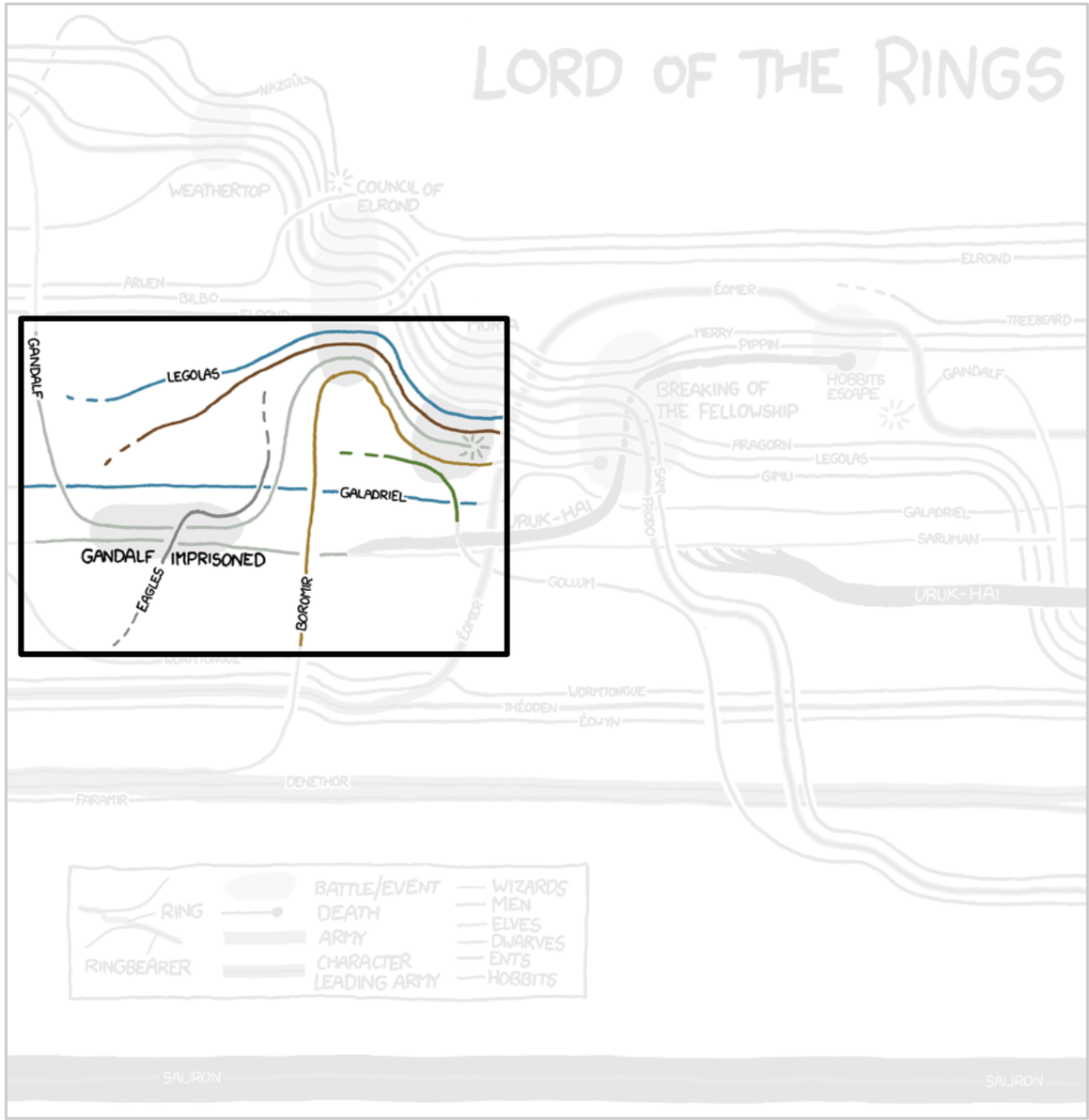

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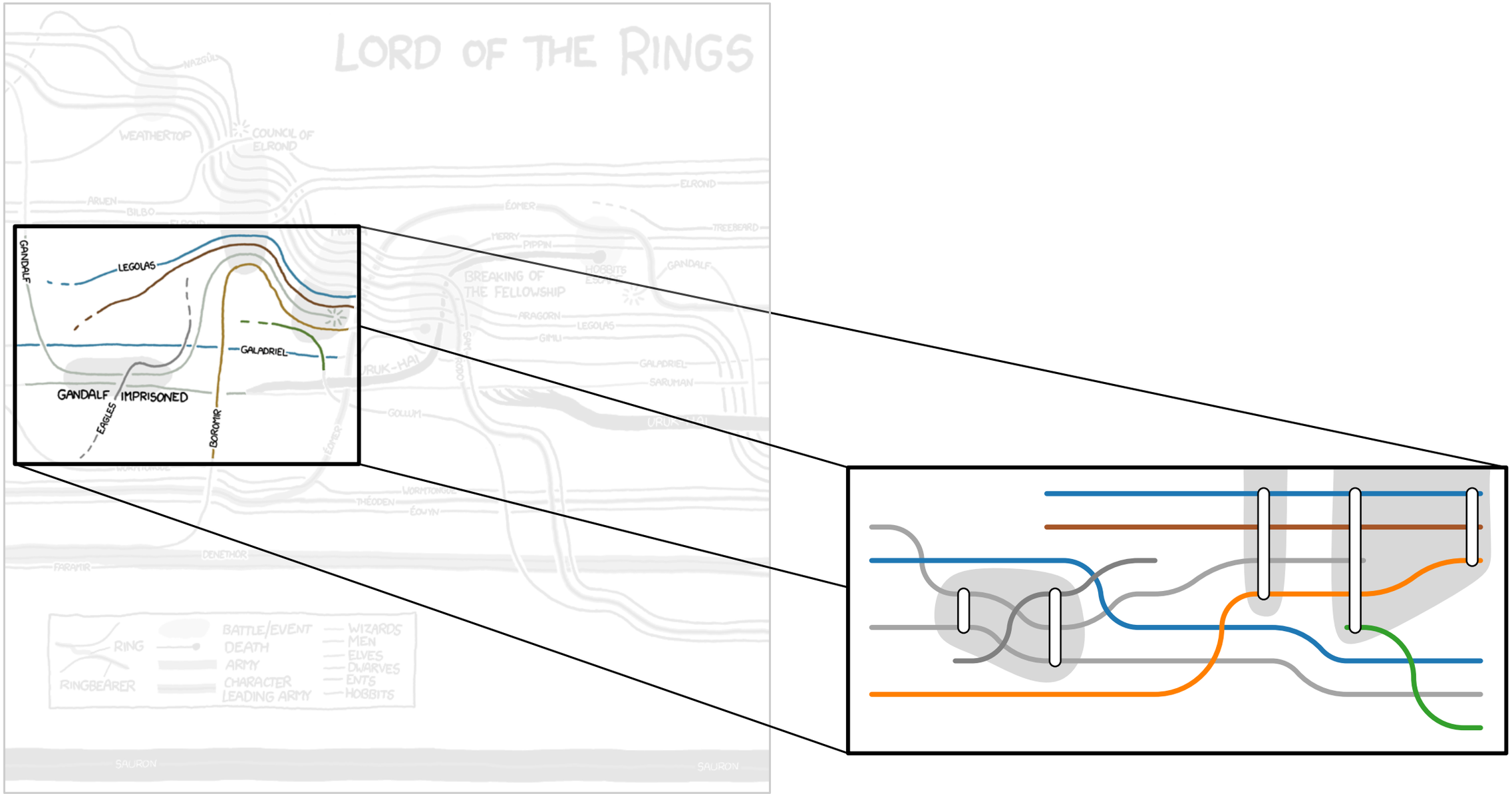

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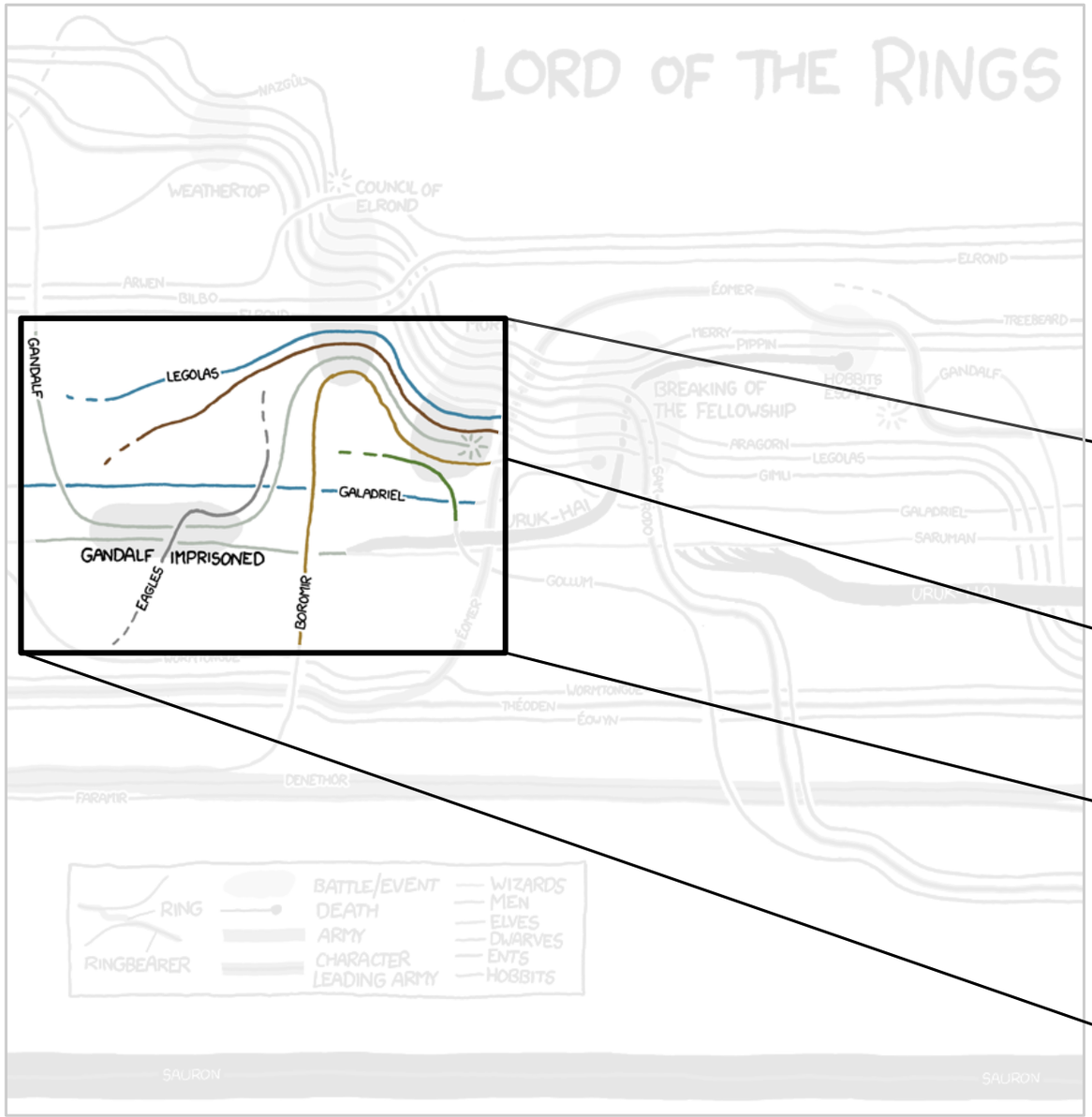
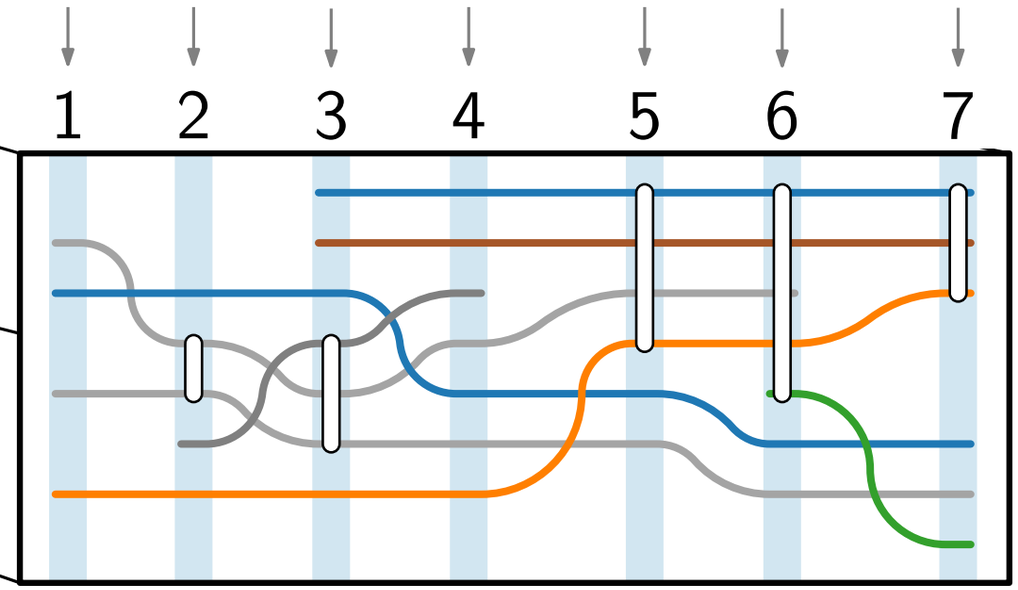


Time steps $T = \{1, 2, \ldots, \ell\}$



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Storylines

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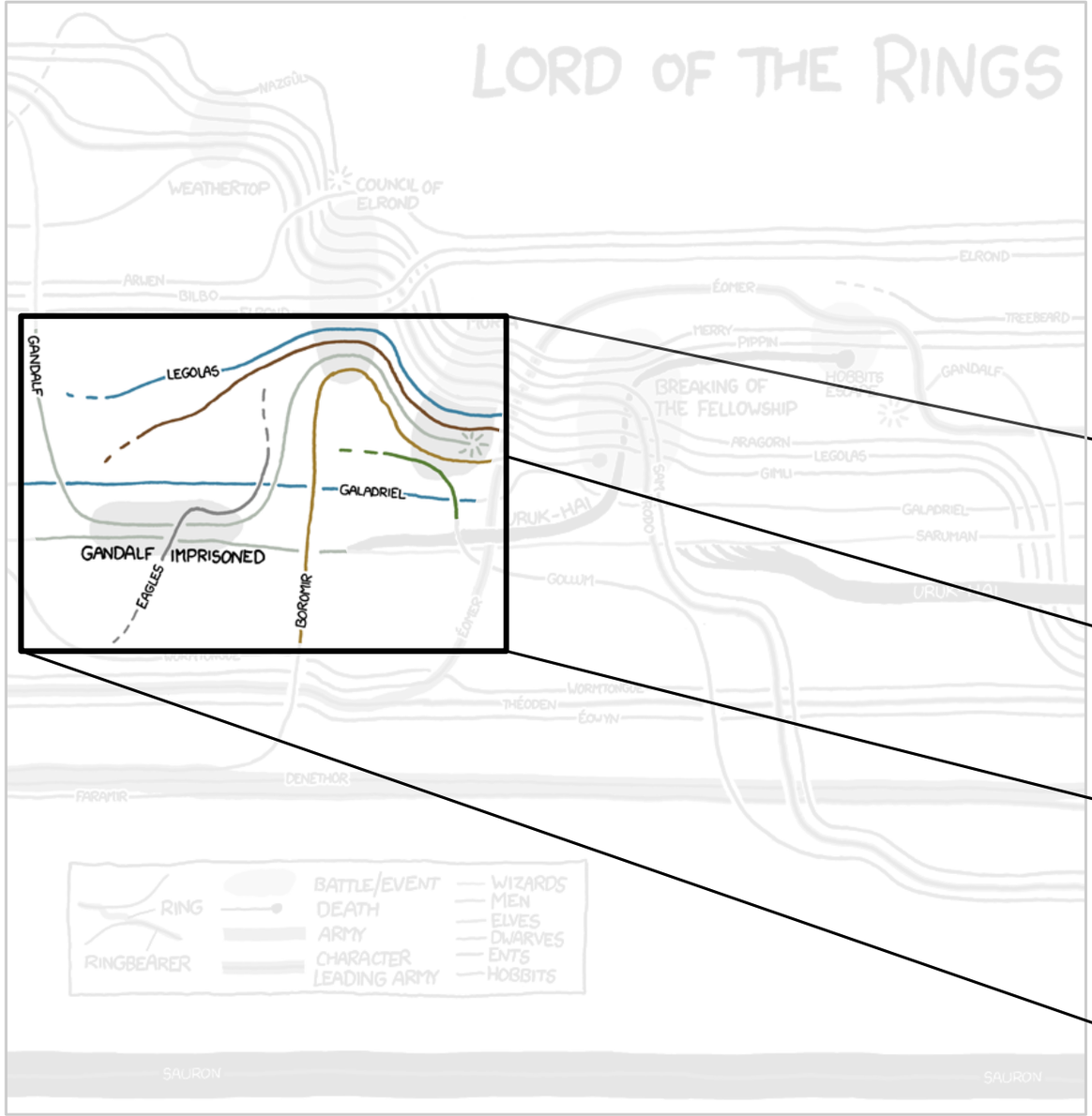
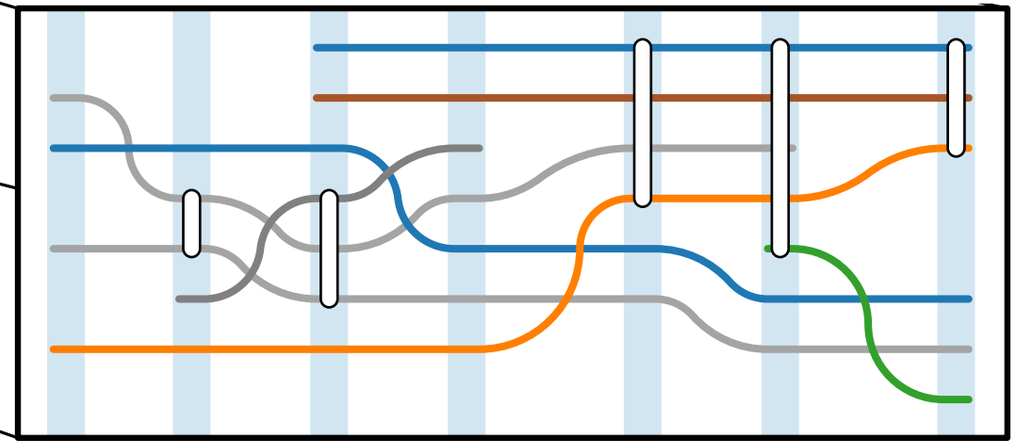
Characters Time steps Meetings

Storyline instance $(\mathcal{C}, T, \mathcal{M}, A)$





[xkcd 657. Munroe. 2009 (clipping)]

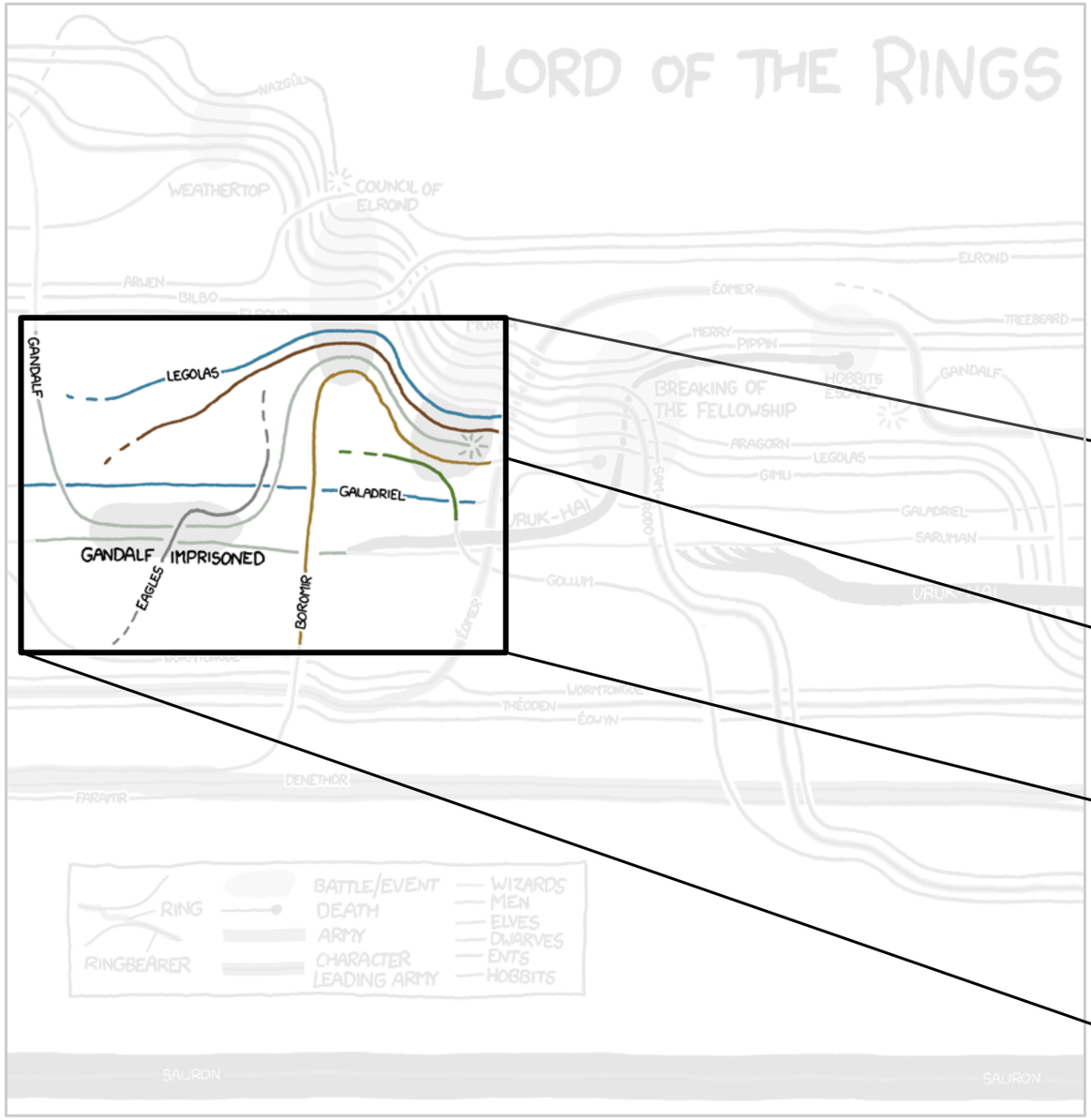
Storylines

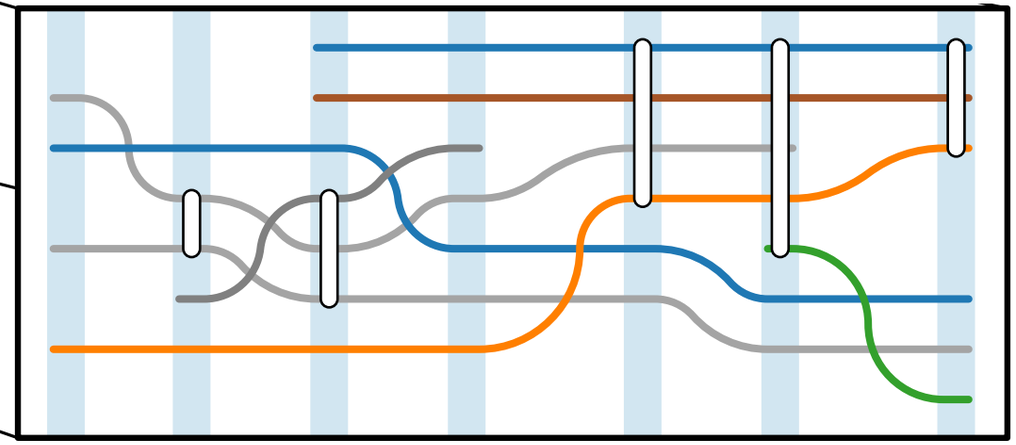
Time

Characters Time steps Meetings

Storyline instance $(\mathcal{C}, T, \mathcal{M}, A)$

$A(c) = \{i, i+1, \ldots, j\}$
with $c \in \mathcal{C}$ and $i \leq j \in T$





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$A(c) = \{i, i+1, \ldots, j\}$
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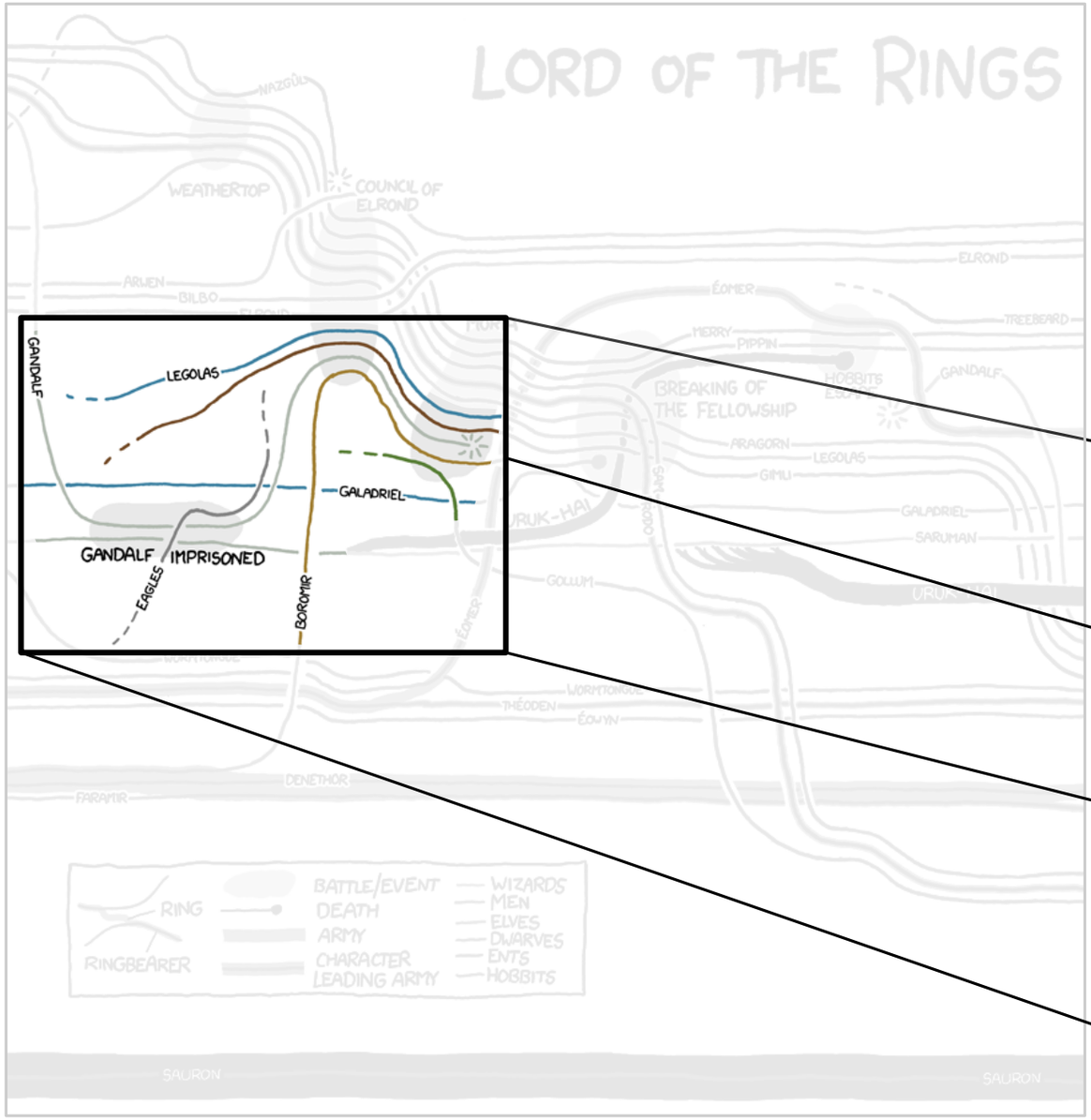
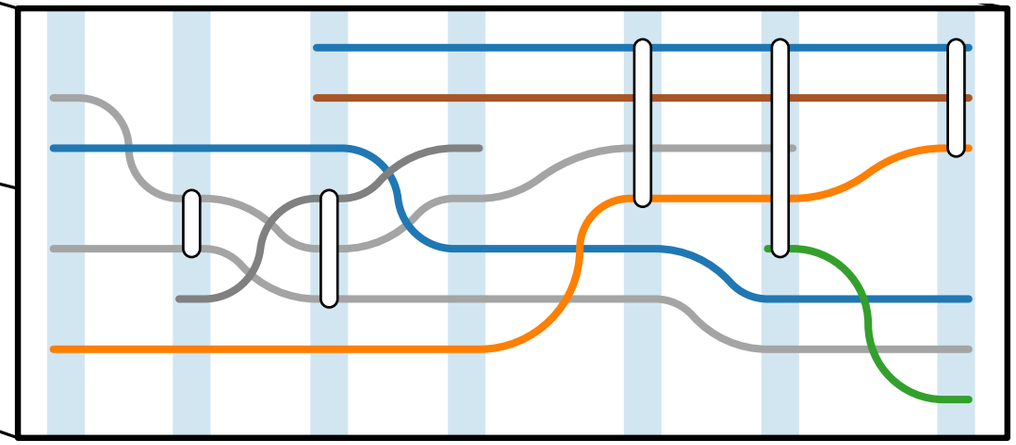
$\mathrm{AC}(t) = \{c \in \mathcal{C} \mid t \in A(c)\}$





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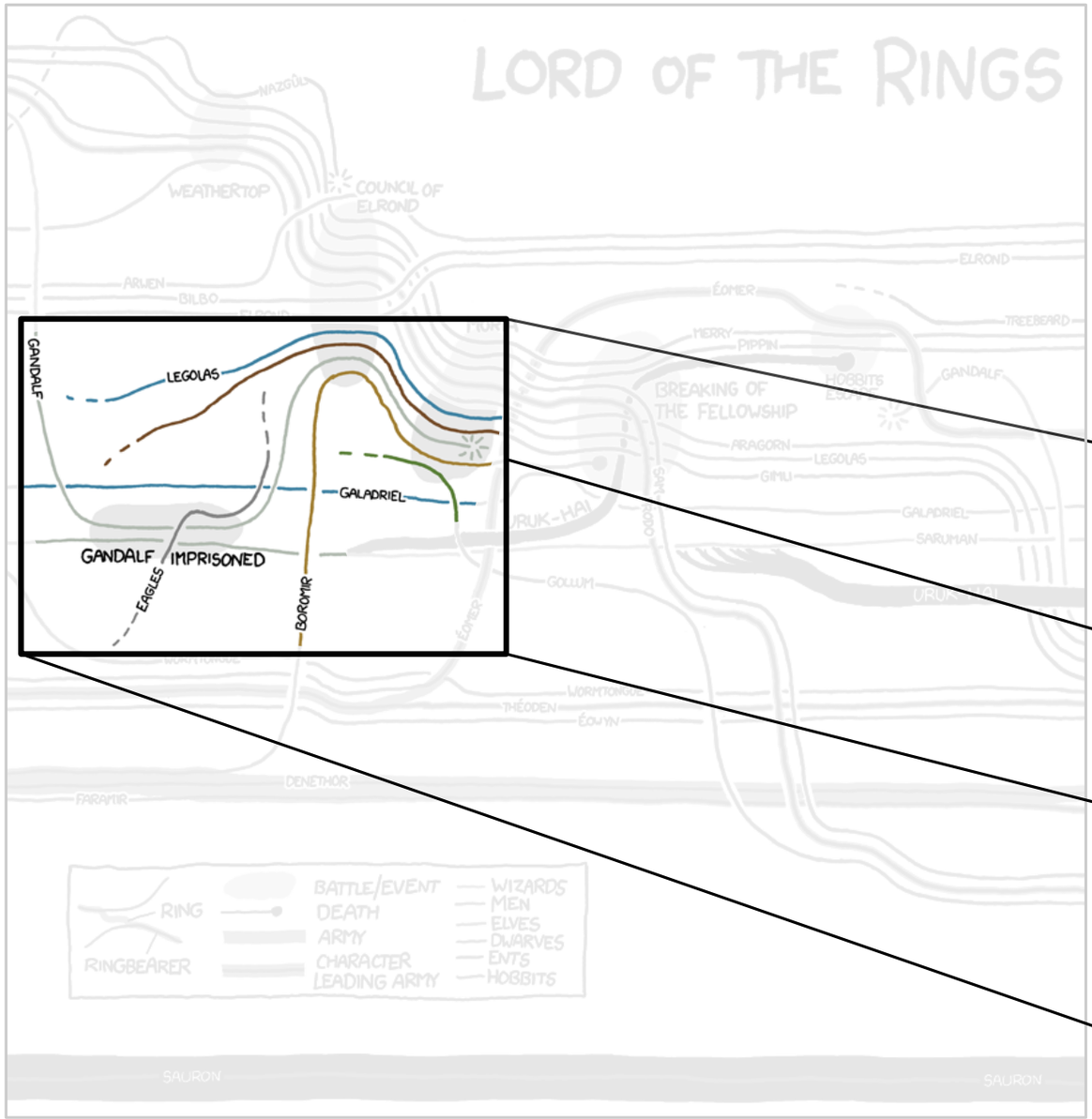
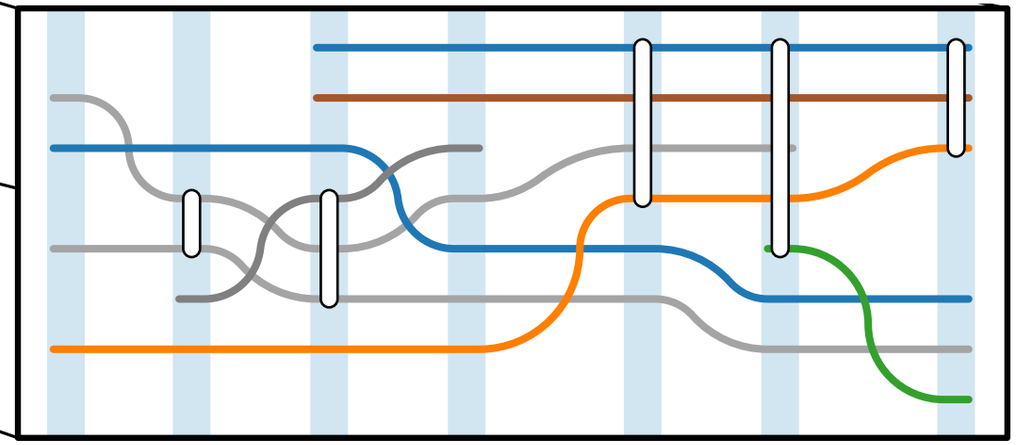
Ordered storyline instance:
Permutation π_t of $\mathrm{AC}(t)$ with $t \in T$





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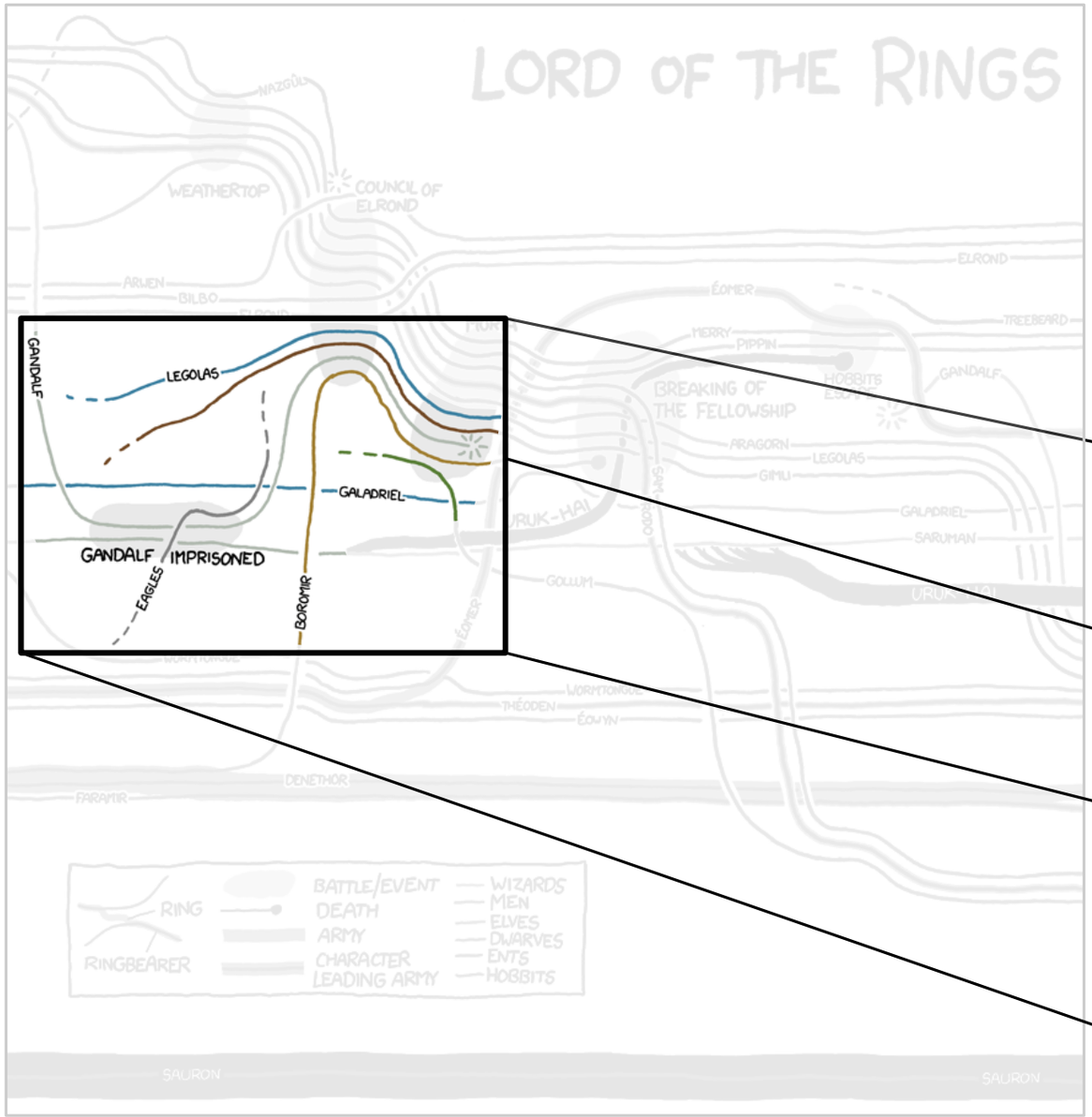

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Ordered storyline instance

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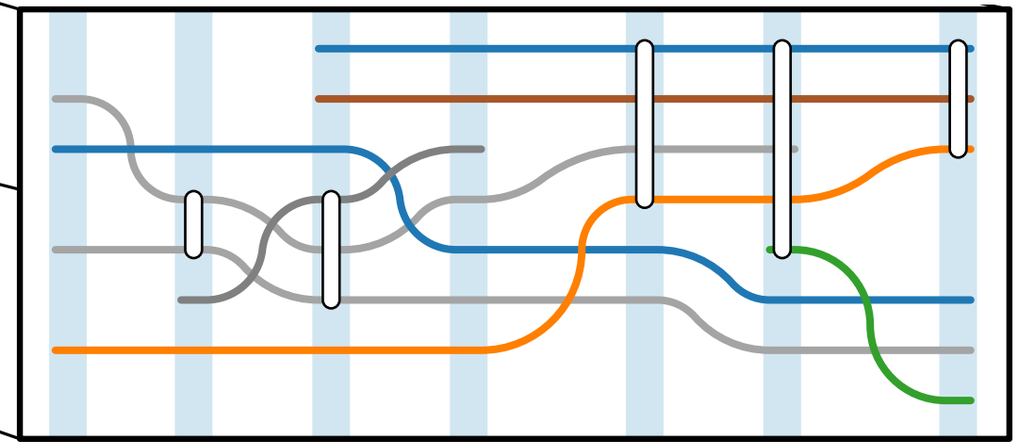

Storylines

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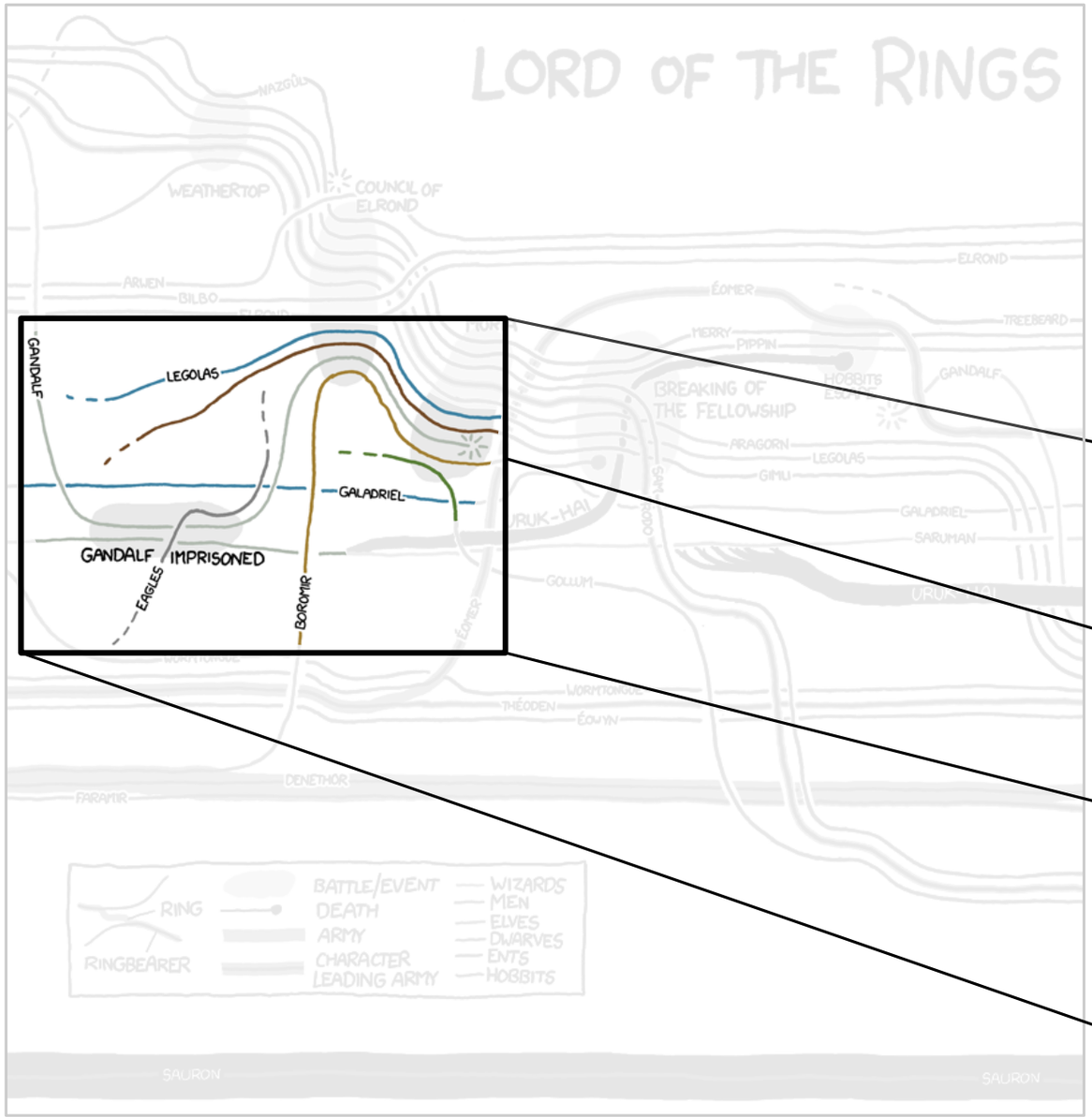


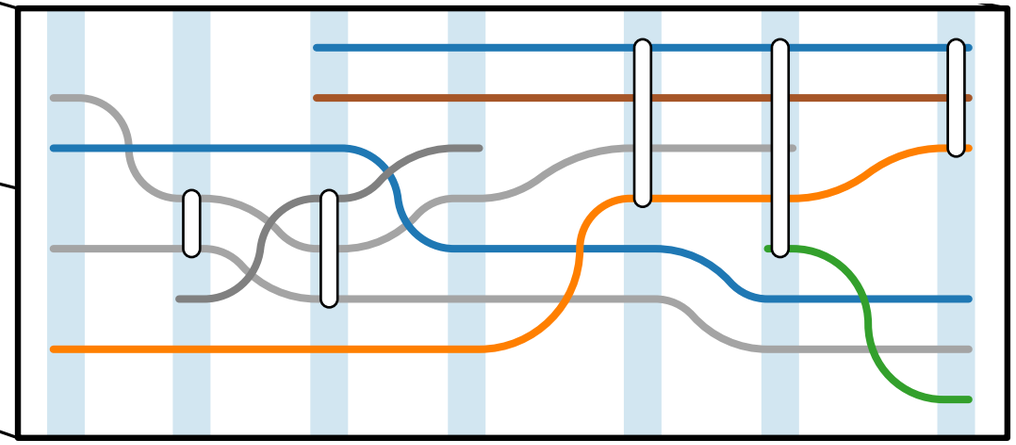
[xkcd 657. Munroe. 2009 (clipping)]

Ordered storyline instance

$(\mathcal{C}, T, \mathcal{M}, A, \pi)$

Coordination:

$y_{t,c} \in \mathbb{R}$ with $t \in T, c \in \mathrm{AC}(t)$



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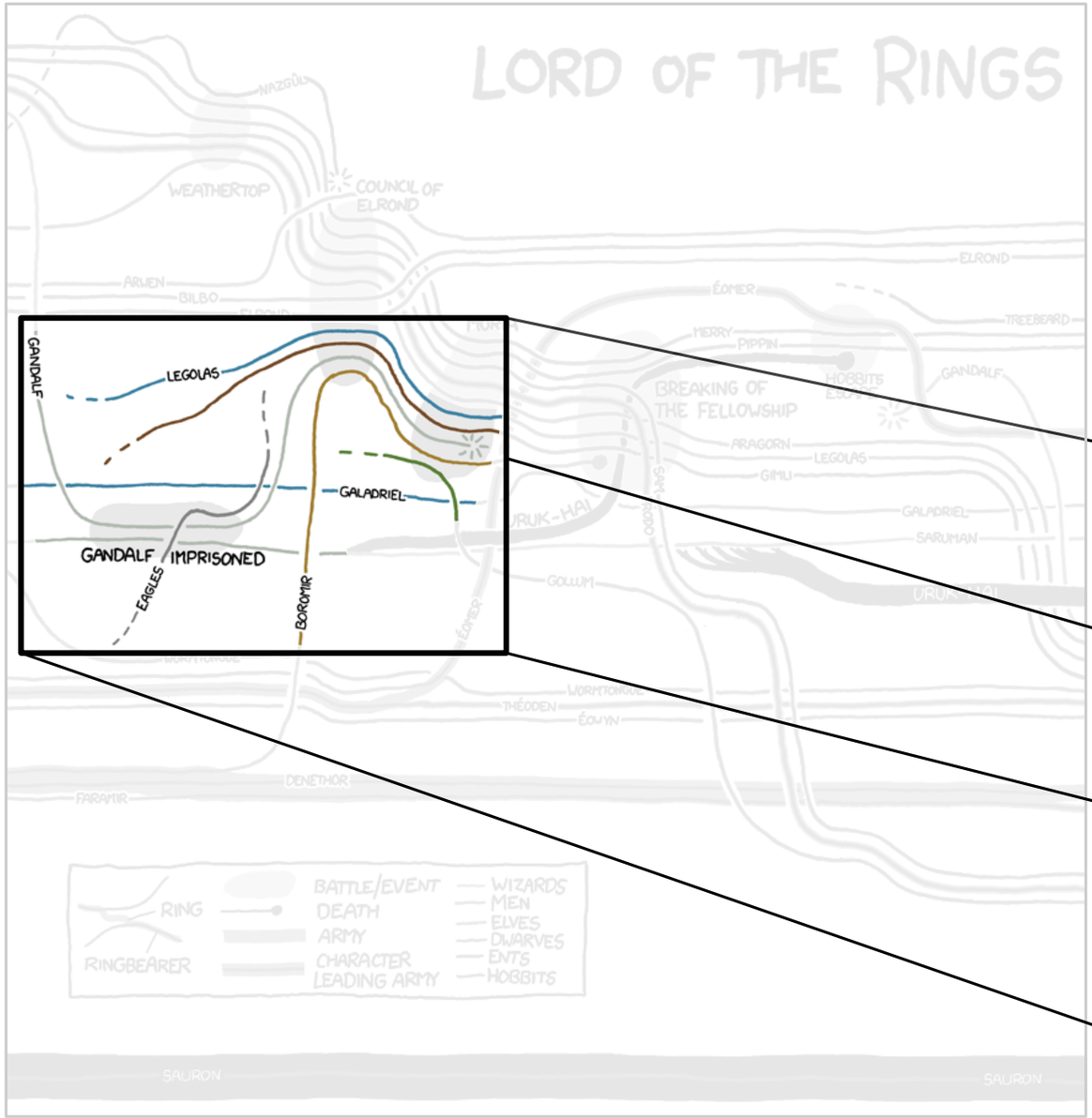

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Ordered storyline instance
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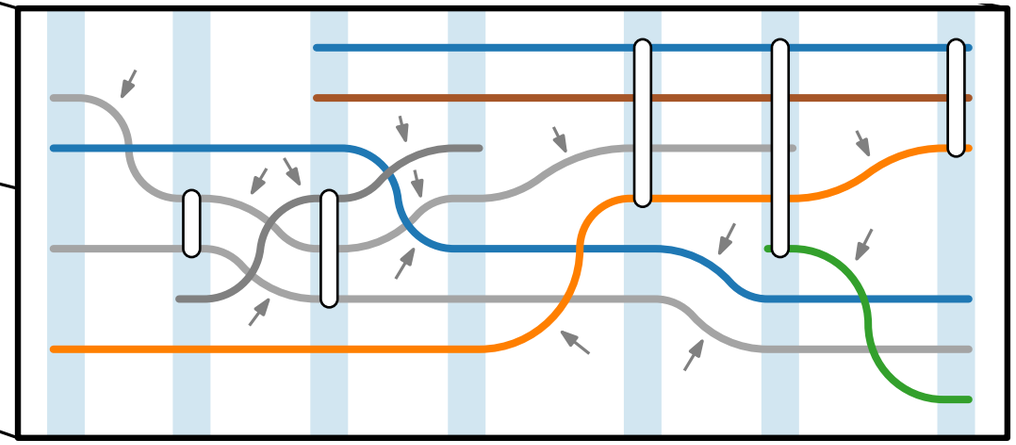
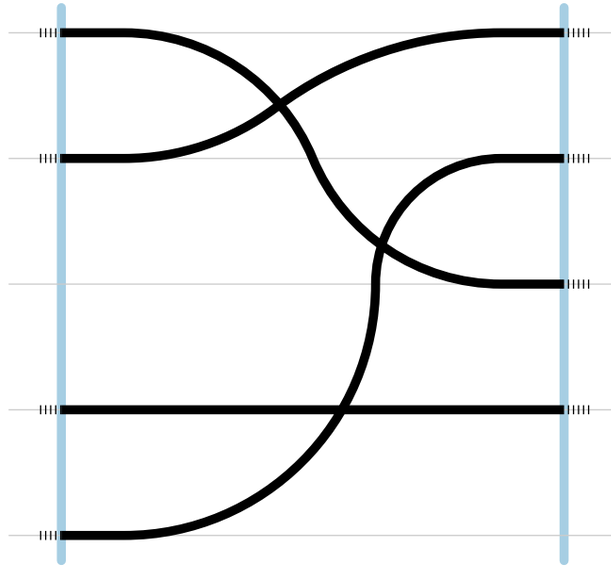
Wiggle $\iff y_{t,c} \neq y_{t+1,c}$



Optimal Wiggle Aesthetics

Given: ordered storyline instance
$(\mathcal{C}, T, \mathcal{M}, A, \pi)$



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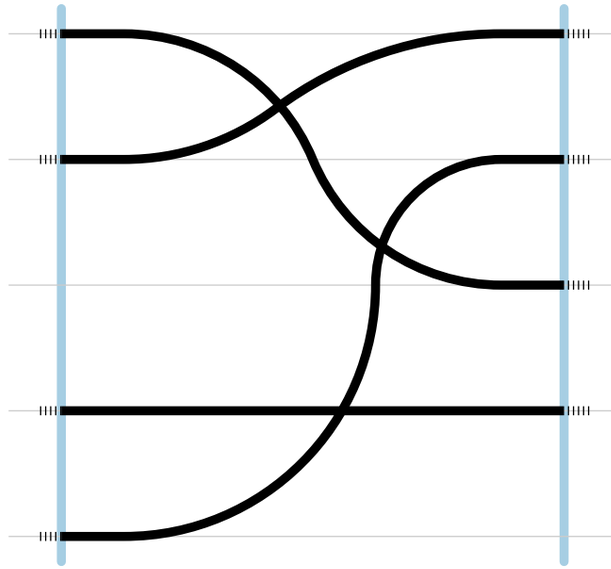

Given: ordered storyline instance
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Problem: find a coordination
minimizing total

- wiggle count (WC):
  $|\{(t, c) \mid 1 \leq t \leq \ell - 1, c \in \widetilde{\mathrm{AC}}(t), y_{t,c} \neq y_{t+1,c}\}|$

with $\widetilde{\mathrm{AC}}(t) = \mathrm{AC}(t) \cap \mathrm{AC}(t+1)$

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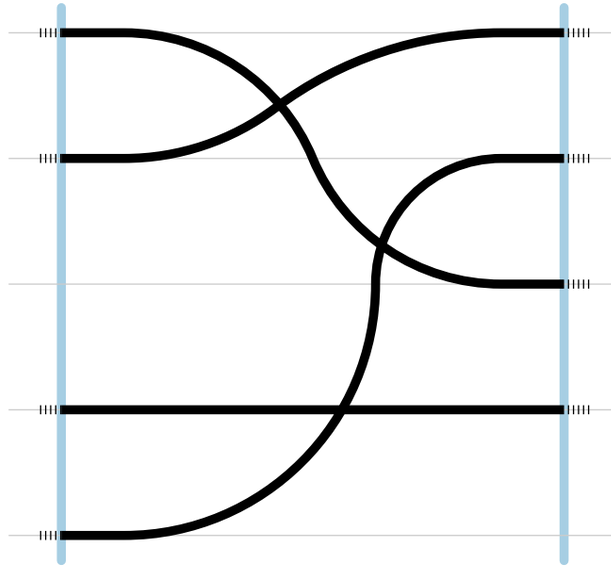

WC: 3

with $\widetilde{\mathrm{AC}}(t) = \mathrm{AC}(t) \cap \mathrm{AC}(t+1)$

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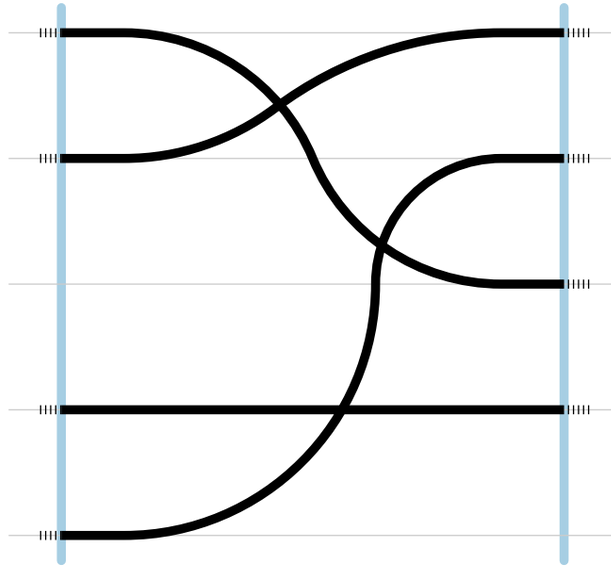

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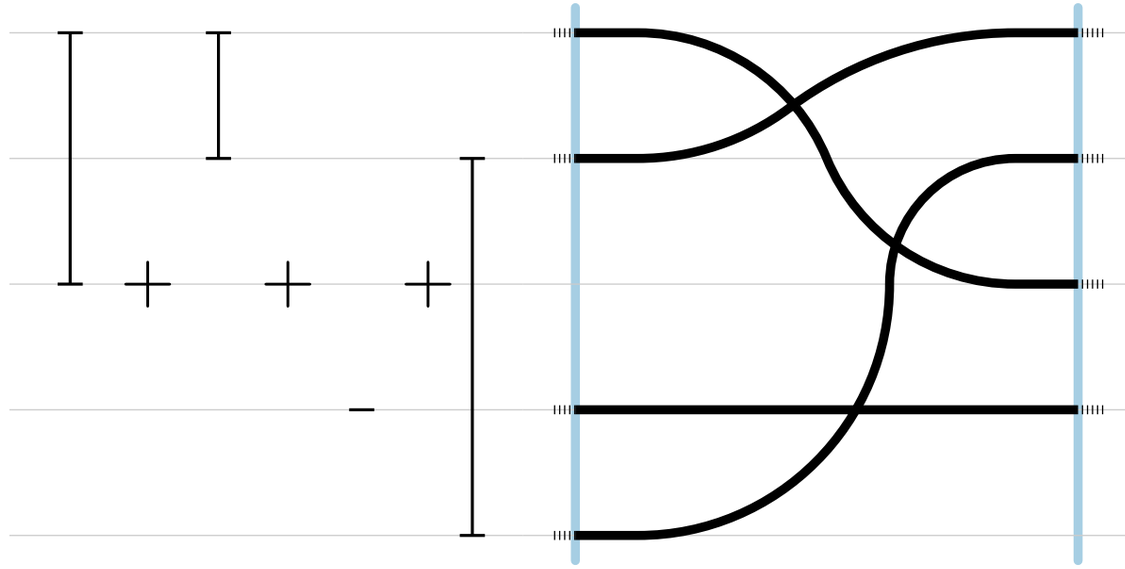

WC: 3
LWH: 6

with $\widetilde{\mathrm{AC}}(t) = \mathrm{AC}(t) \cap \mathrm{AC}(t+1)$

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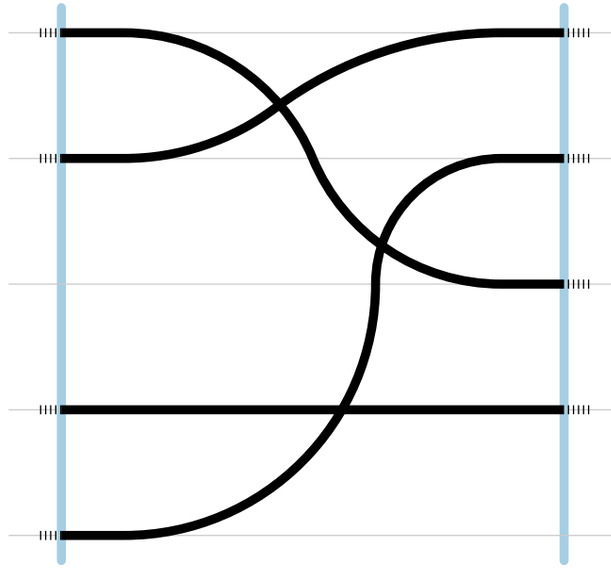

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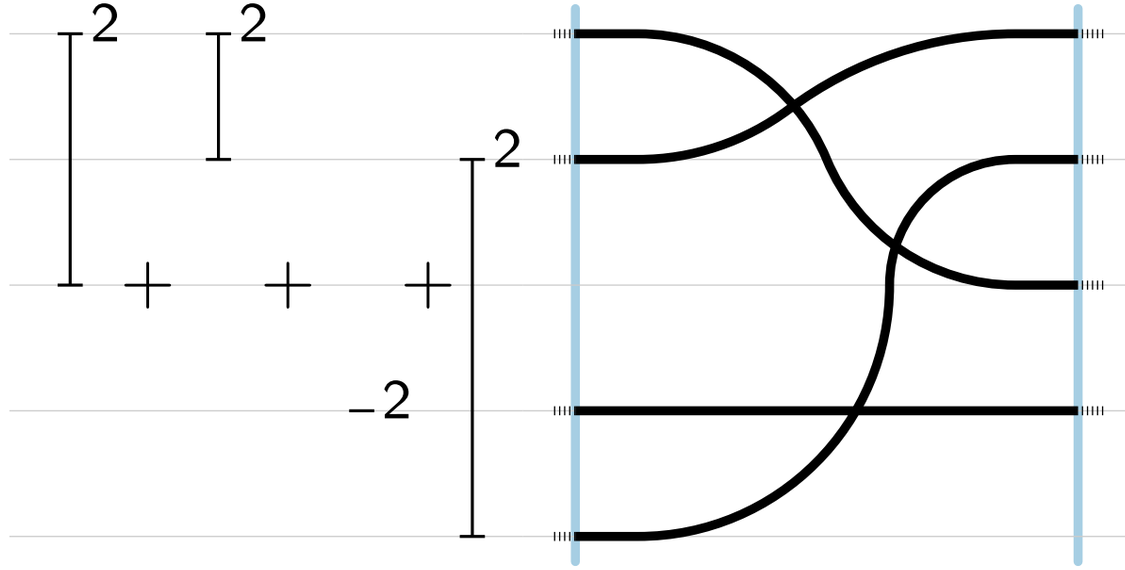

WC: 3
LWH: 6
QWH: 14

Optimal Wiggle Aesthetics

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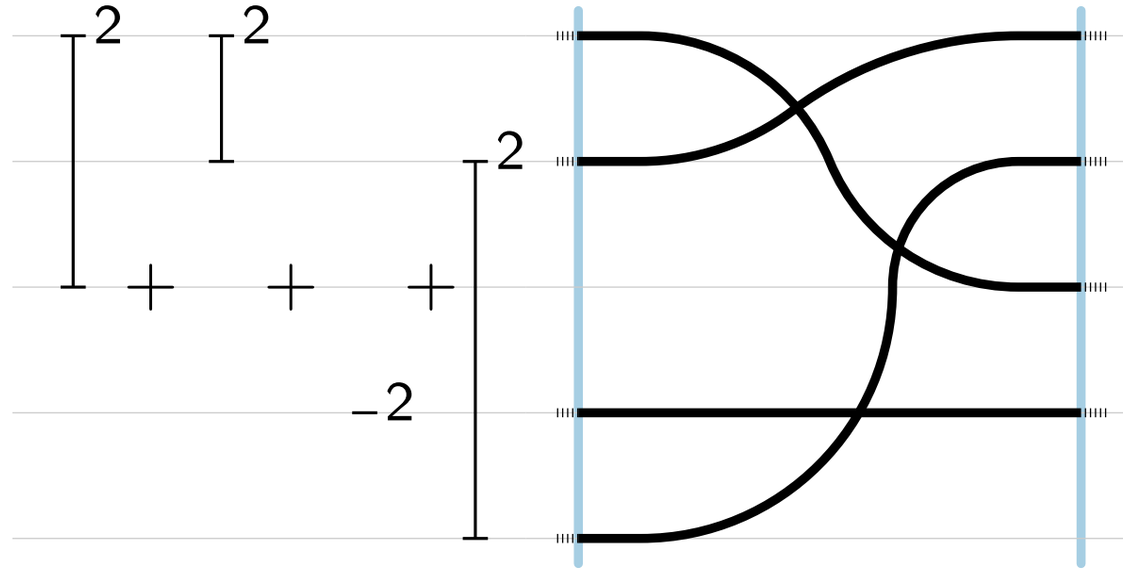

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Integer programming

Linear programming

Convex quadratic programming

Our Contribution

- Mathematical programming solutions for three variants of Storyline Wiggle Minimization

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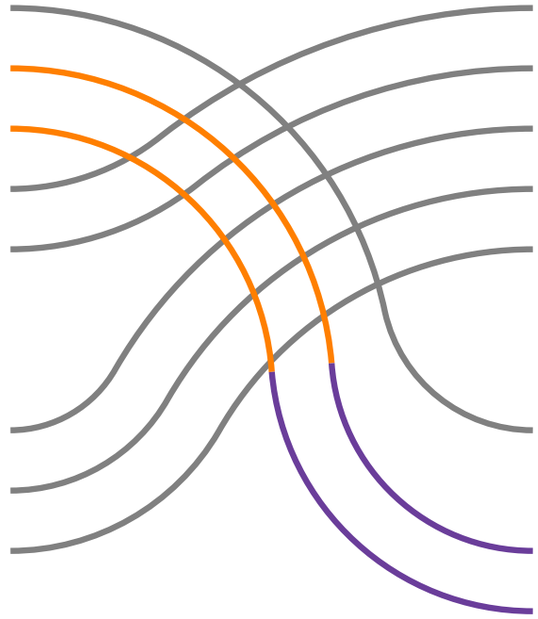
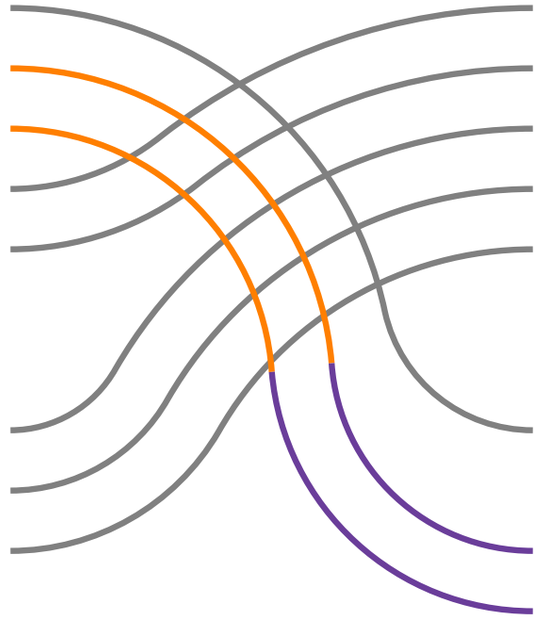
- New method for routing character curves that run in parallel



Our Contribution

- Mathematical programming solutions for three variants of Storyline Wiggle Minimization
- Minimizing total wiggle count is NP-complete
- New method for routing character curves that run in parallel
- Case study: experimental evaluation on 17 well-known benchmark instances



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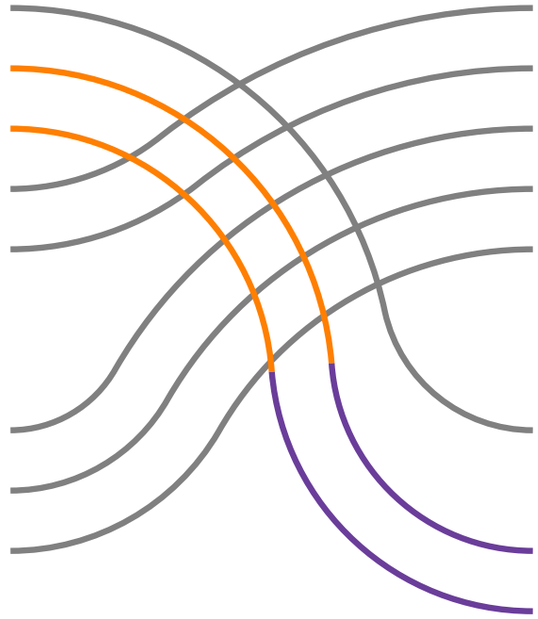
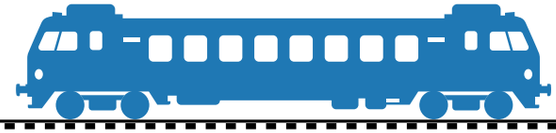
- New method for routing character curves that run in parallel
- Case study: experimental evaluation on 17 well-known benchmark instances
- Case study: visualizing rolling stock schedules in railway planning





Minimizing Total Wiggle Count is Hard

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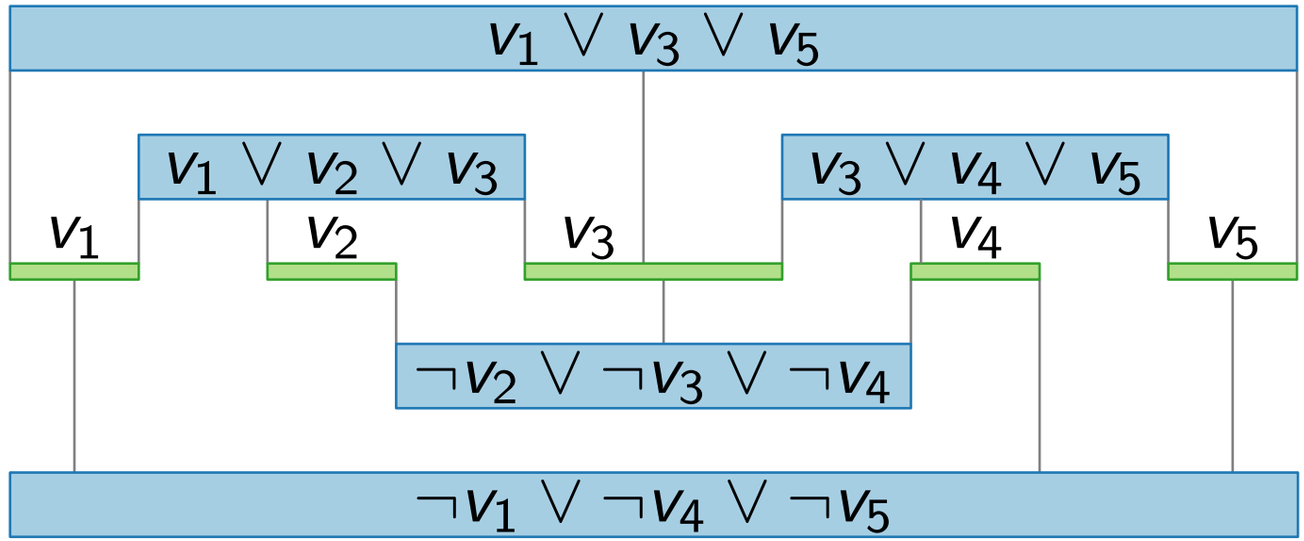
Reduction from Planar Monotone 3-Sat (PM3-Sat):



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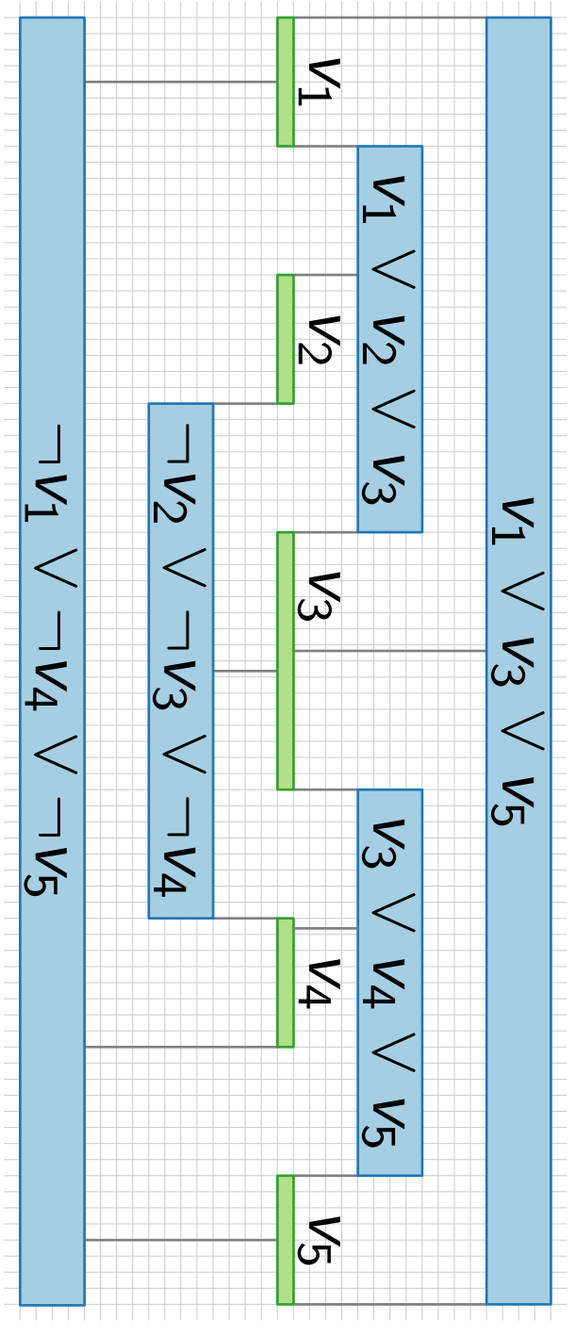

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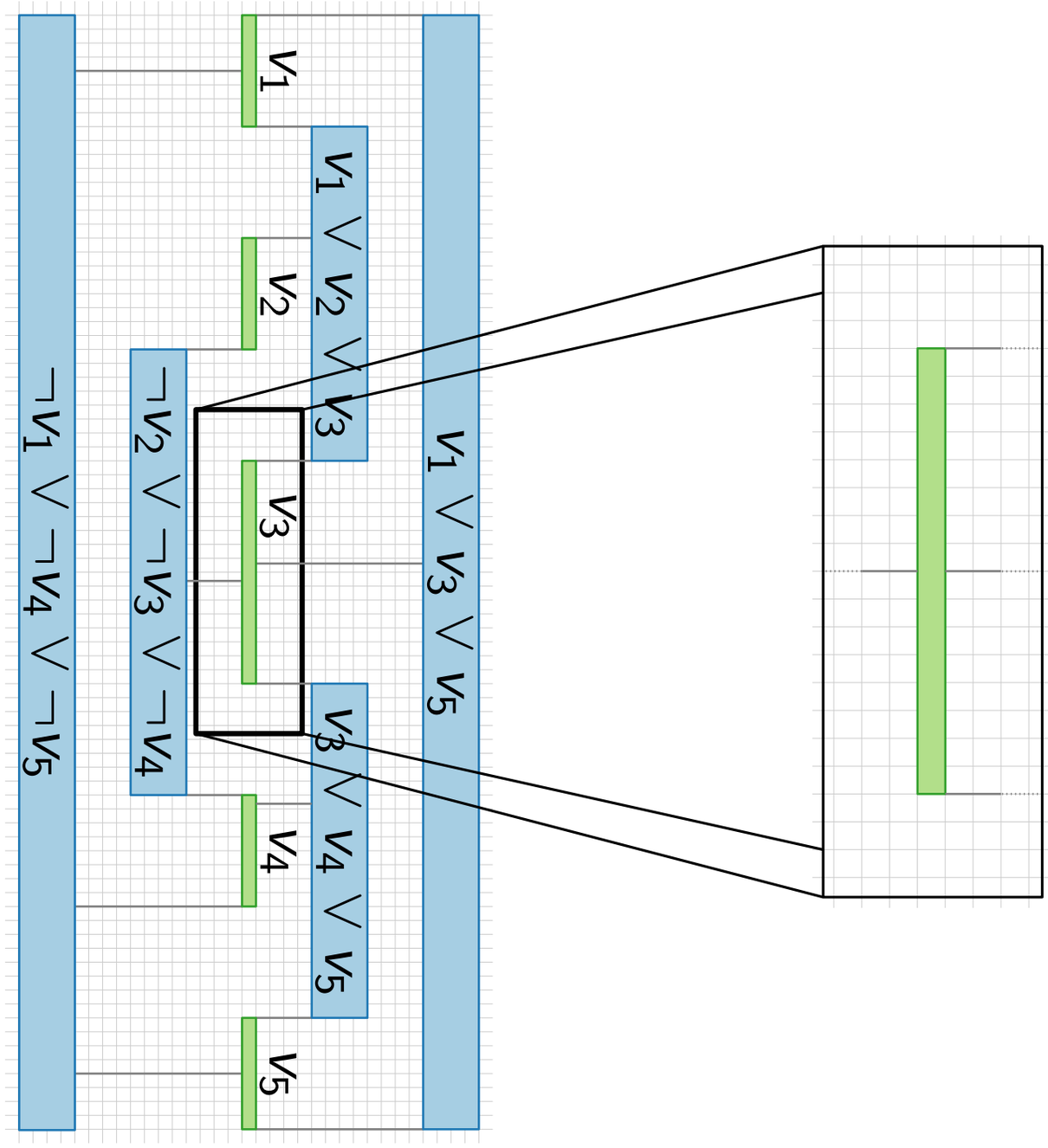

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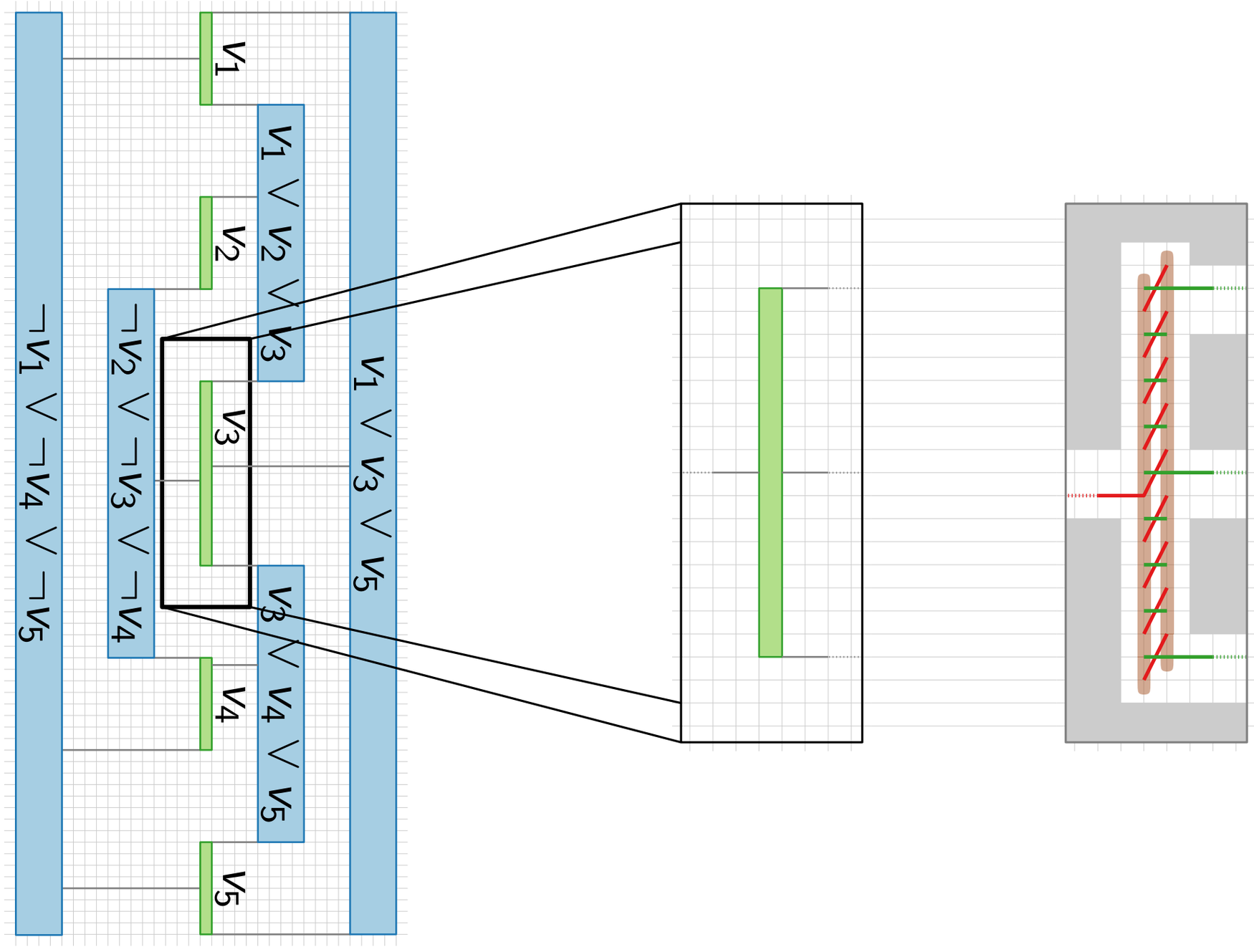

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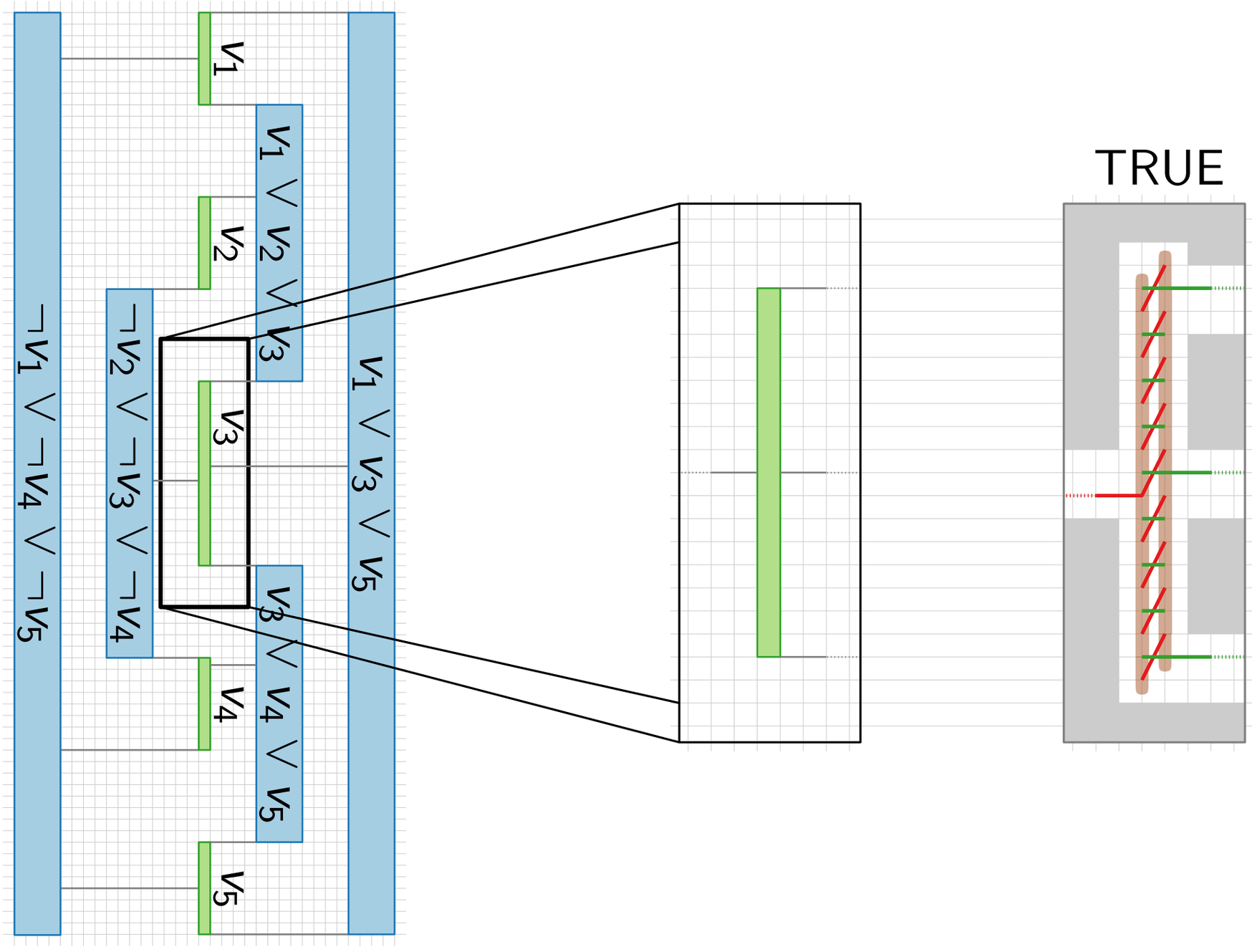
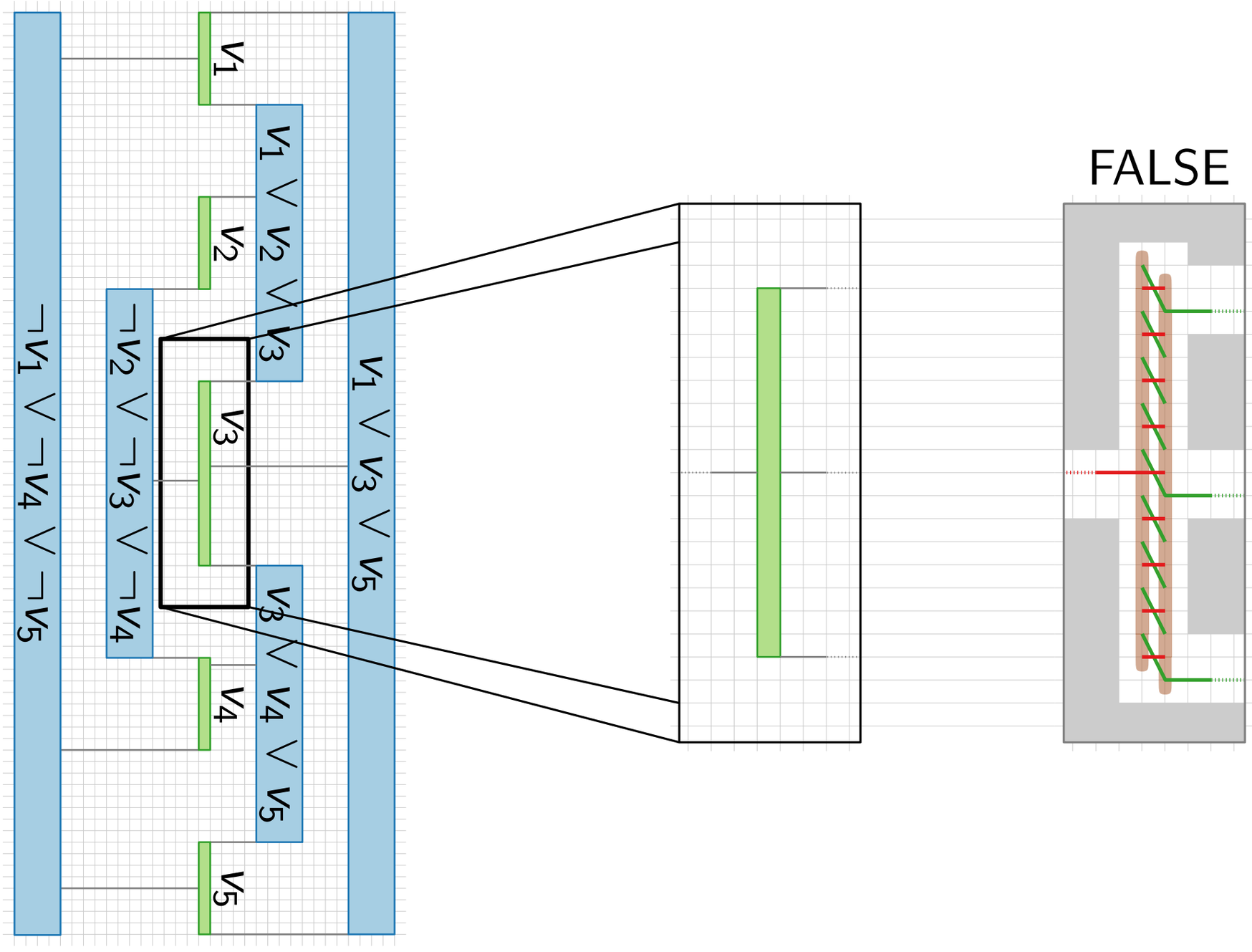
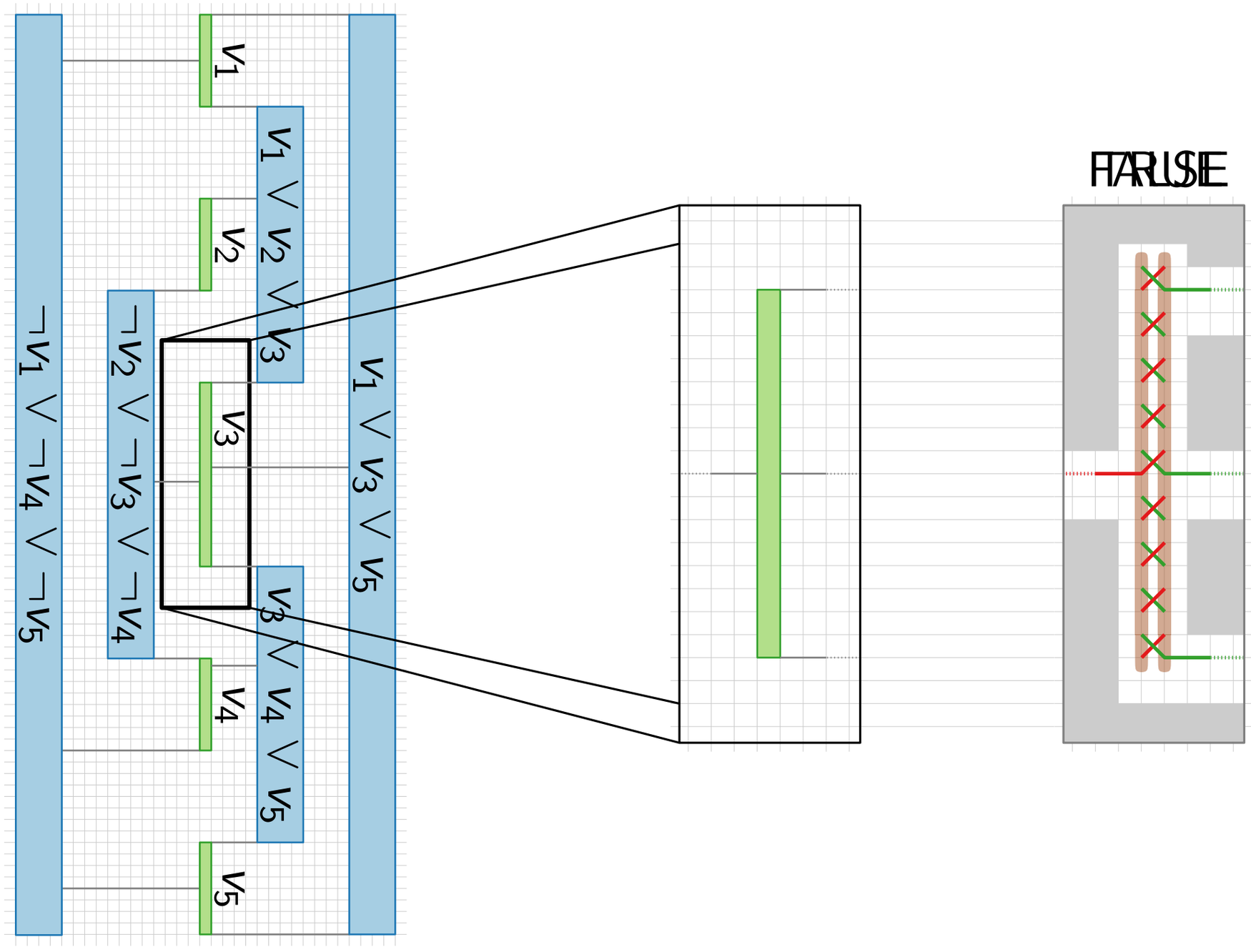


Minimizing Total Wiggle Count is Hard



Minimizing Total Wiggle Count is Hard



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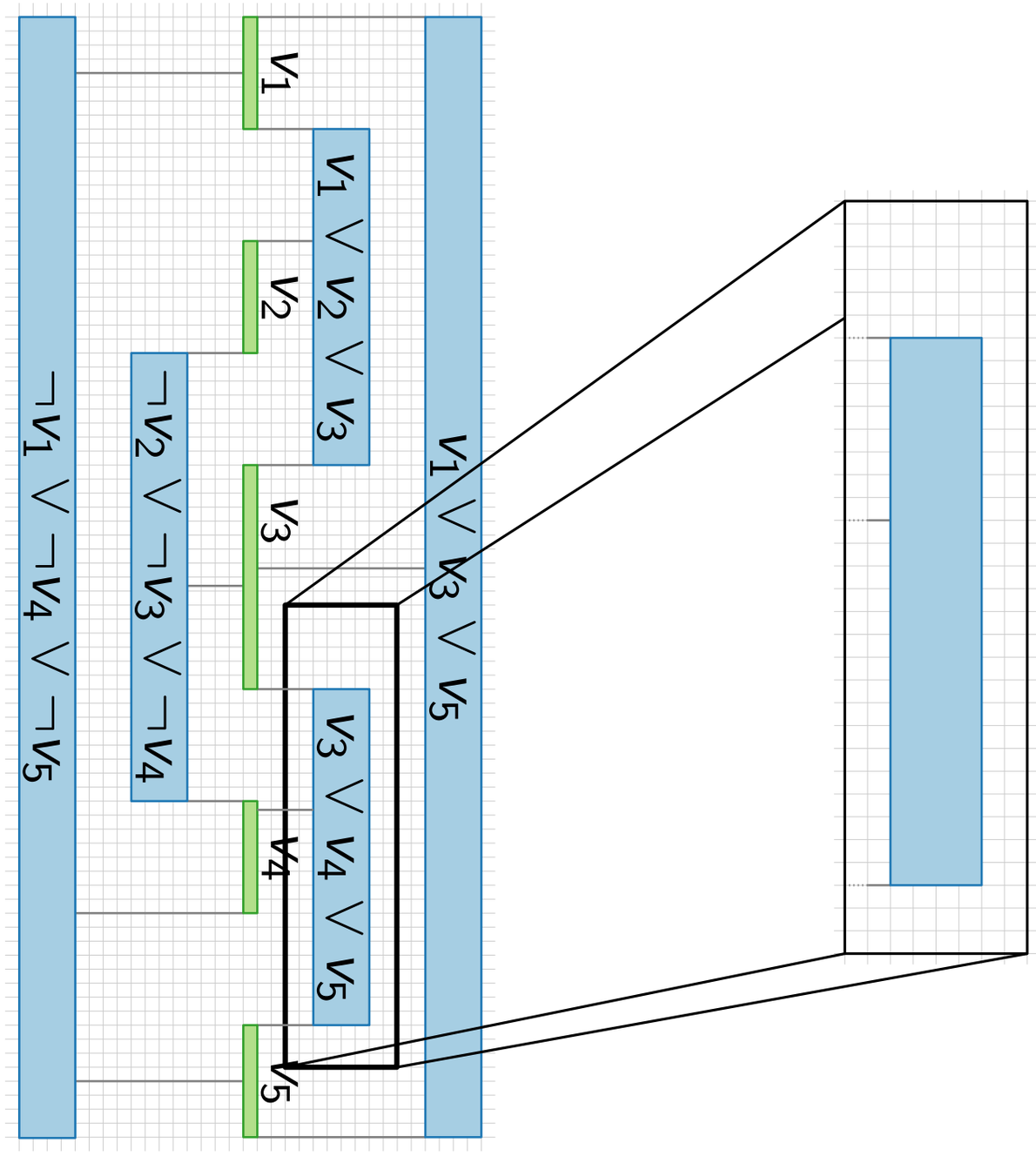

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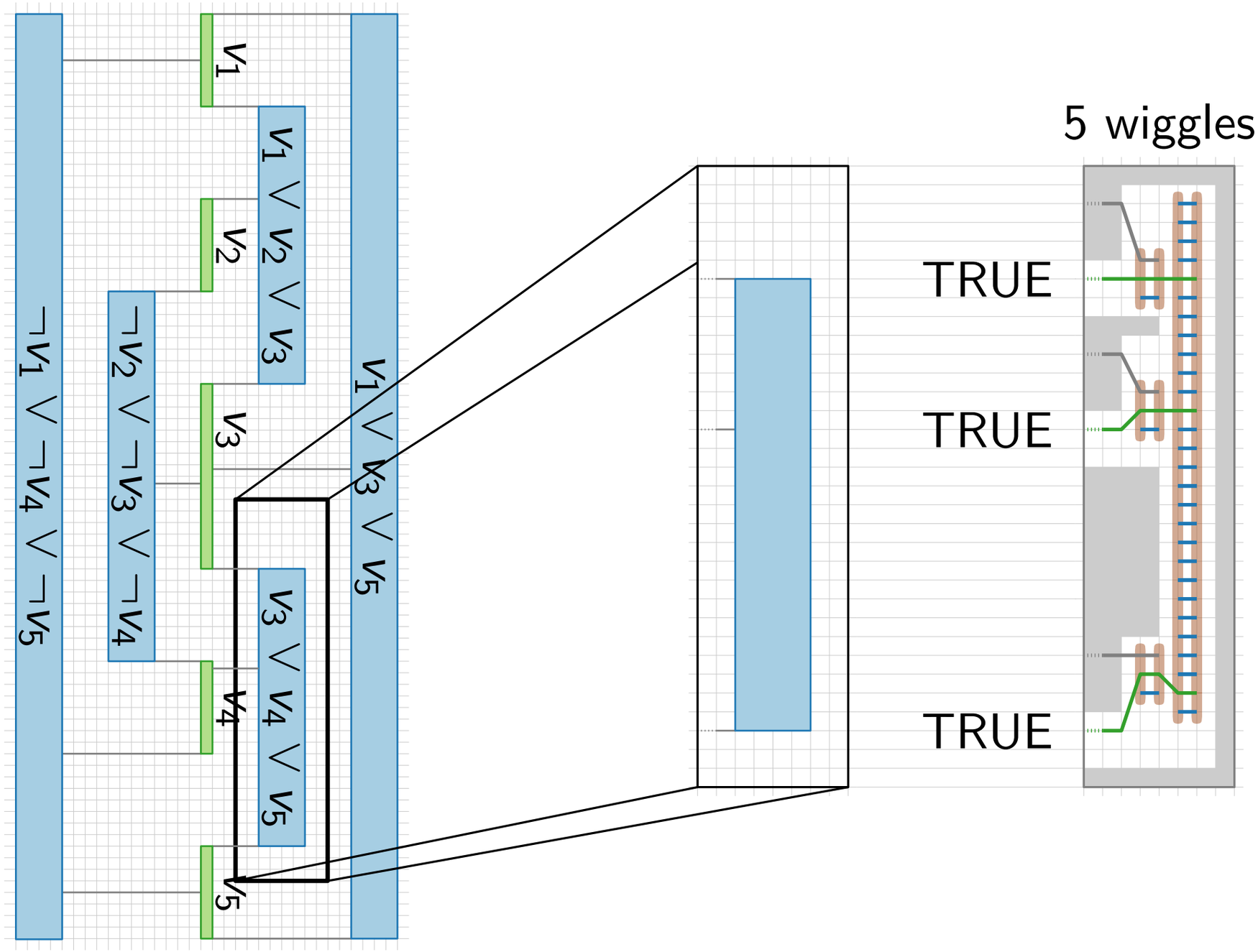

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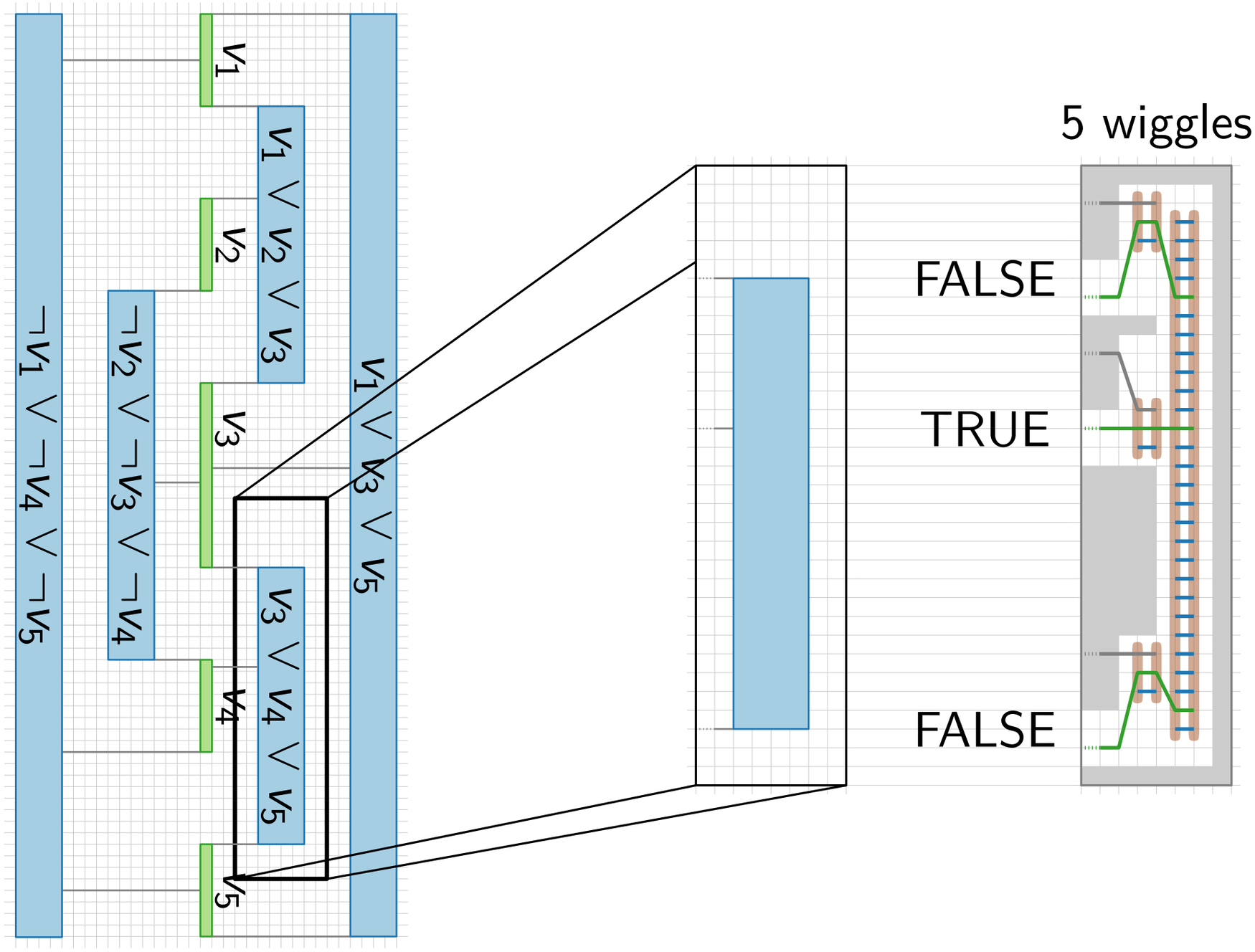
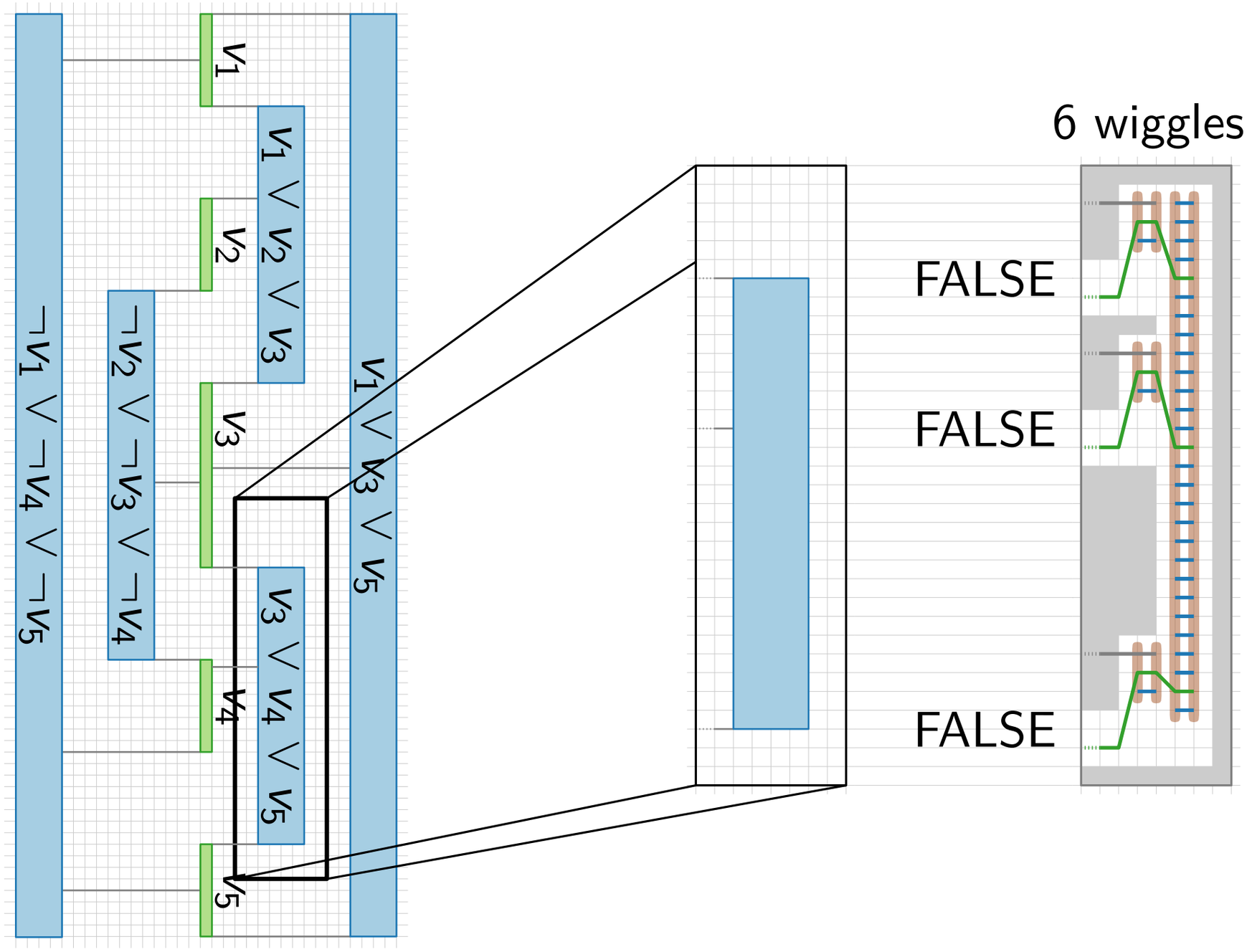
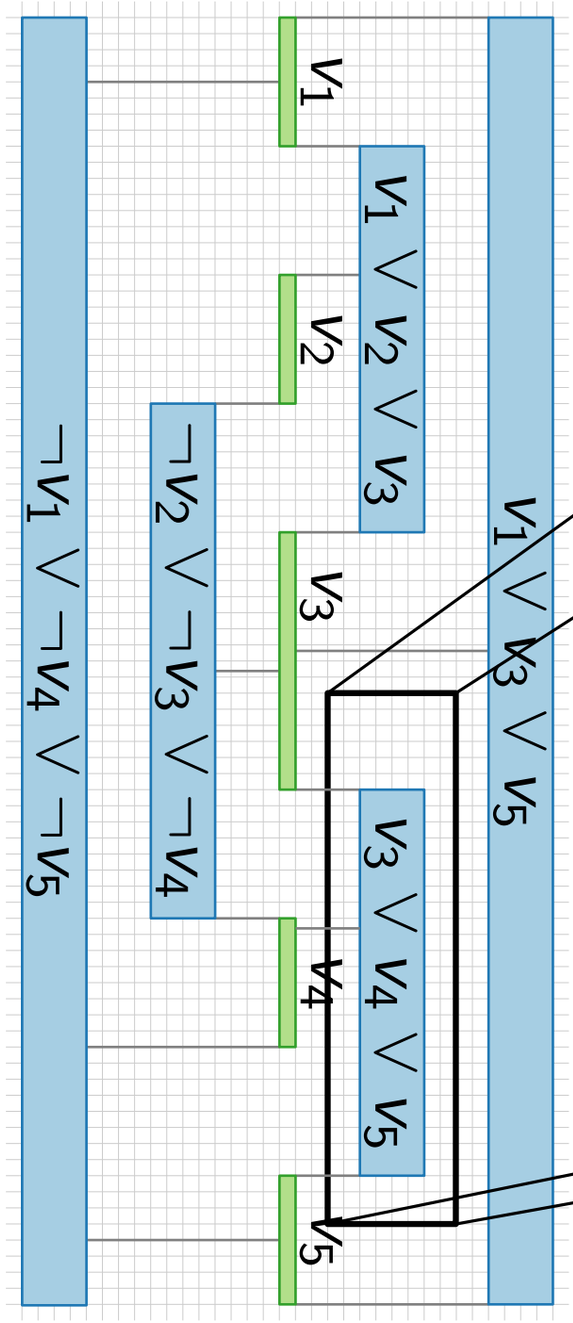
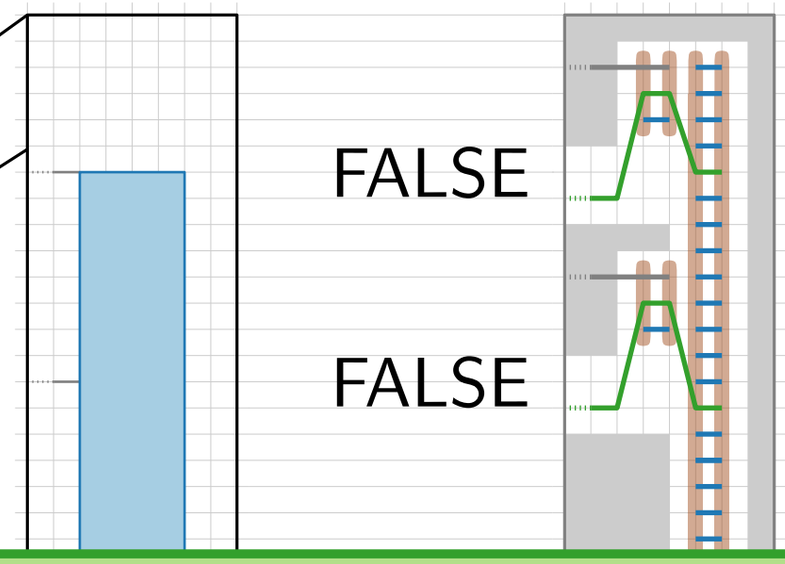


Minimizing Total Wiggle Count is Hard



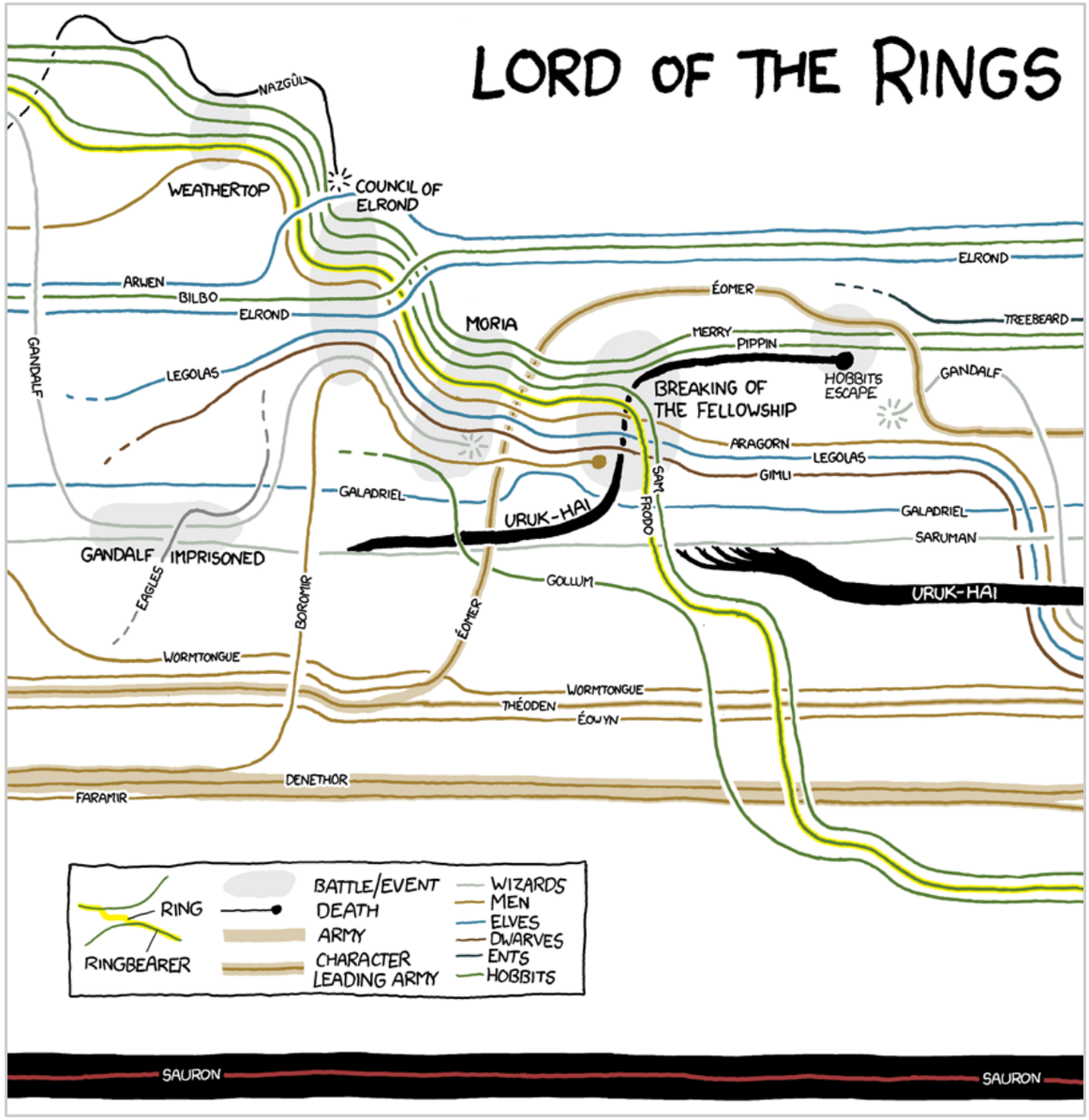
Minimizing Total Wiggle Count is Hard





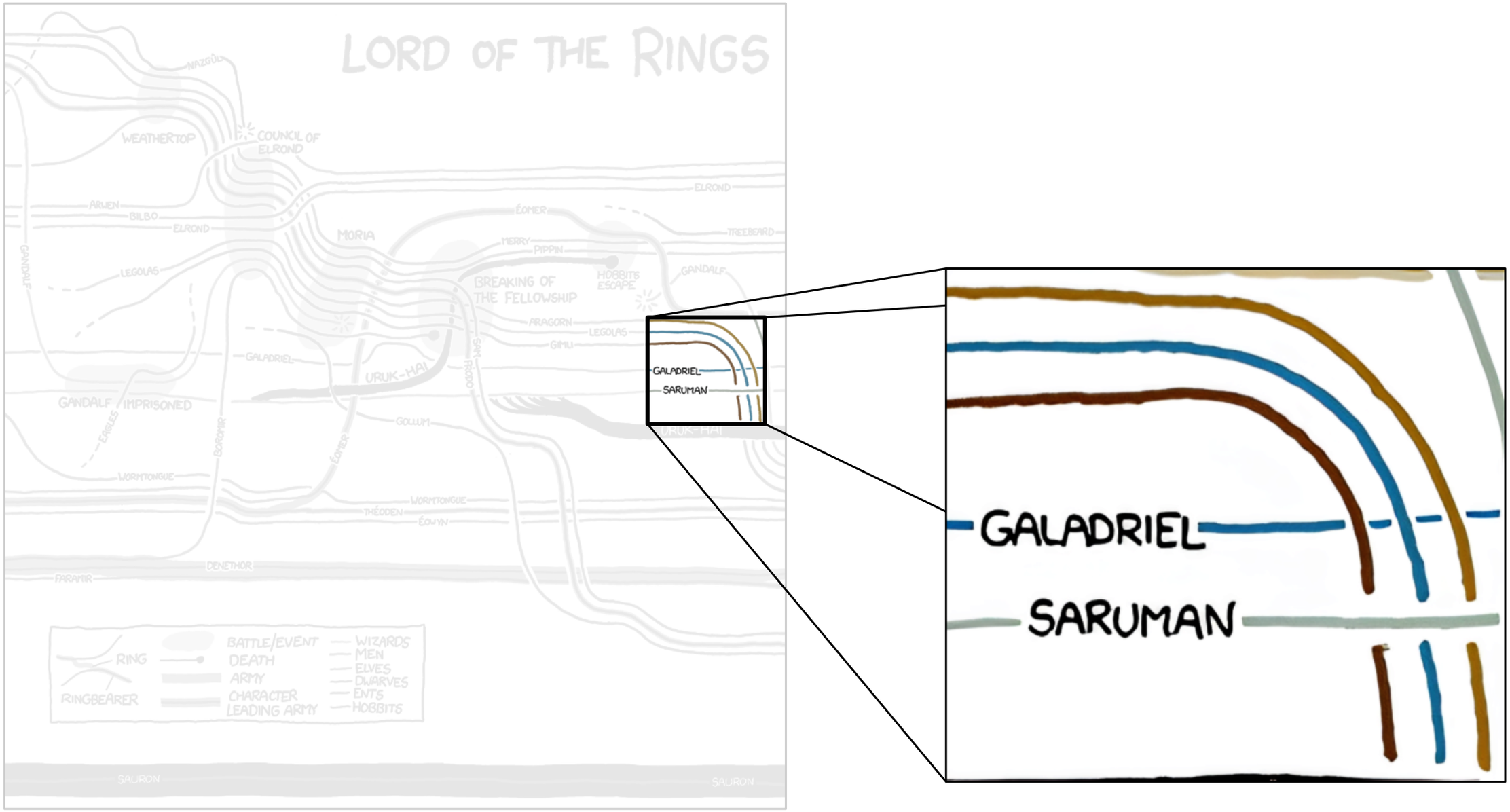
Theorem.
Given an ordered storyline instance and a number k, it is NP-complete to decide whether there is a coordination with total wiggle count k.

Drawing Wiggles

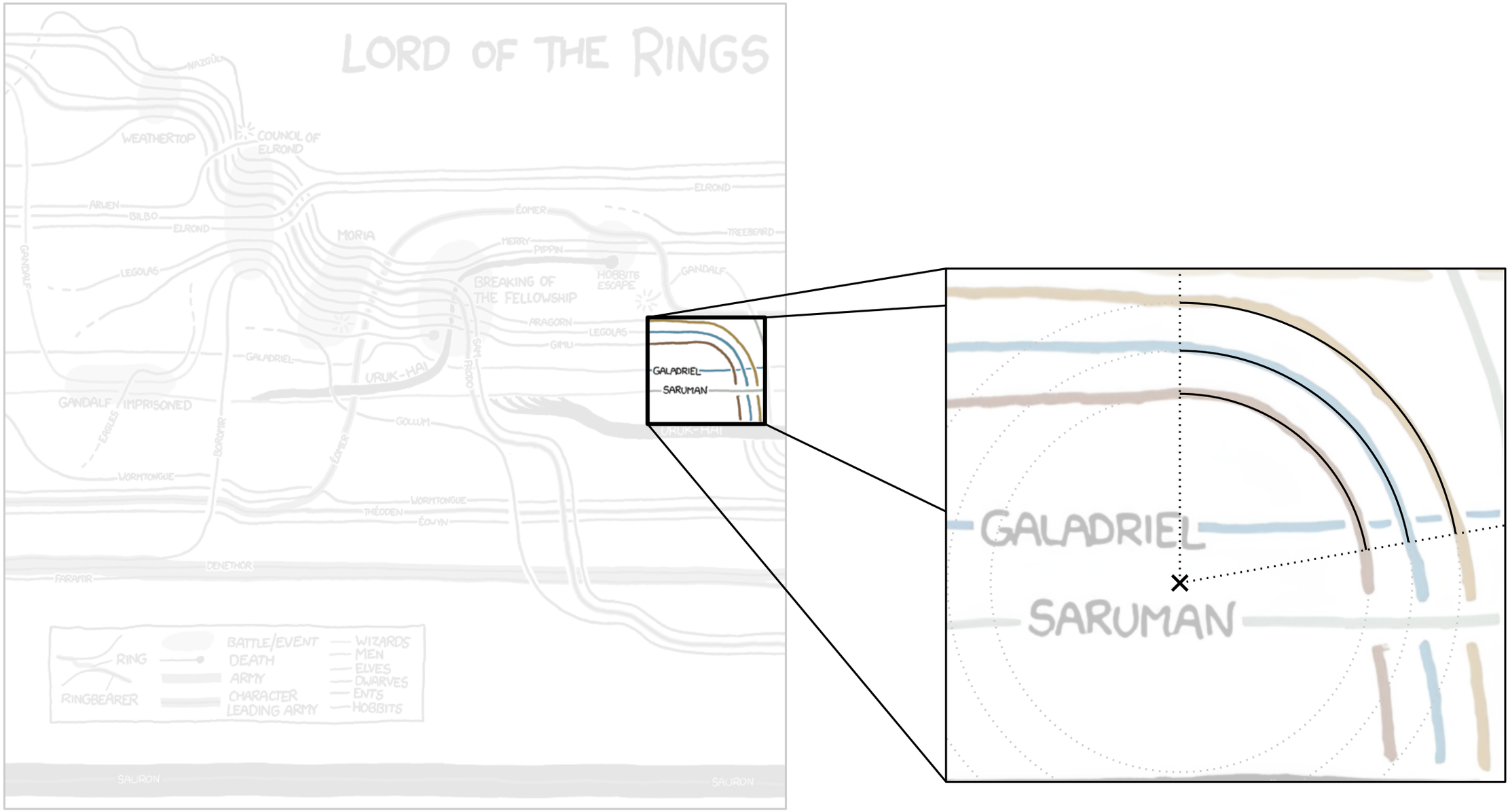


[xkcd 657. Munroe. 2009 (clipping)]

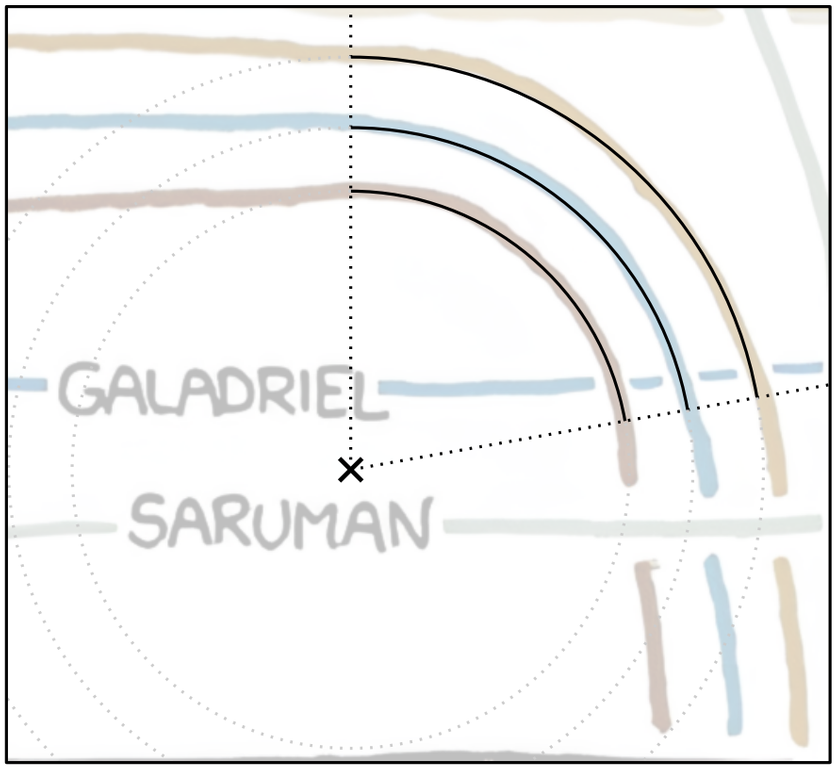
Drawing Wiggles

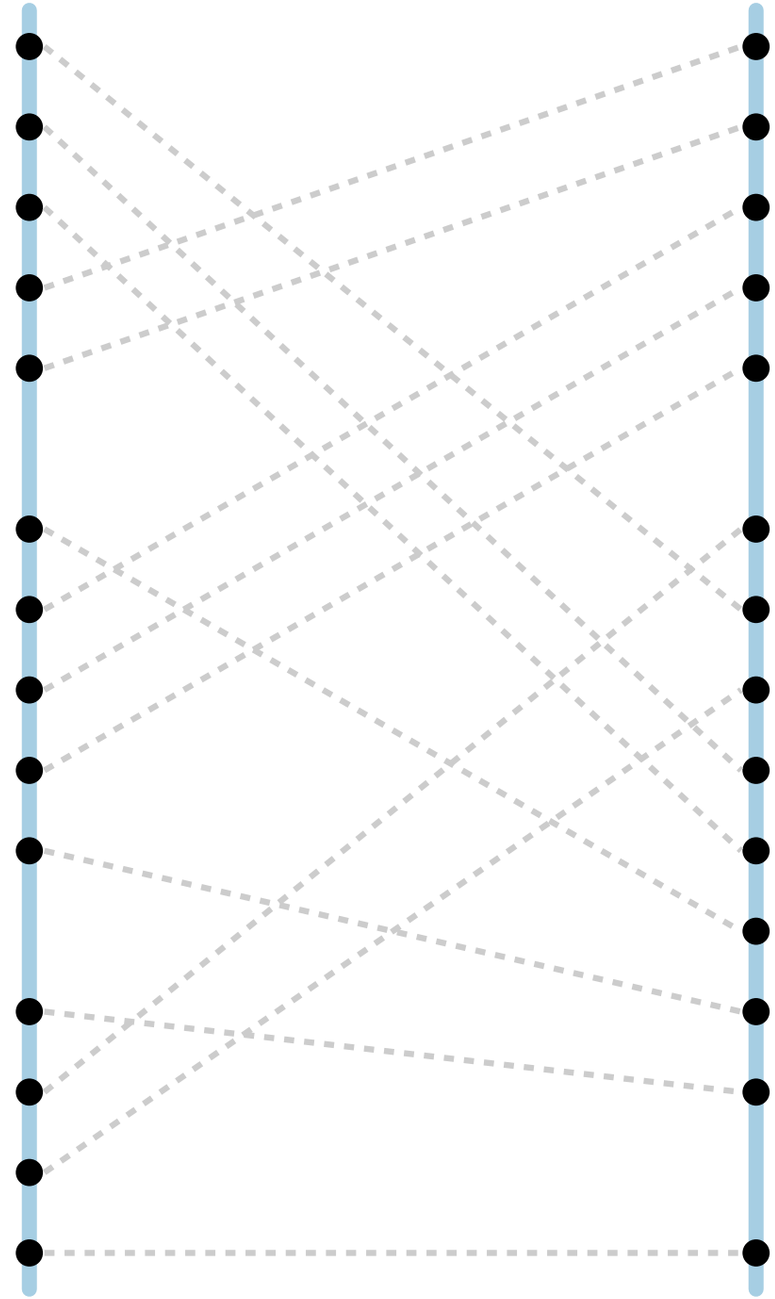


Drawing Wiggles

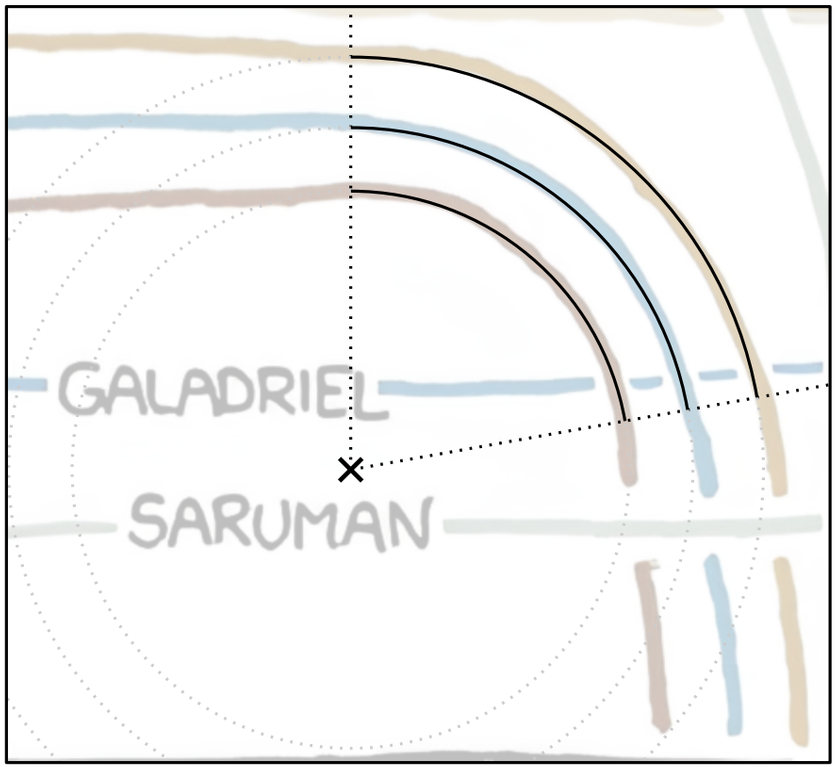


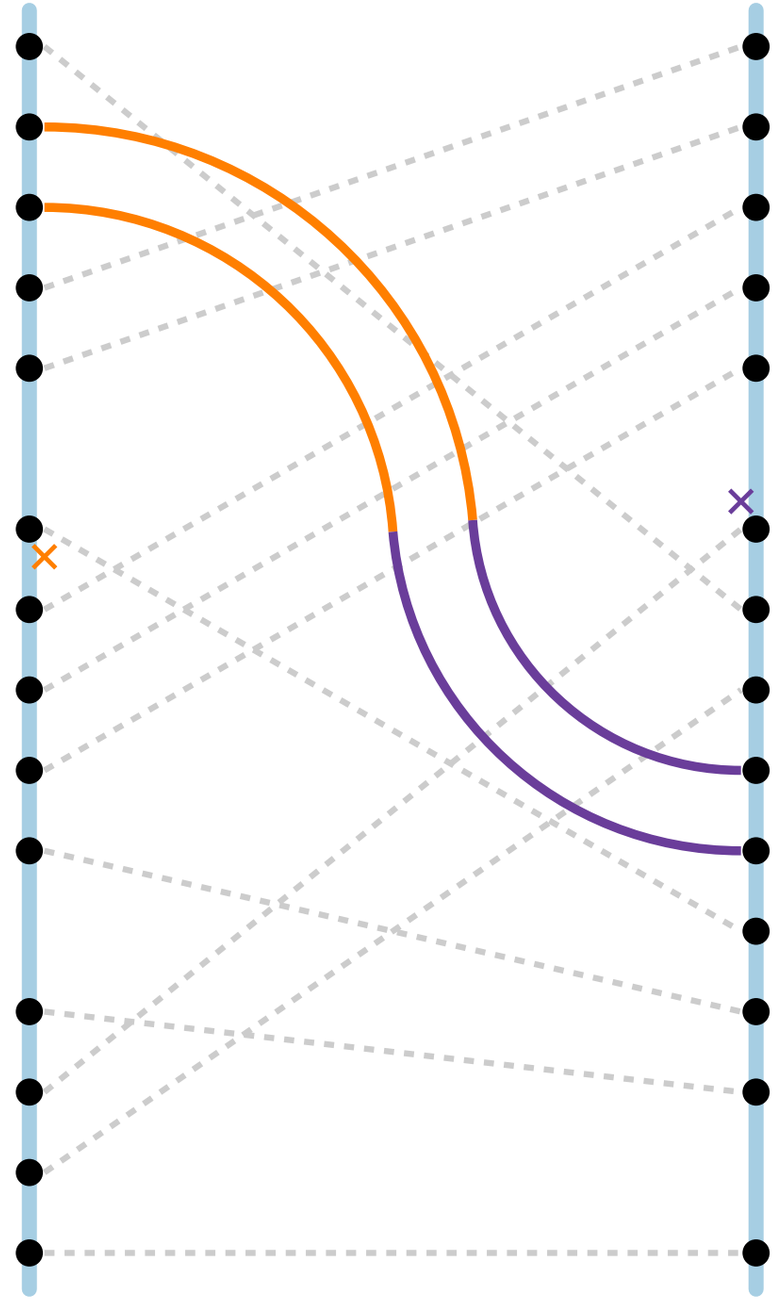
Drawing Wiggles



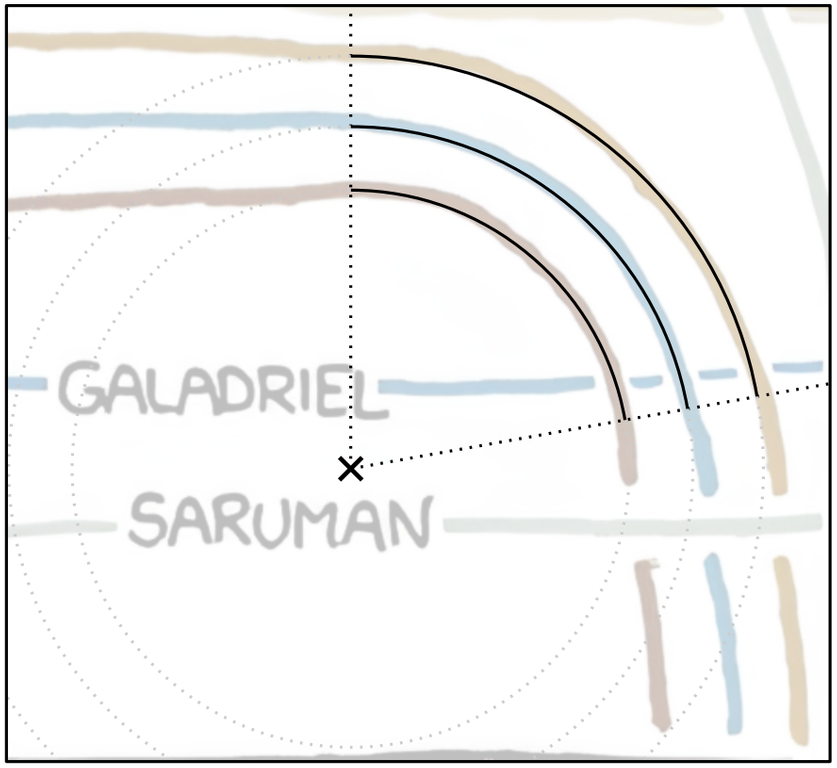


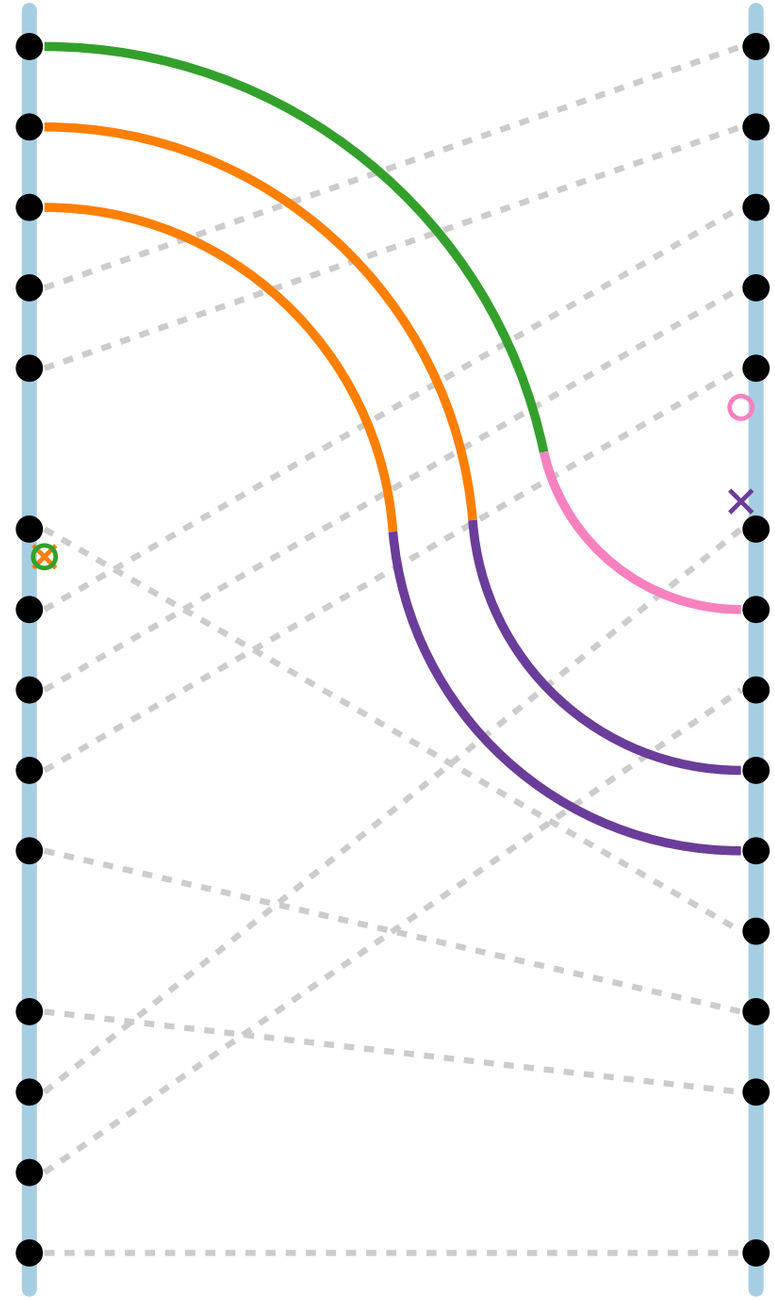
Drawing Wiggles



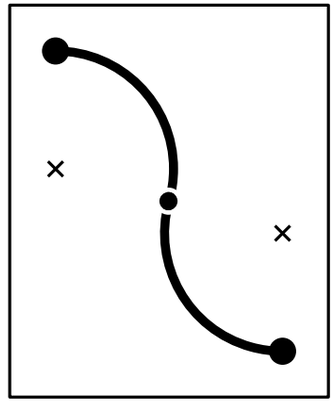


Drawing Wiggles

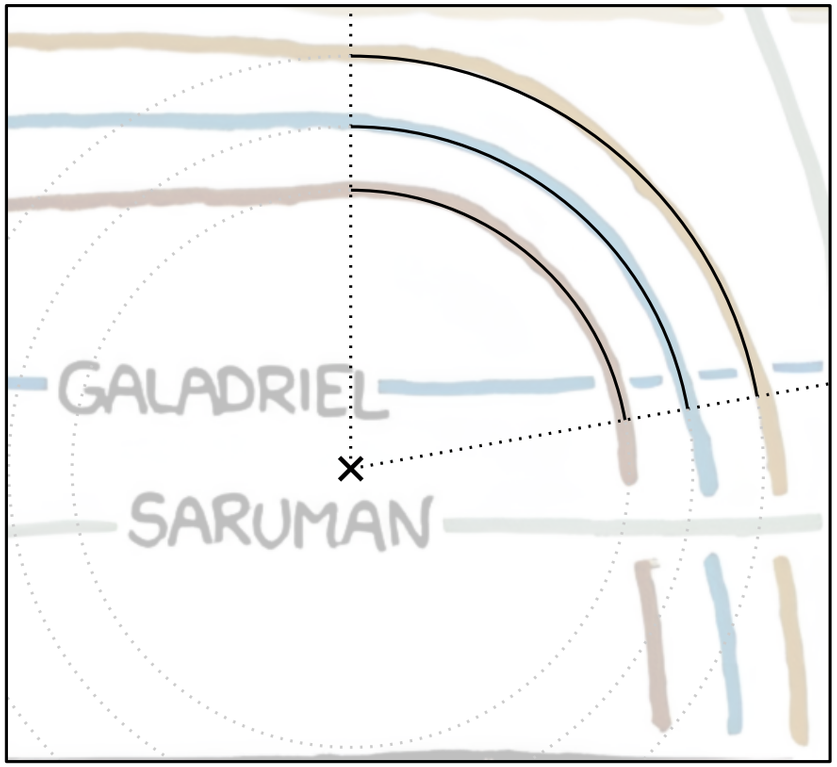


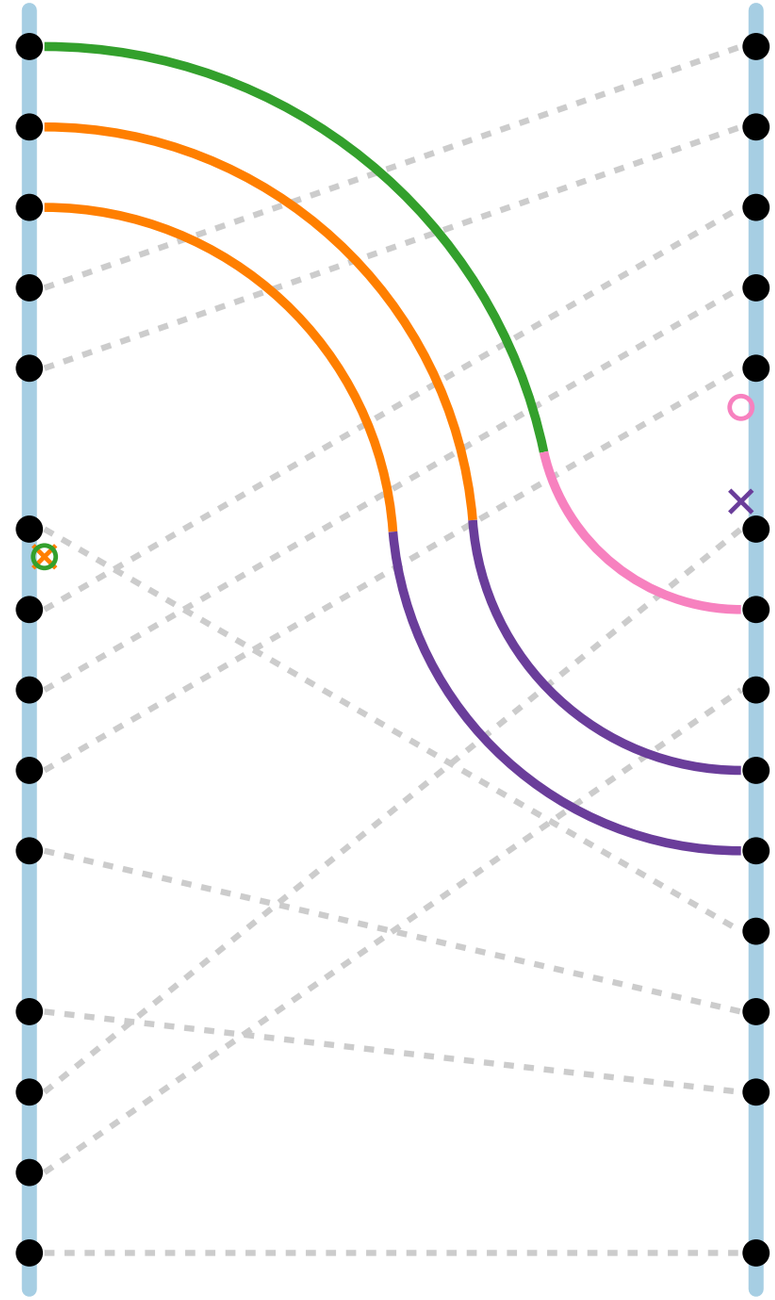


Drawing Wiggles

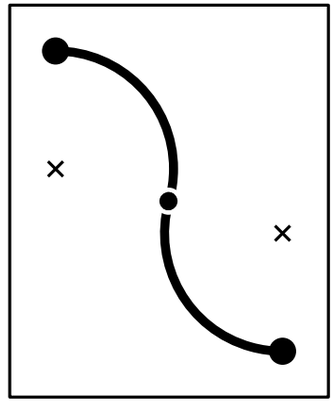


Not x-monotone!





Drawing Wiggles



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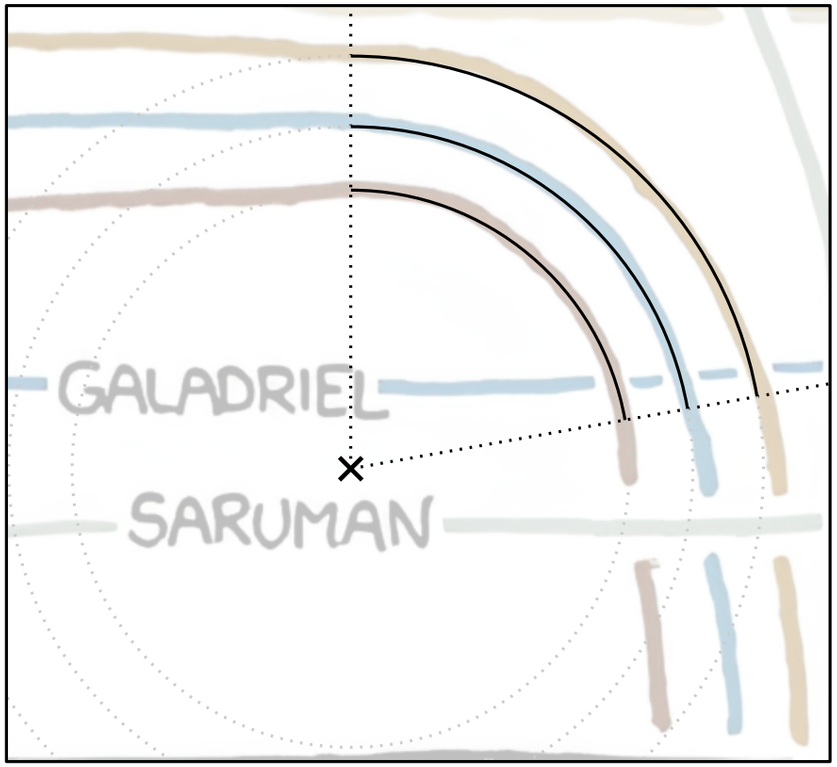
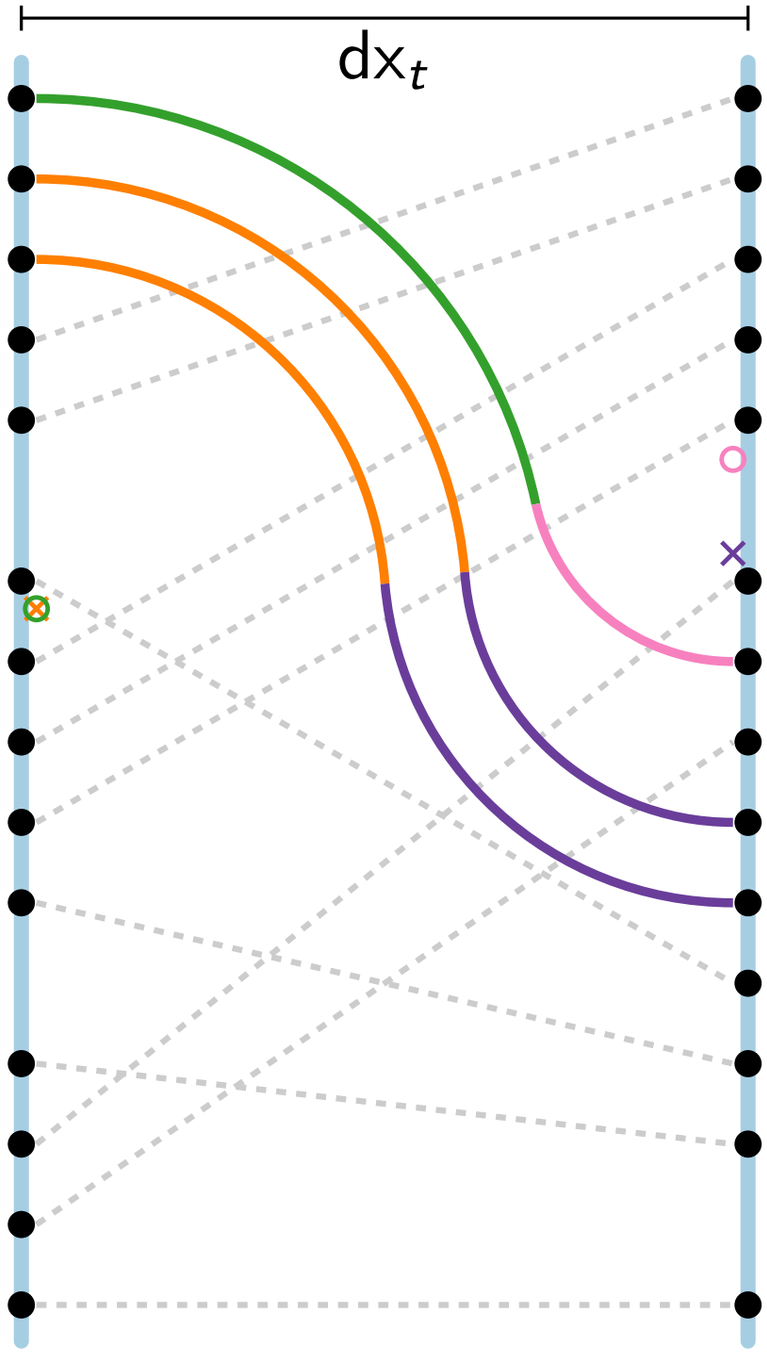
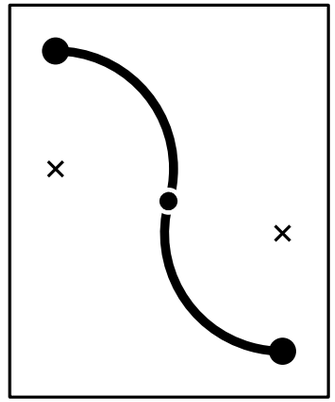




Drawing Wiggles



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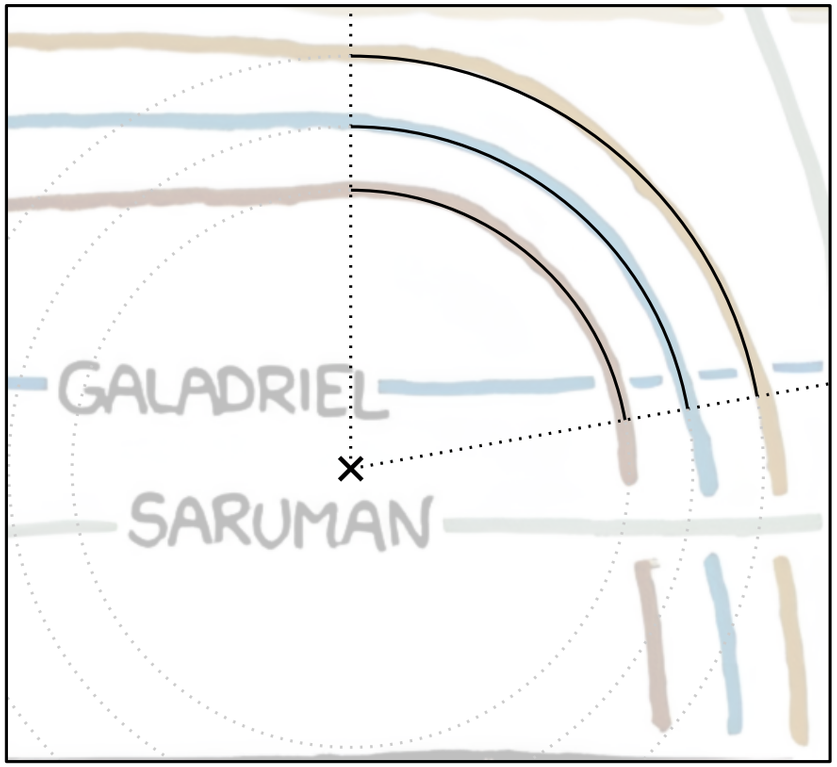
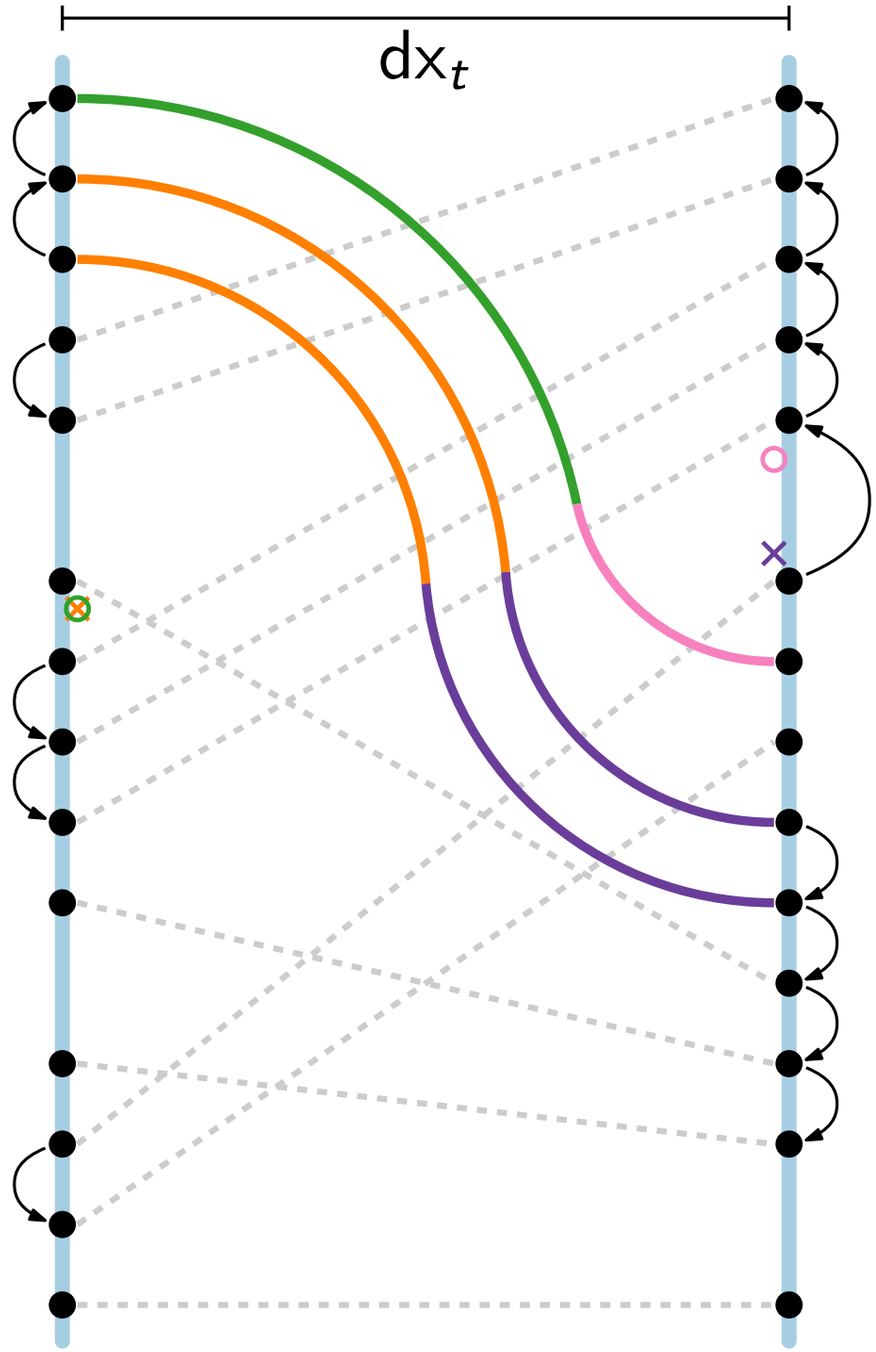
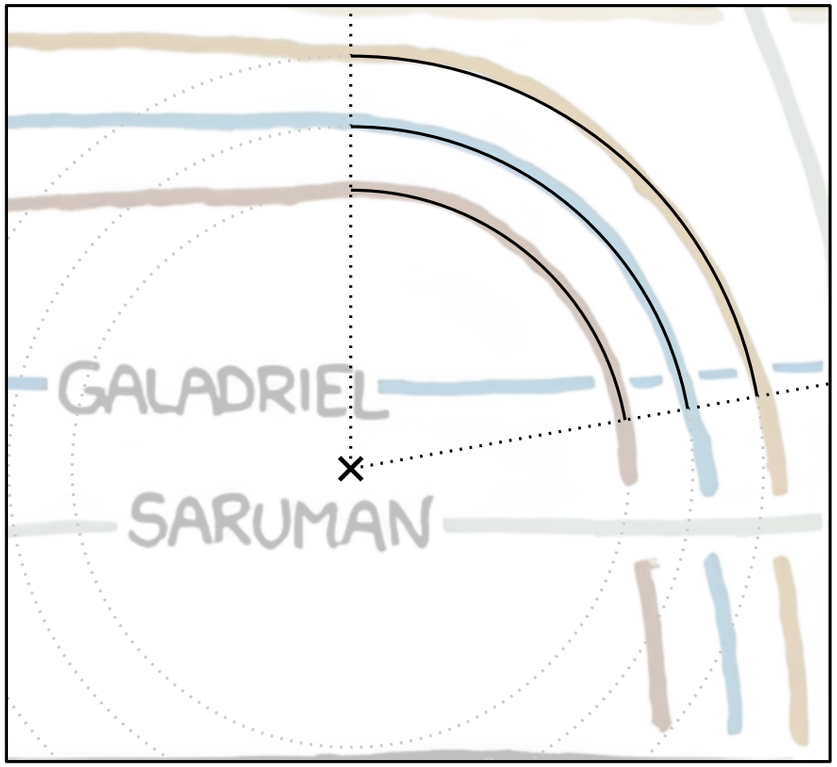
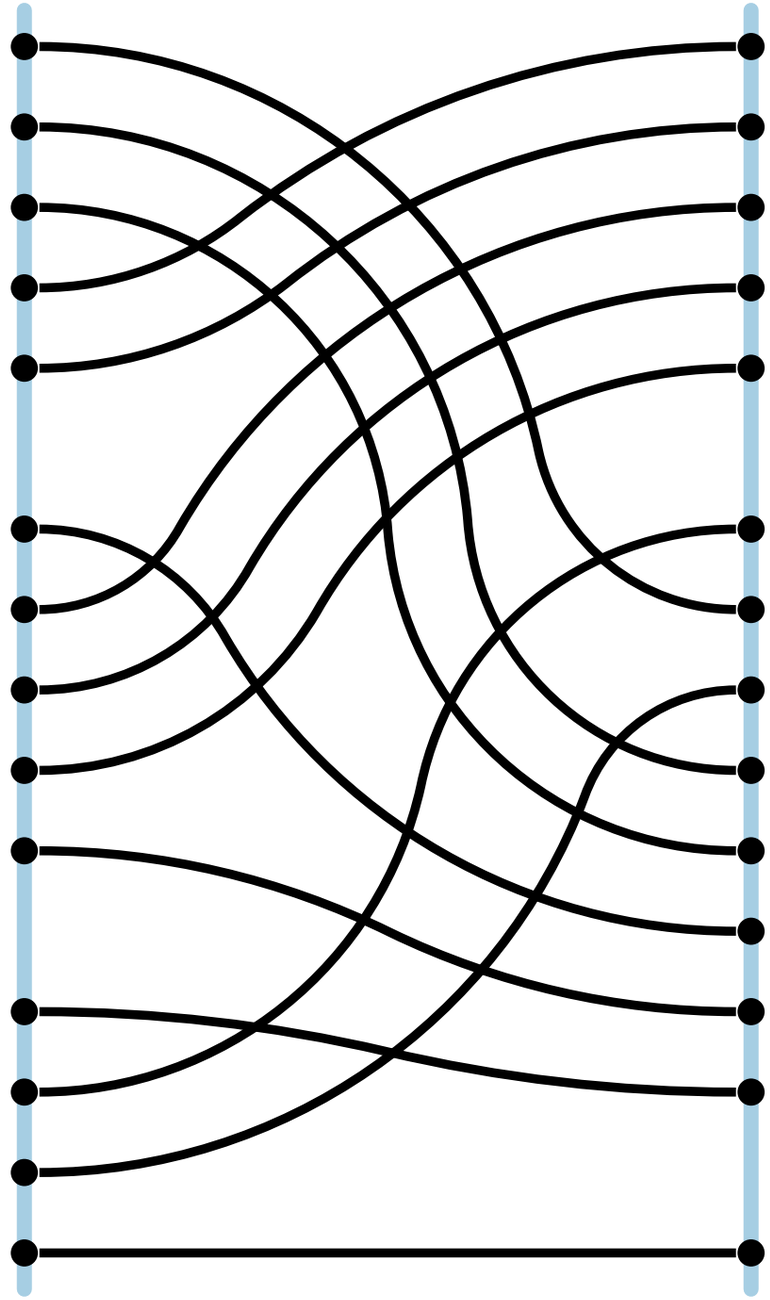




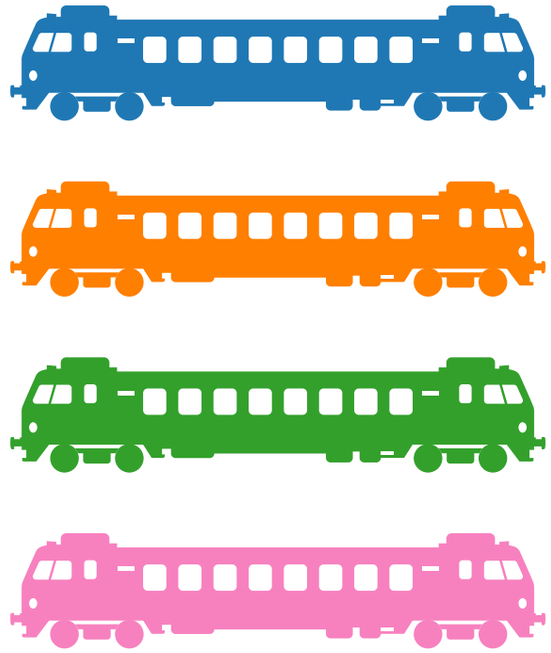
Drawing Wiggles





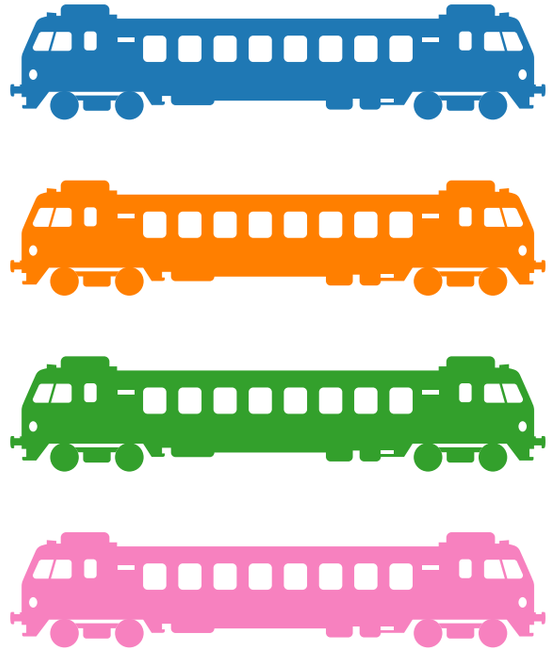
Visualizing Rolling Stock Schedules

Visualizing Rolling Stock Schedules

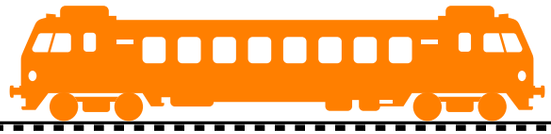


Rolling stock

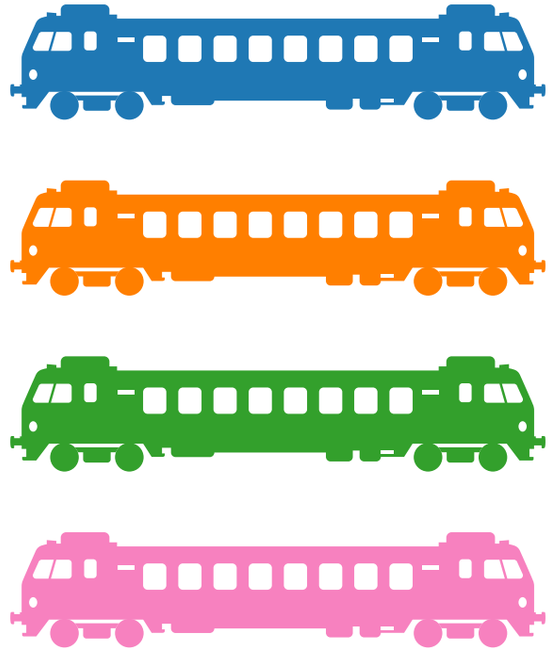
Visualizing Rolling Stock Schedules



Rolling stock



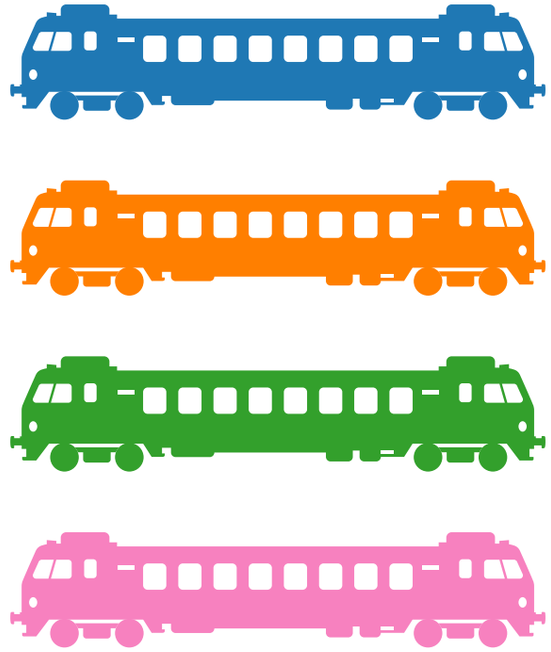
Visualizing Rolling Stock Schedules



Rolling stock

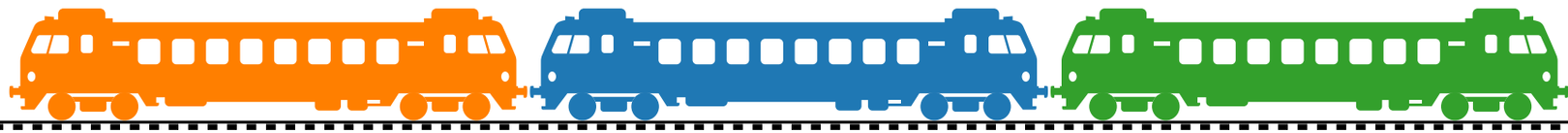
Composition

Visualizing Rolling Stock Schedules



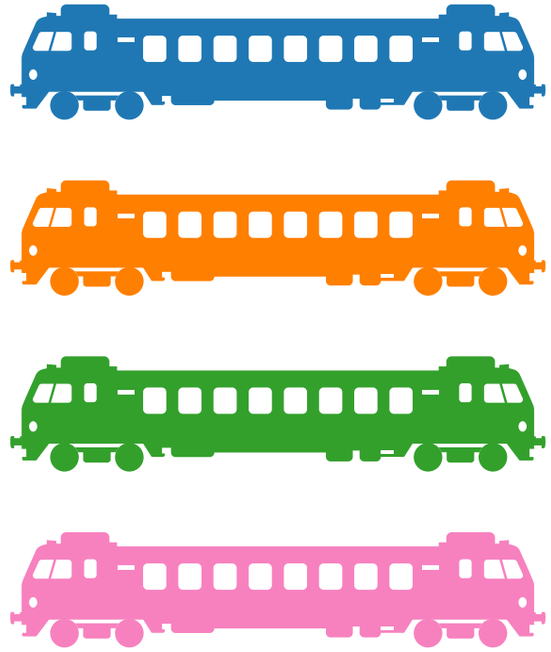
Rolling stock

Composition



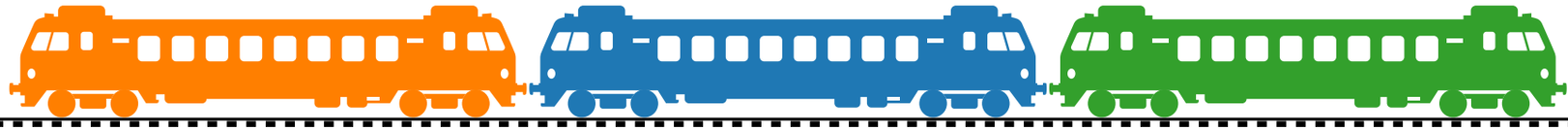
Visualizing Rolling Stock Schedules

Departure		Arrival		Demand	Train units
08^{00}	A	08^{45}	B		
08^{00}	A	08^{30}	C		
08^{00}	C	08^{15}	D		
08^{30}	D	08^{45}	C		
08^{45}	C	09^{00}	E		
09^{00}	B	09^{15}	E		
09^{30}	E	09^{45}	C		
10^{15}	C	10^{45}	A		
		...			



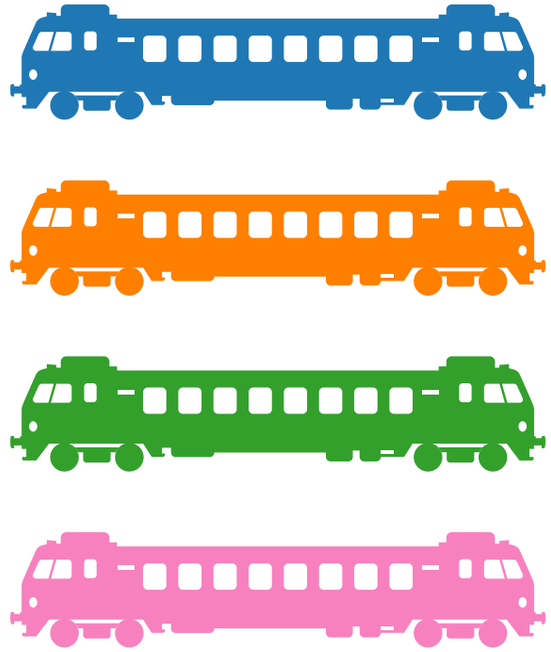
Rolling stock

Composition



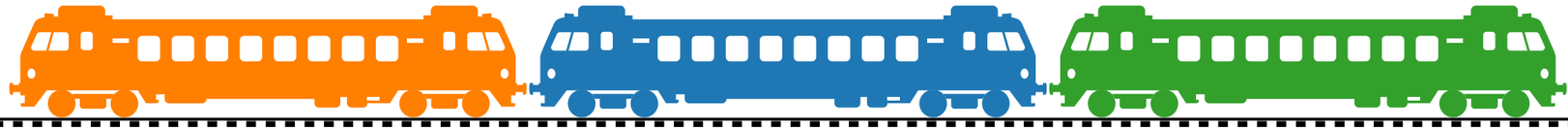
Visualizing Rolling Stock Schedules

Departure		Arrival		Demand	Train units
08^{00}	A	08^{45}	B		
08^{00}	A	08^{30}	C		
08^{00}	C	08^{15}	D		
08^{30}	D	08^{45}	C		
08^{45}	C	09^{00}	E		
09^{00}	B	09^{15}	E		
09^{30}	E	09^{45}	C		
10^{15}	C	10^{45}	A		
		...			



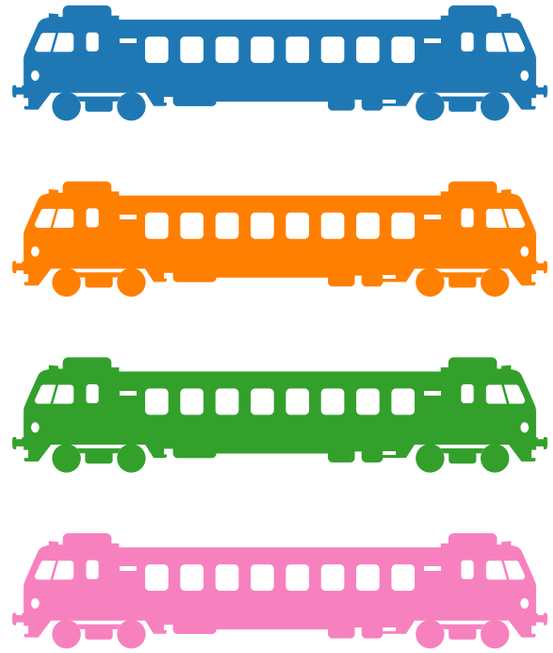
Rolling stock

Composition



Visualizing Rolling Stock Schedules

Departure		Arrival		Demand	Train units
08^{00}	A	08^{45}	B		

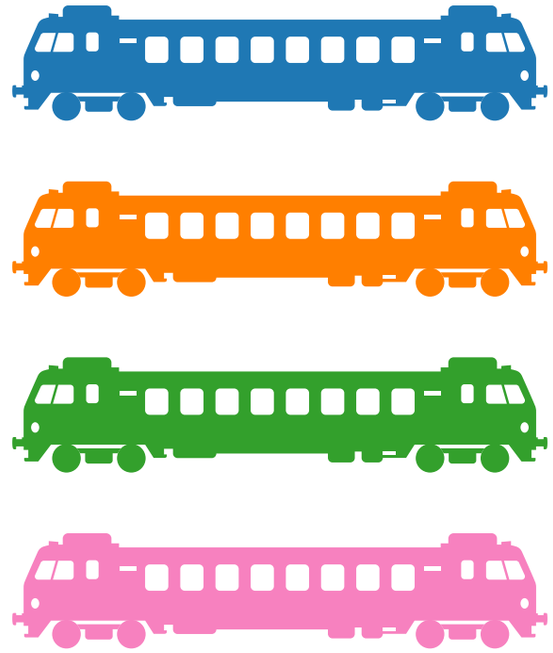


$\rightarrow$ Characters

Visualizing Rolling Stock Schedules

Departure		Arrival		Demand	Train units
08^{00}	A	08^{45}	B		

→ Meetings



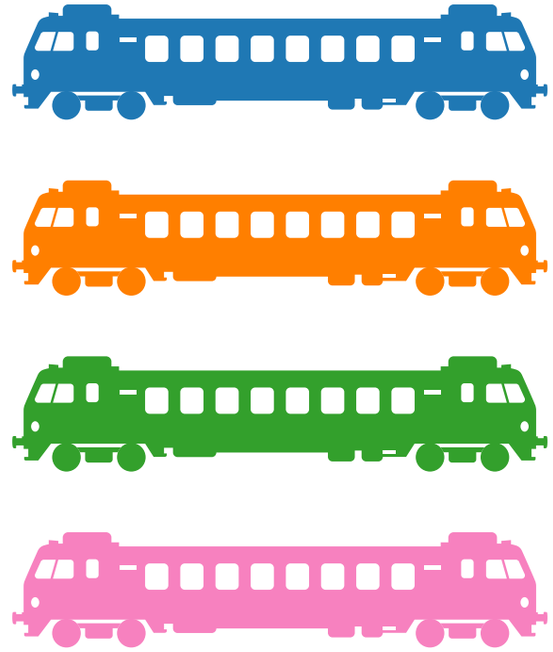
→ Characters

Visualizing Rolling Stock Schedules

Departure		Arrival		Demand	Train units
08^{00}	A	08^{45}	B		

→ Meetings

1 departure meeting



→ Characters

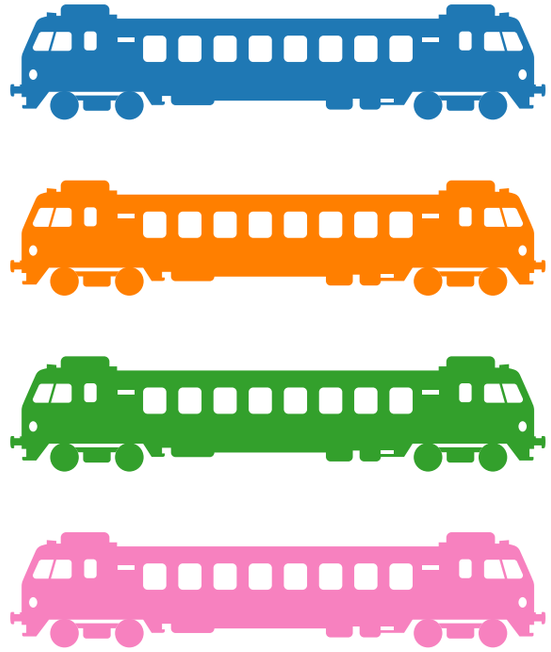
Visualizing Rolling Stock Schedules

Departure	Arrival	Demand	Train units
08^{00} A	08^{45} B		

→ Meetings

1 arrival meeting

1 departure meeting



→ Characters

Visualizing Rolling Stock Schedules

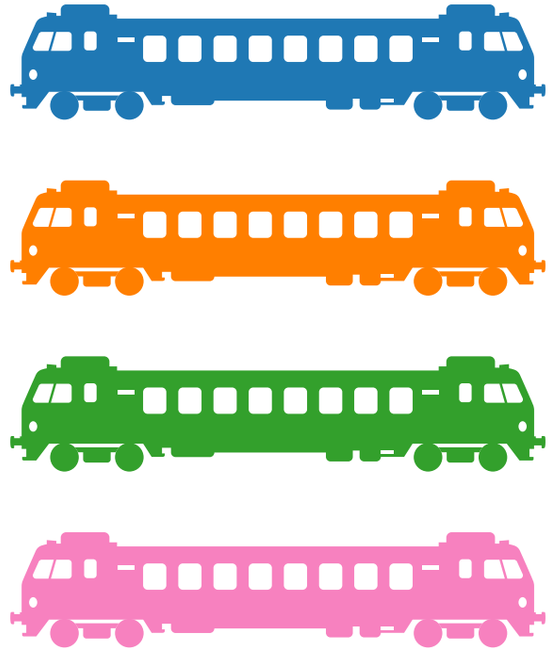
Departure	Arrival	Demand	Train units
08^{00} A	08^{45} B		

→ Meetings

1 arrival meeting

1 departure meeting

intermediate meetings



→ Characters

Visualizing Rolling Stock Schedules

Departure	Arrival	Demand	Train units
08^{00} A	08^{45} B		

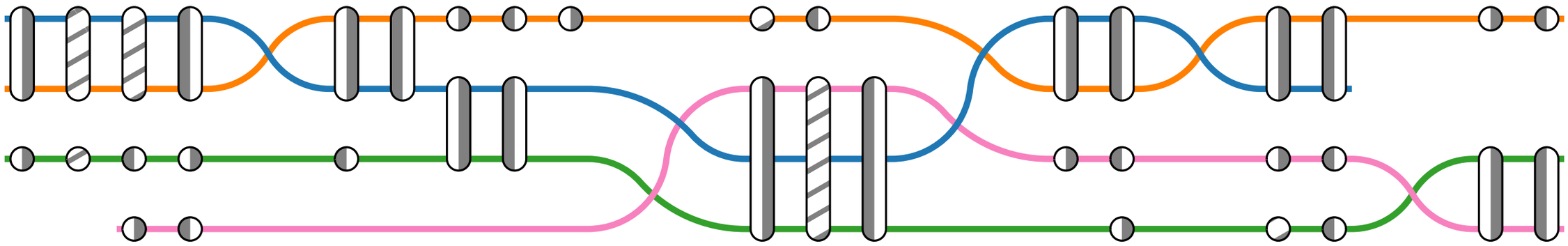
→ Meetings

1 arrival meeting

1 departure meeting

intermediate meetings

→ Characters



Visualizing Rolling Stock Schedules

Departure	Arrival	Demand	Train units
08^{00} A	08^{45} B		

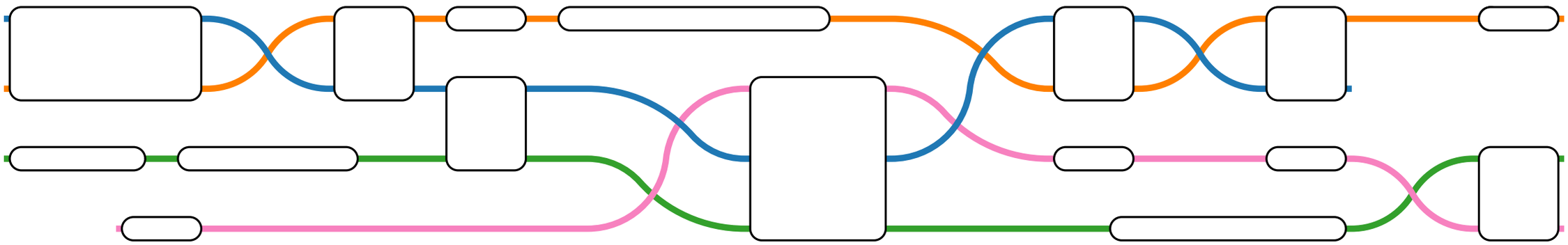
→ Meetings

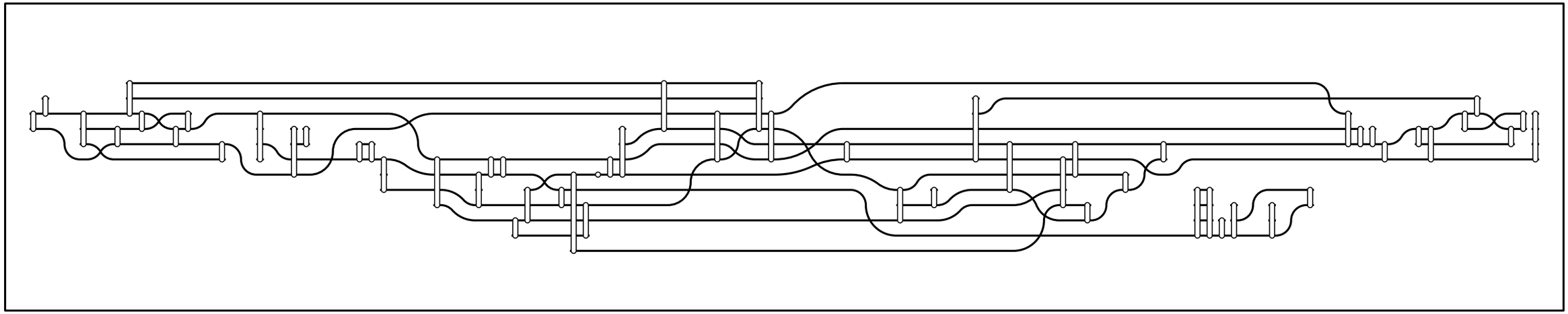
1 arrival meeting

1 departure meeting

intermediate meetings

→ Characters





Open Questions

- Is Wiggle Count Minimization in FPT with respect to the number of time steps? Can it be approximated?
- Is there a more compact geometric solution that maintains the advantages of our method?
- What are the effects on readability of crossing and wiggle optimization in storylines?