Reconfiguration in Curve Arrangements to Reduce Self-Intersections and Popular Faces

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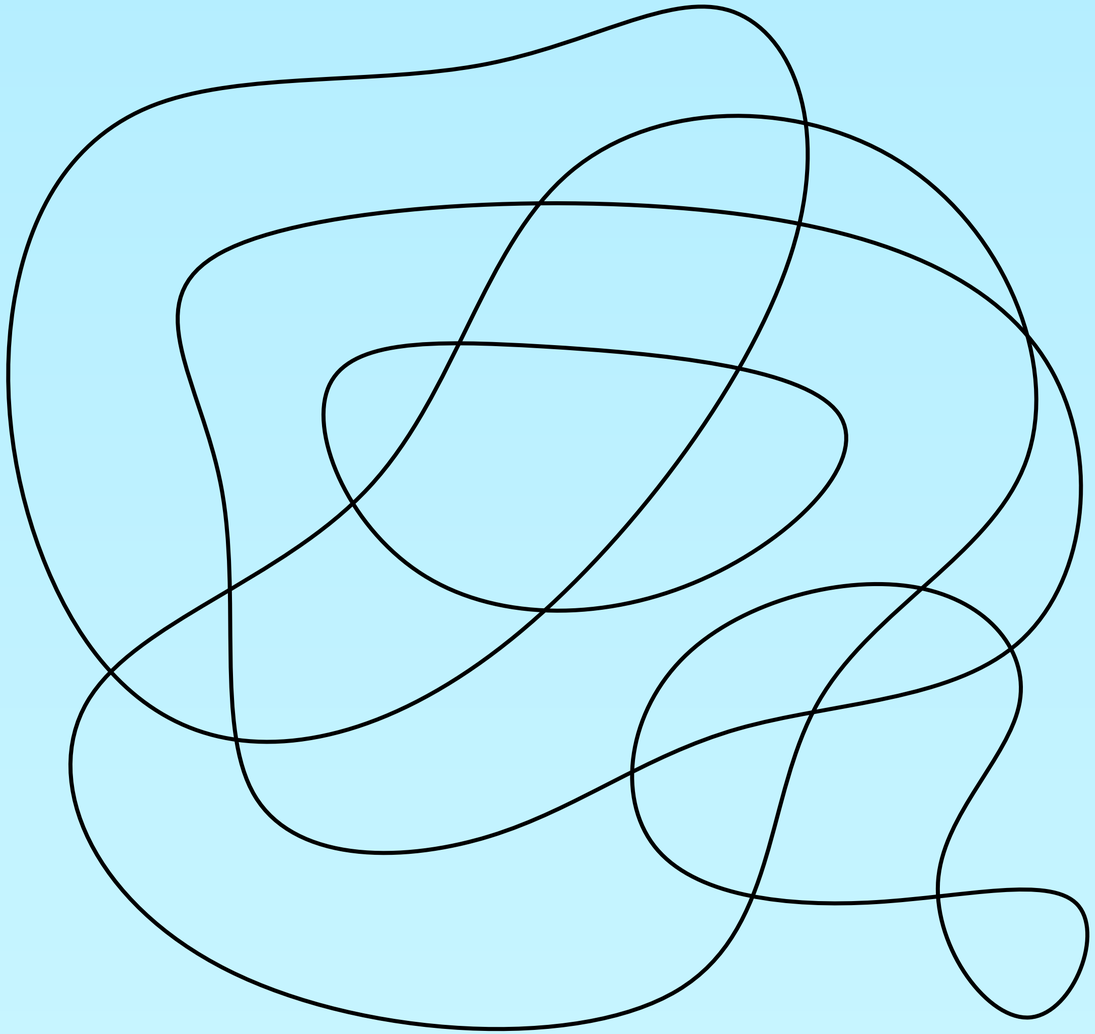
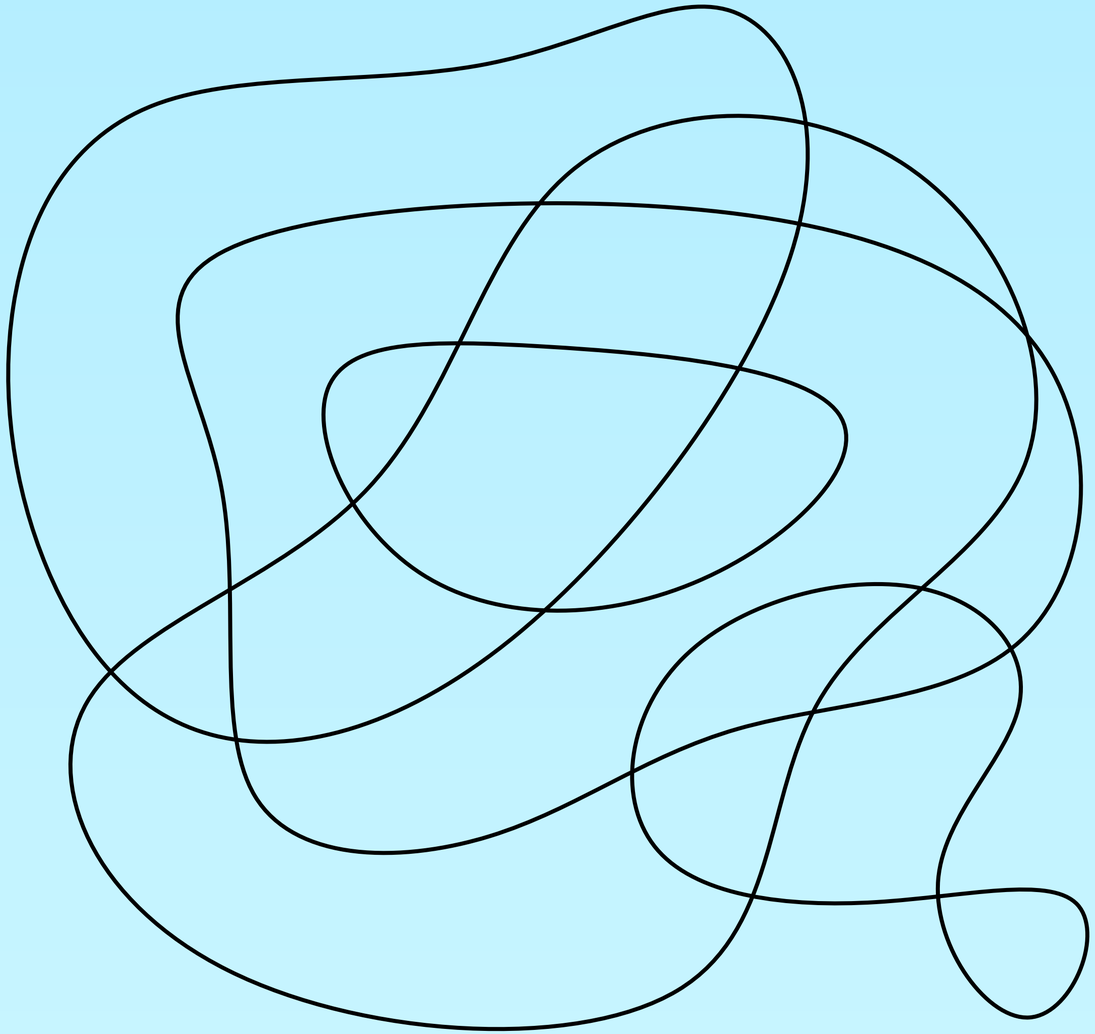
Utrecht University / Tulane University

Joint work with

Florestan Brunck
Hsien-Chih Chang
Tim Ophelders
Lena Schlipf



Consider an arrangement of closed curves in the plane.



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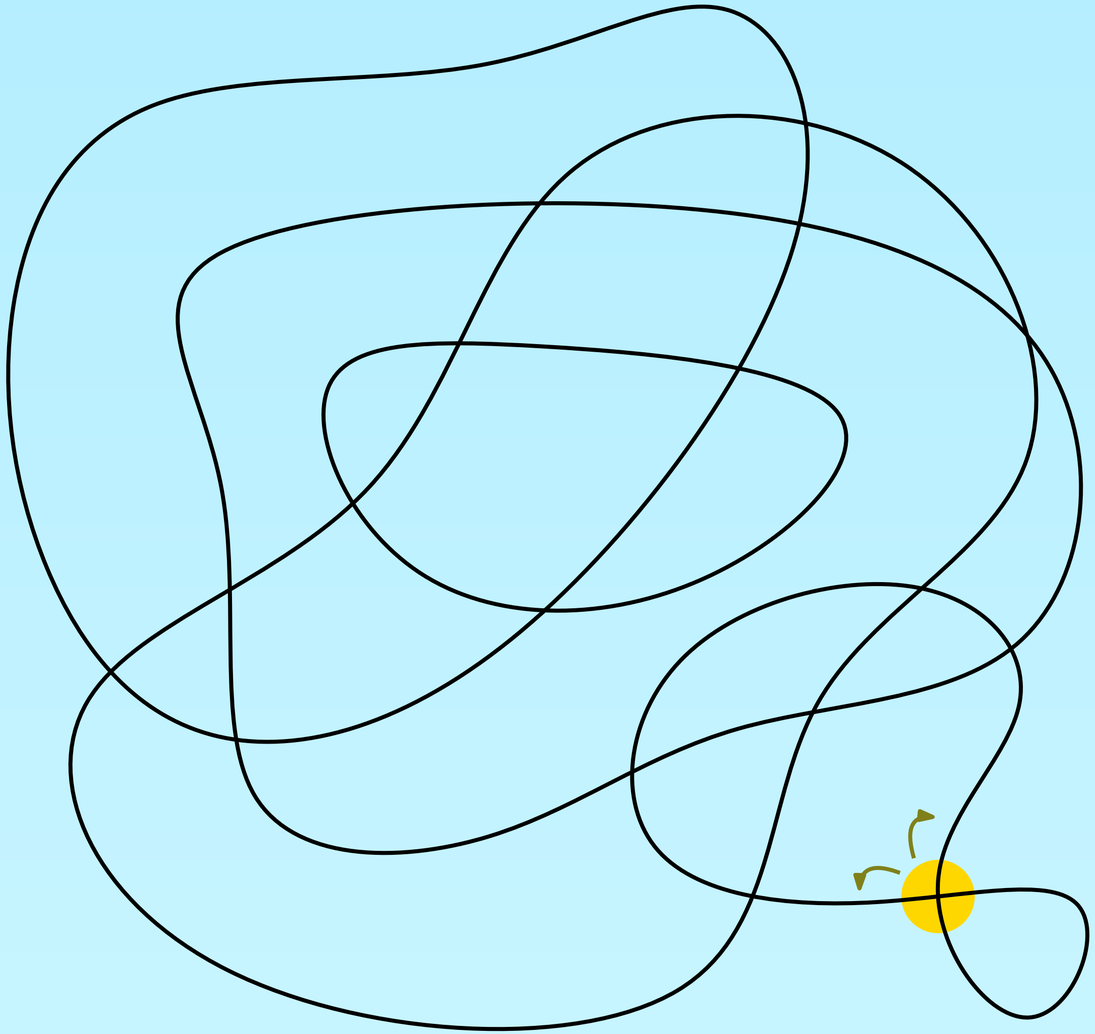
A vertex of the arrangement that is incident to the same curve twice is a self-intersection.



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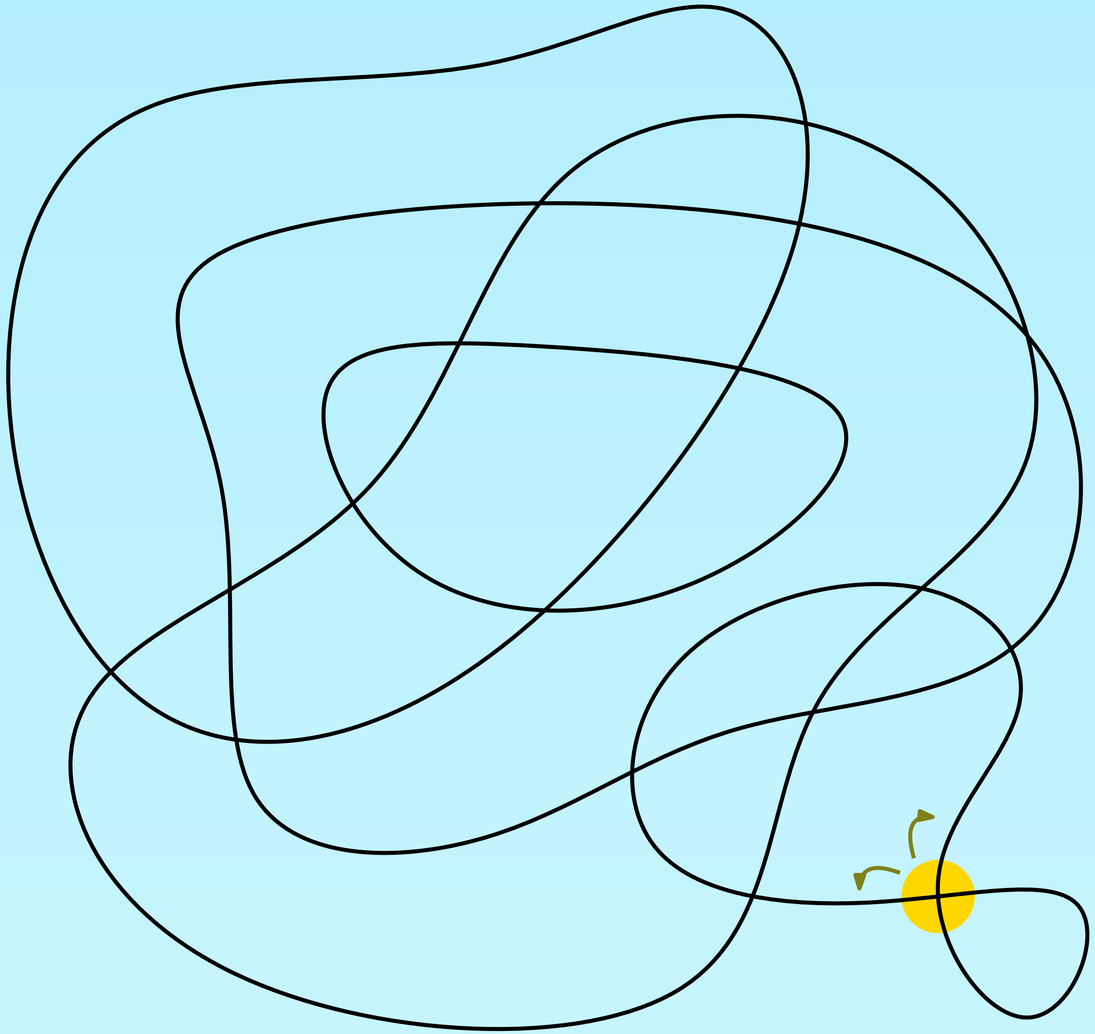

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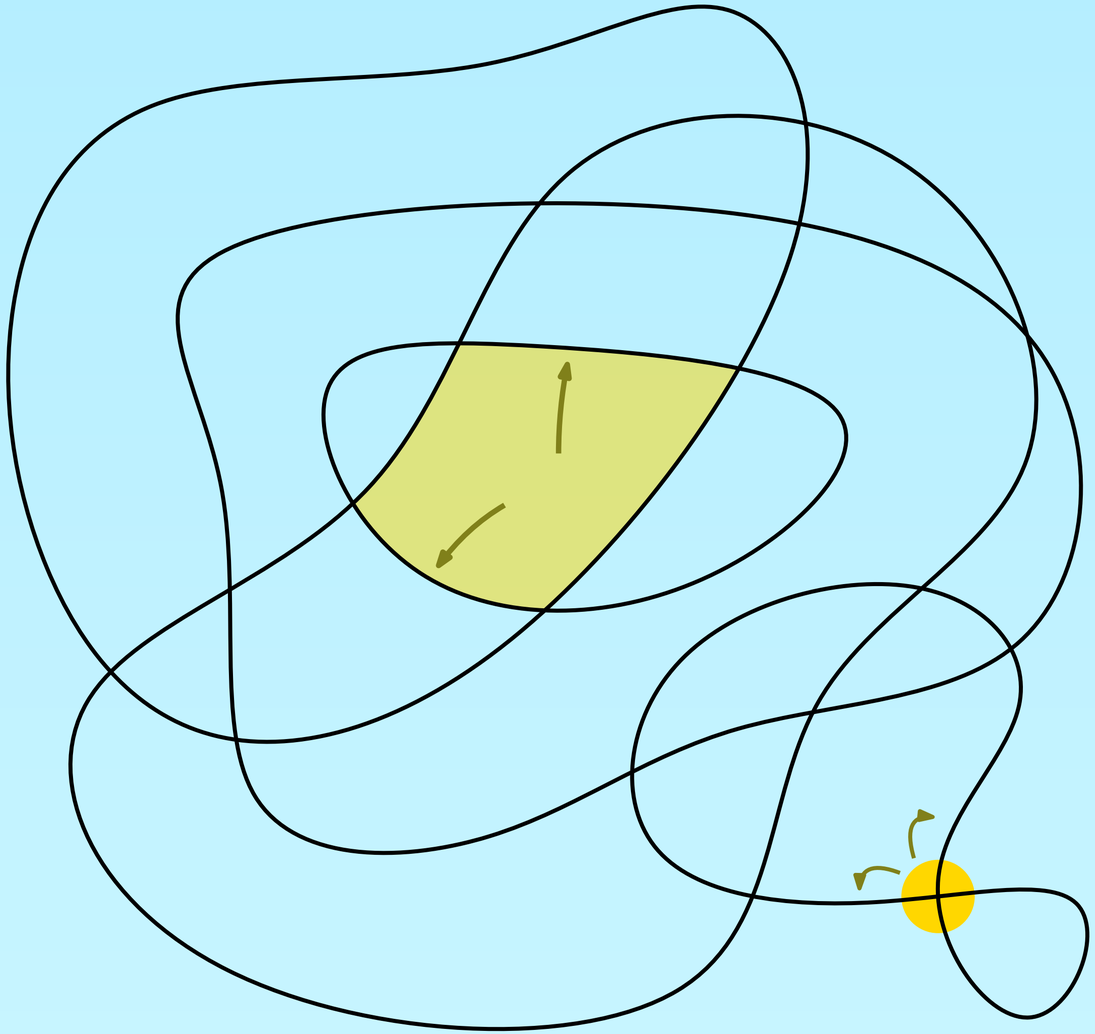

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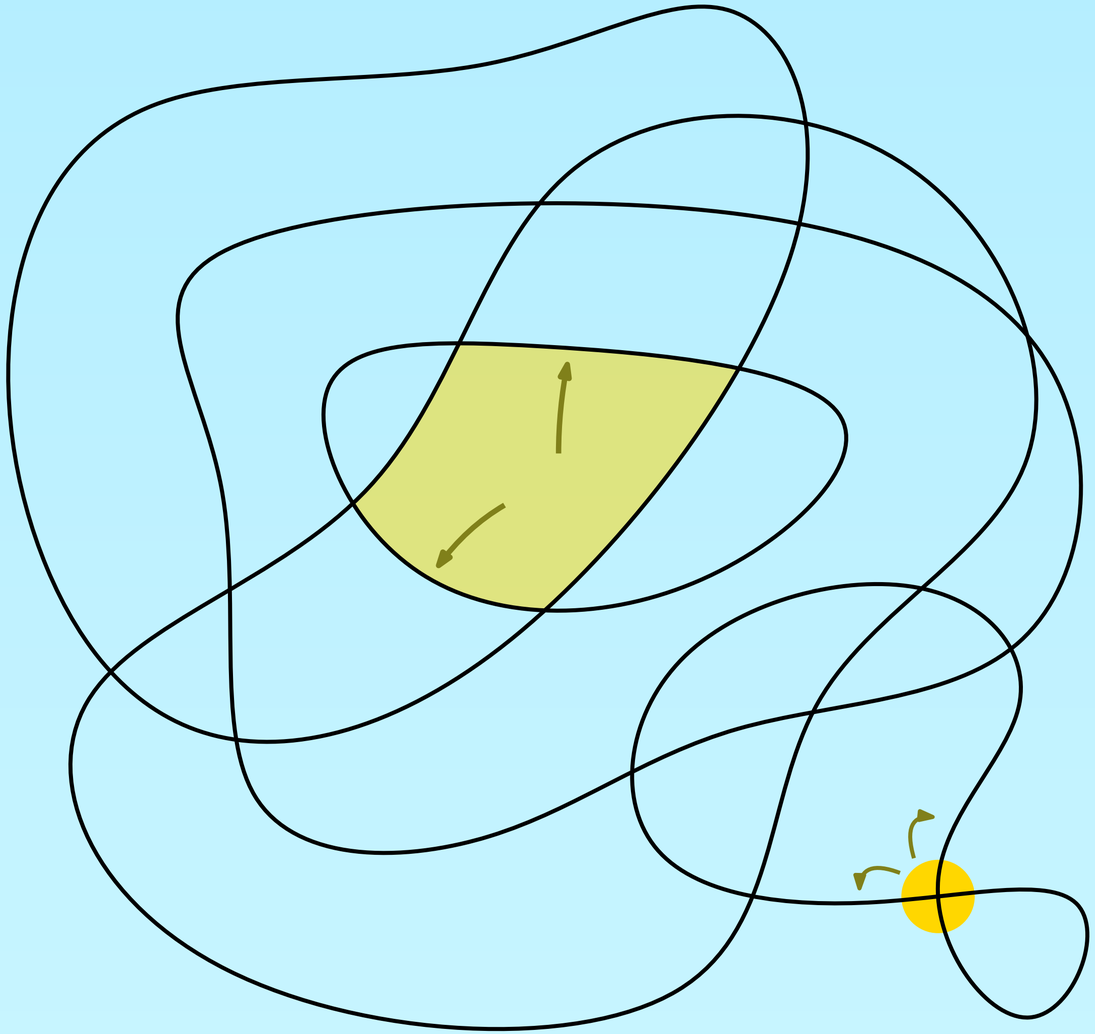

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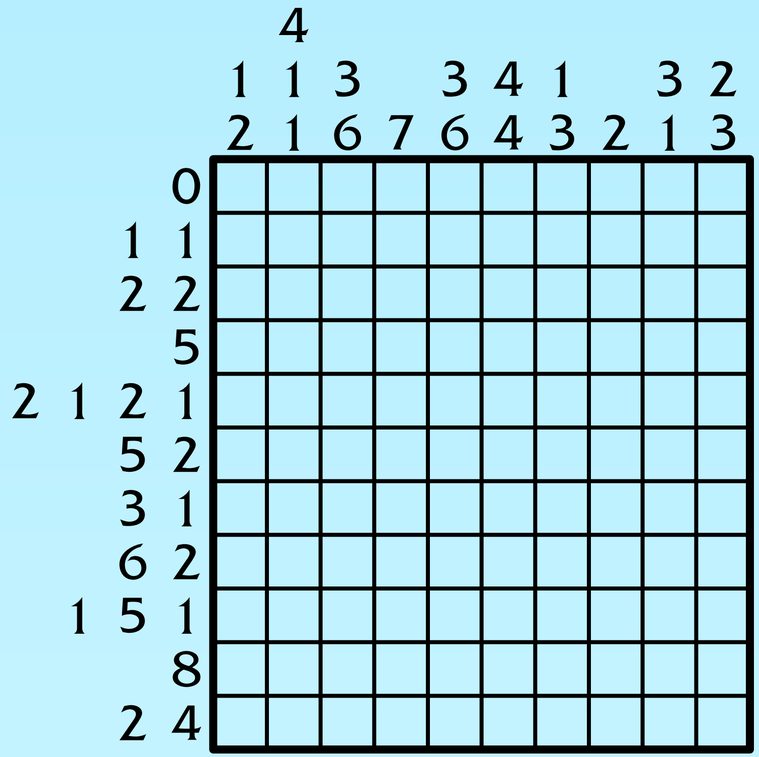
We want to get rid of self-intersections and popular faces.

Part 1

WHY?

Let's talk about Nonograms (a.k.a. Griddlers, Japanese puzzles, Hanjie, Picross, ...)



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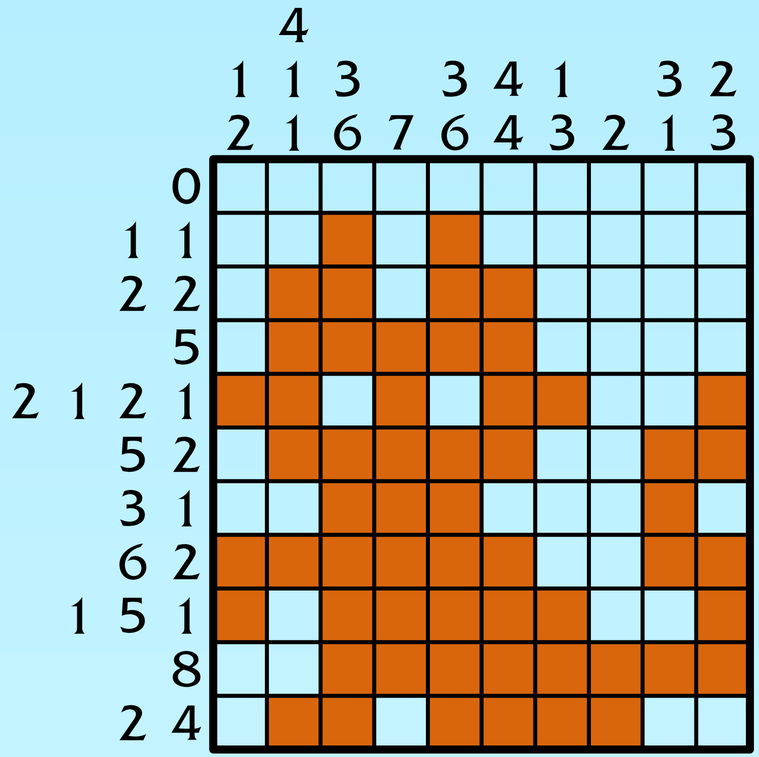

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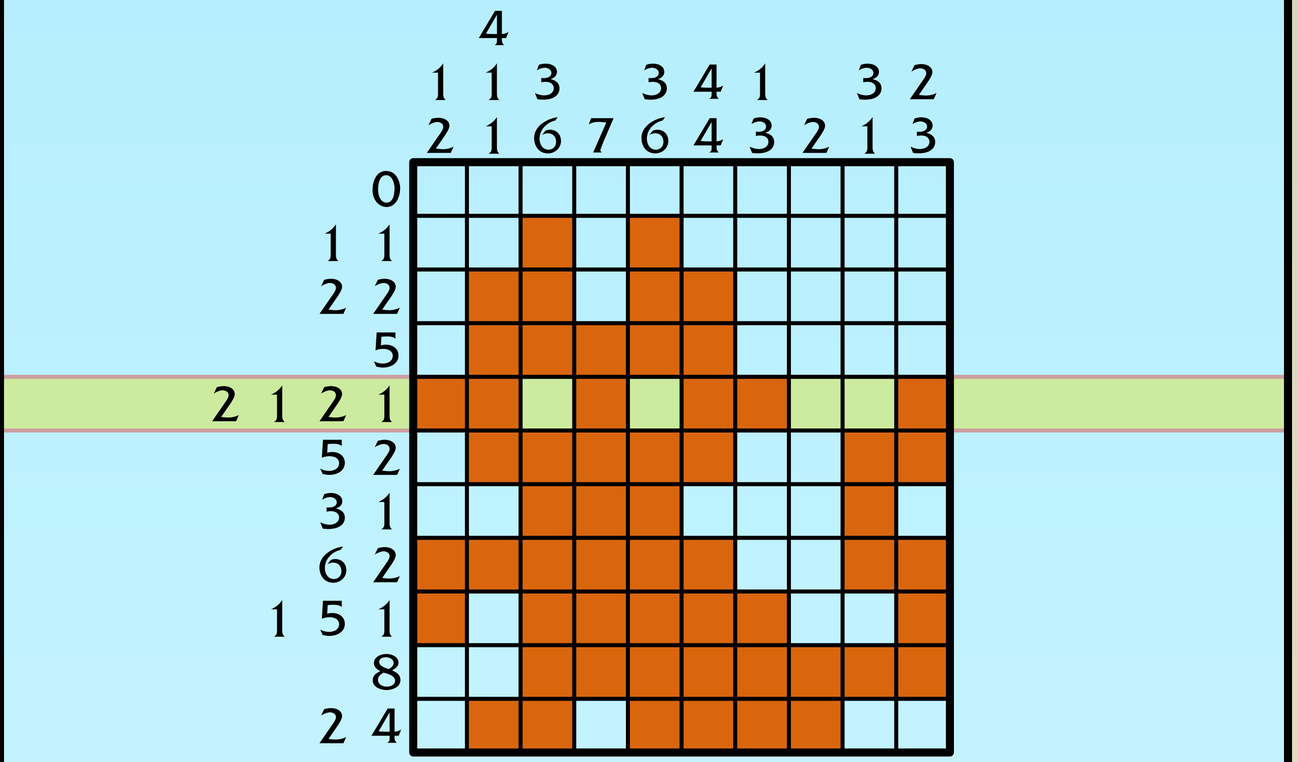

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Input: a grid with some numbers next to and above it.

Output: a black/white coloring of the grid cells.

Rule: in each row and column, the numbers correspond to the sizes of the consecutive blocks of black cells.

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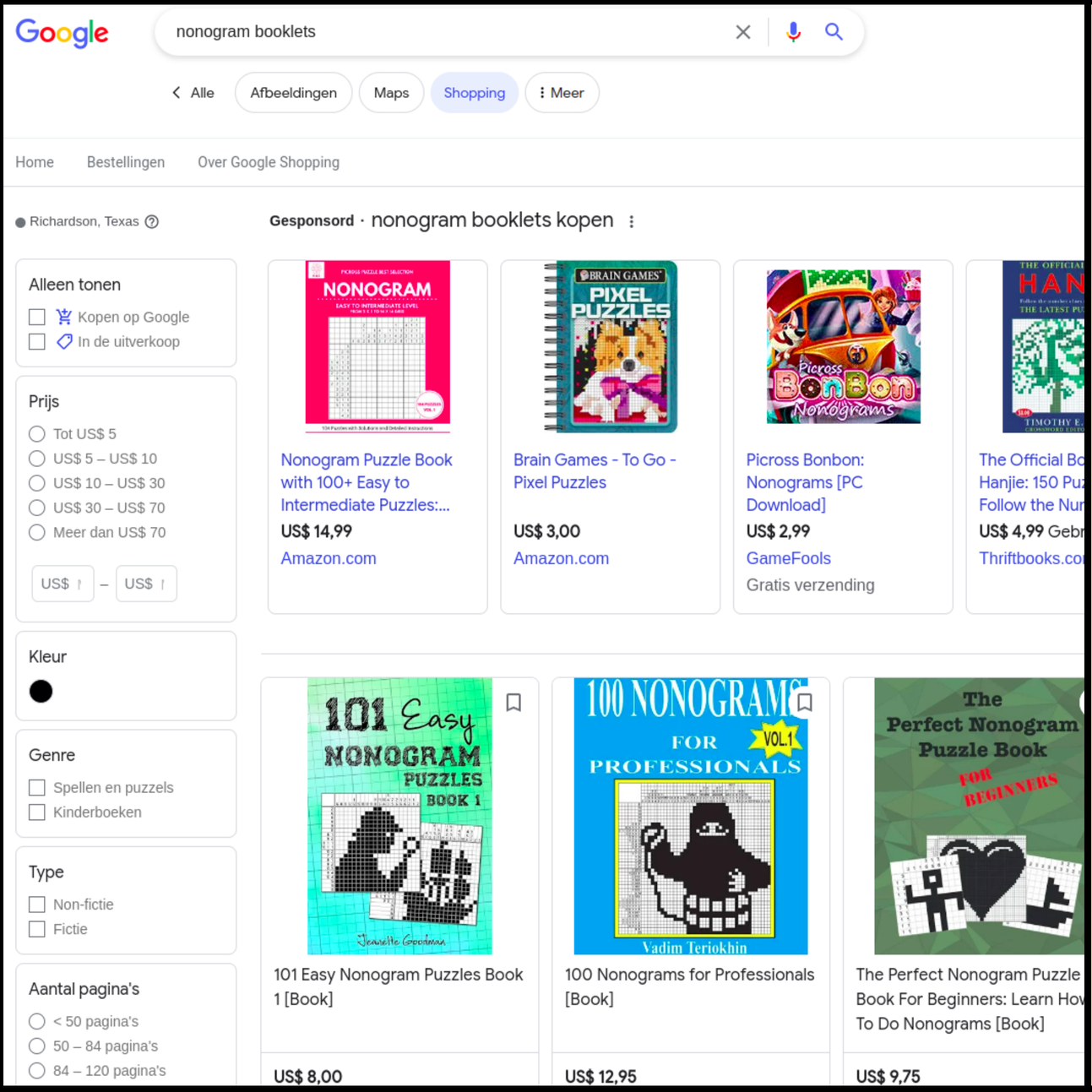

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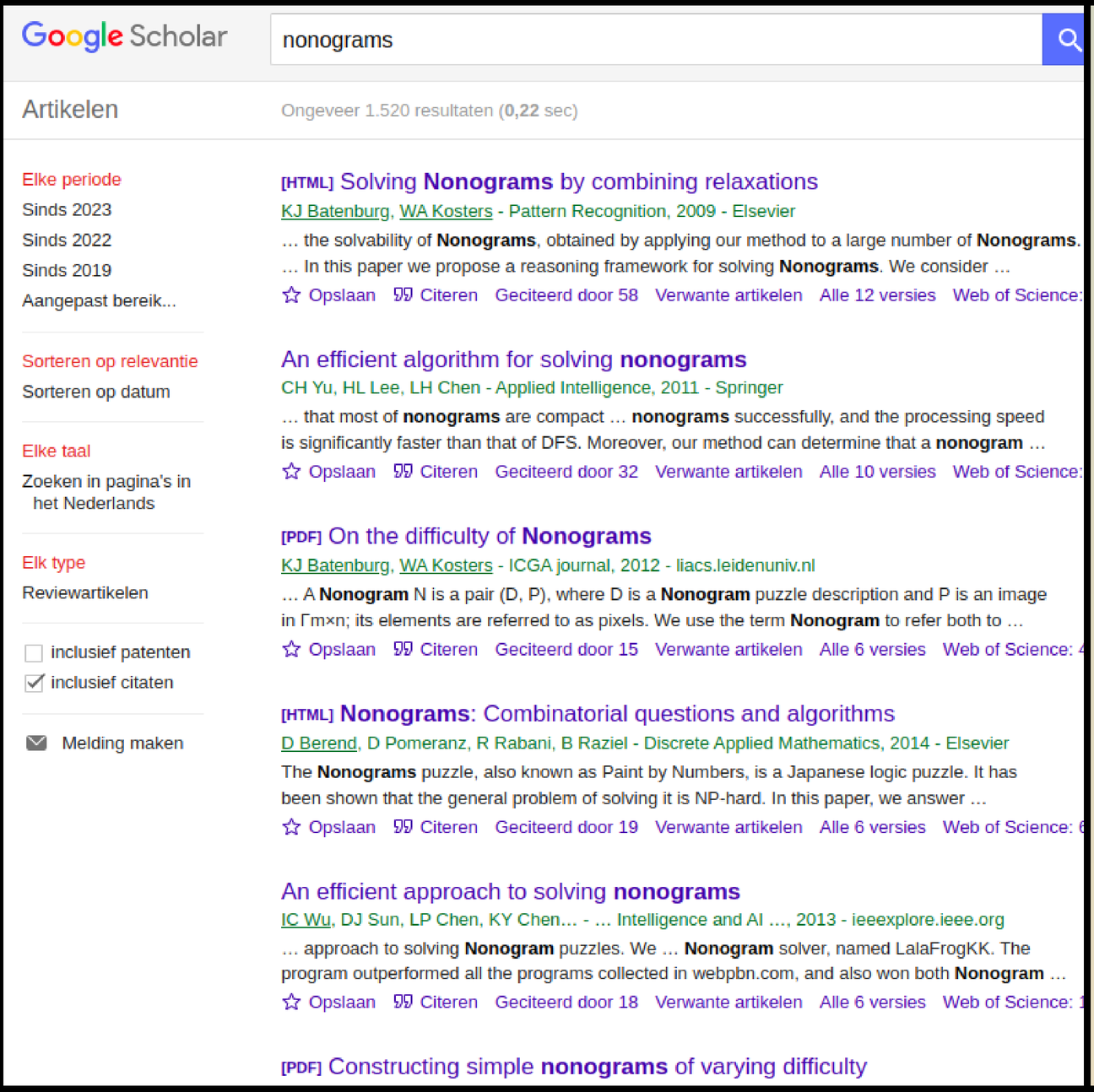

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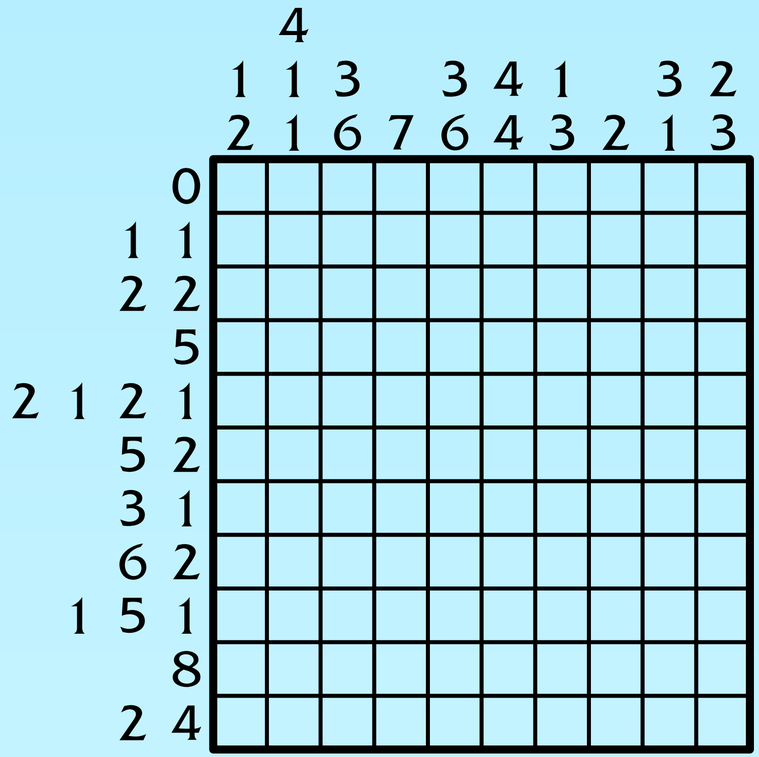
Very popular with consumers ...

... and scientists.



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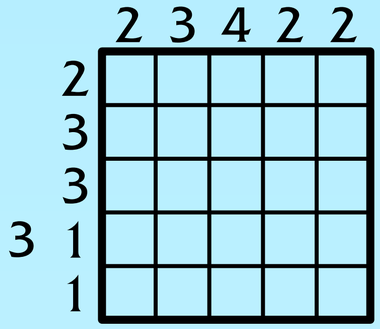

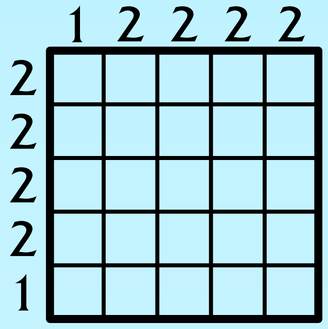

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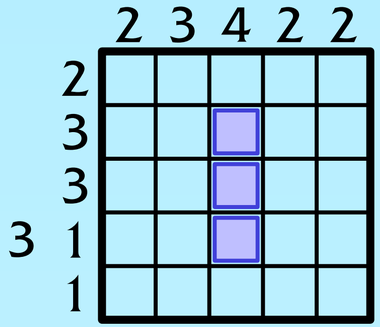

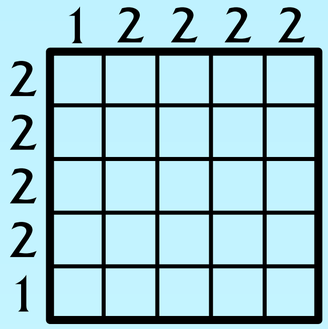

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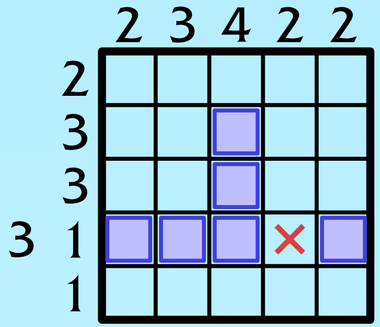

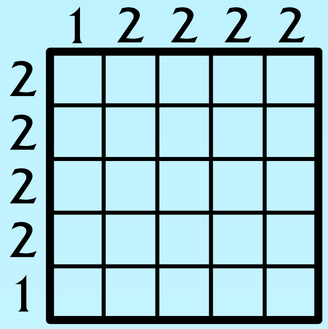

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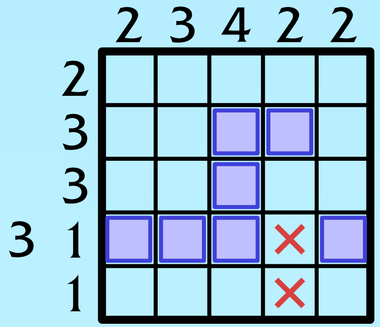

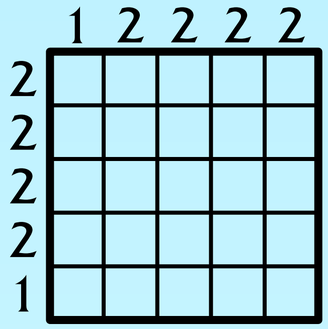

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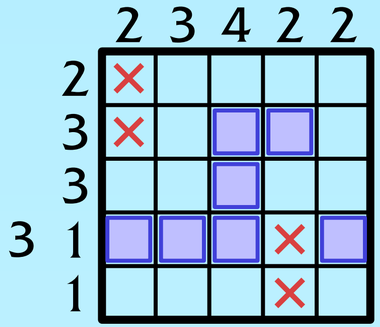

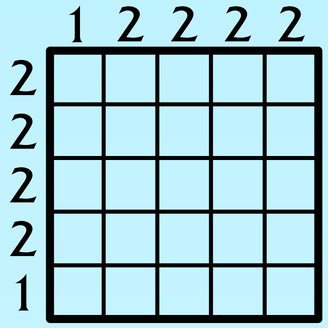

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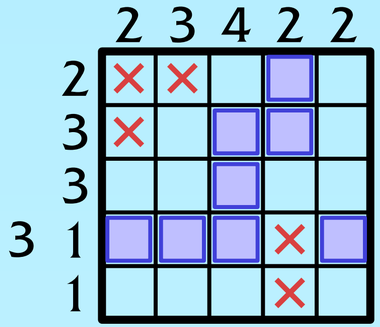

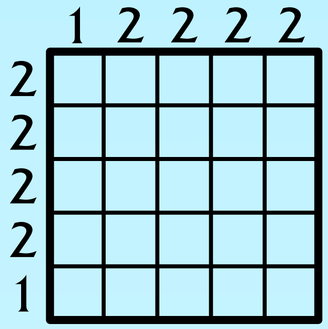

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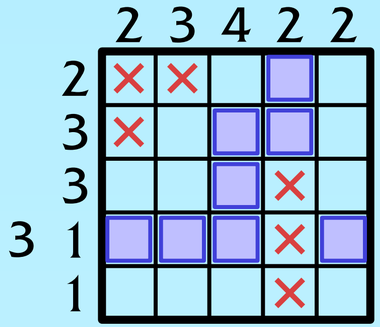

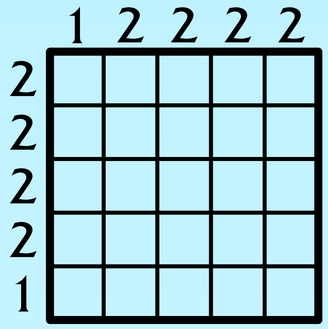

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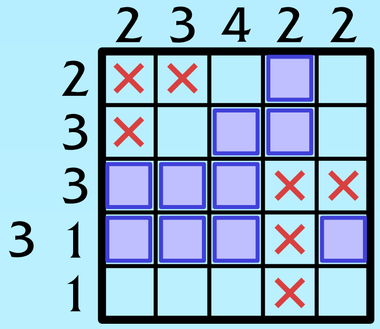

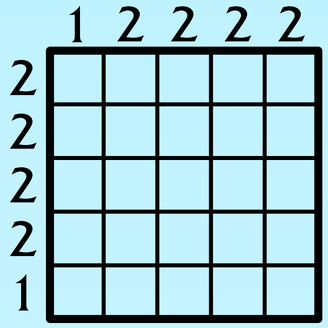

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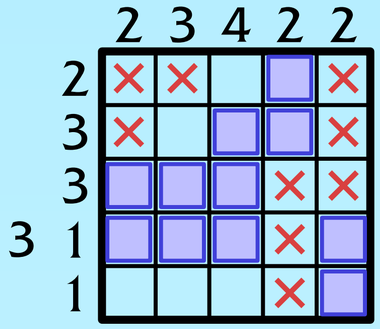

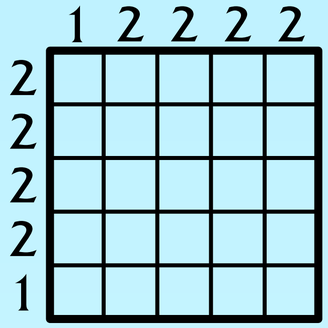

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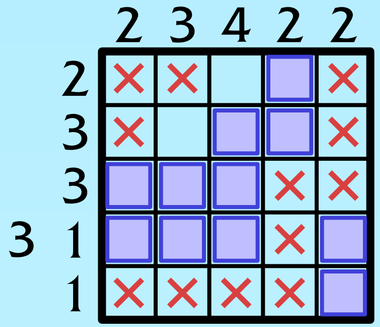

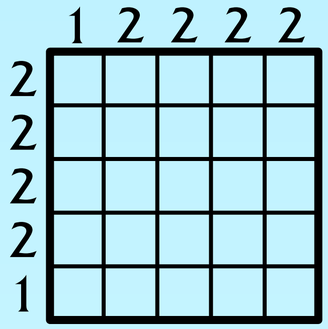

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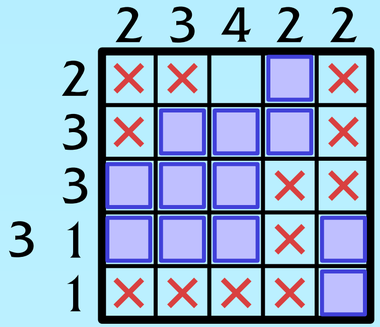

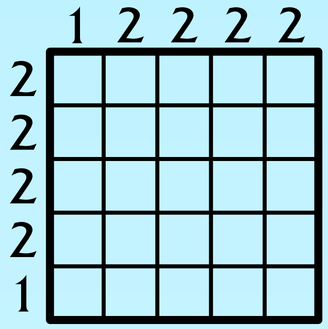

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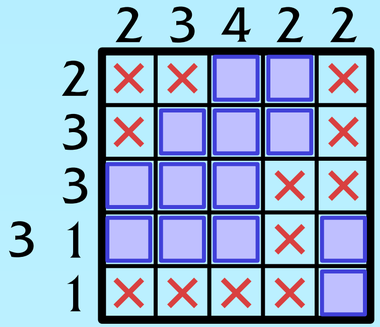

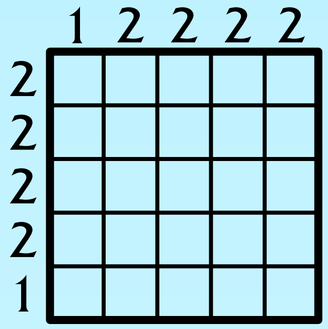

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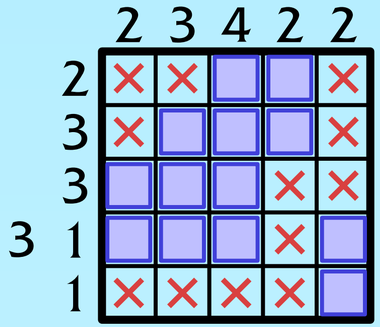

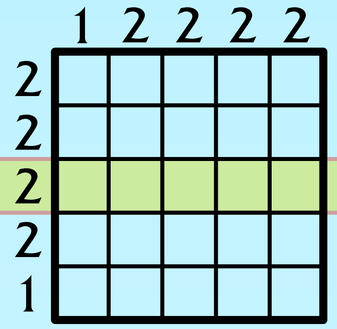

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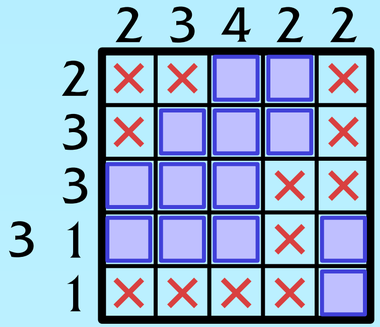

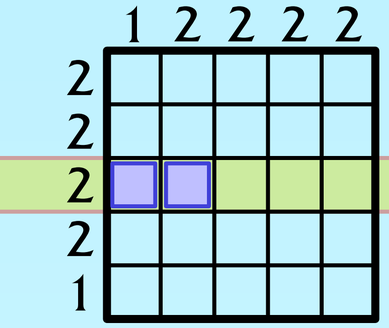

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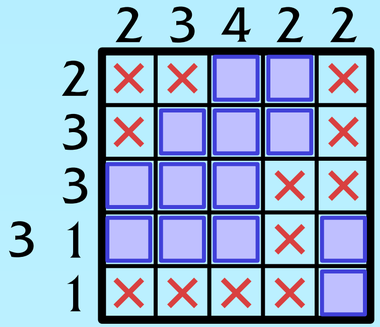

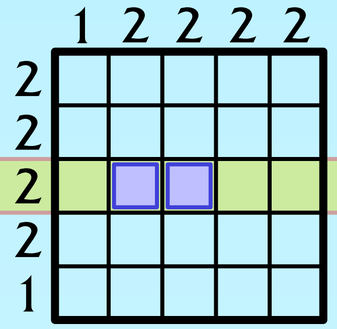

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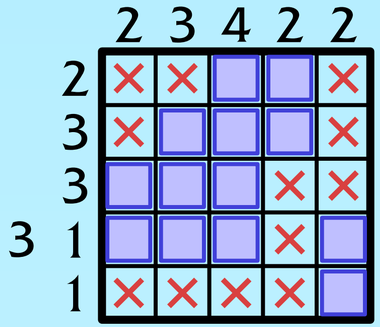

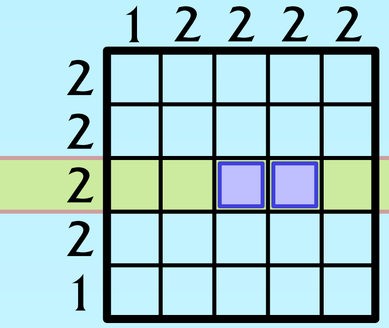

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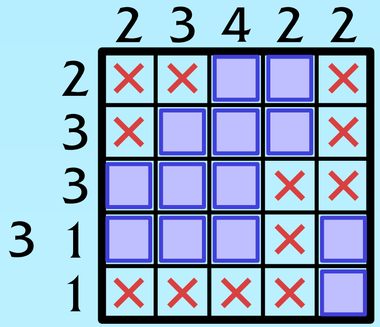

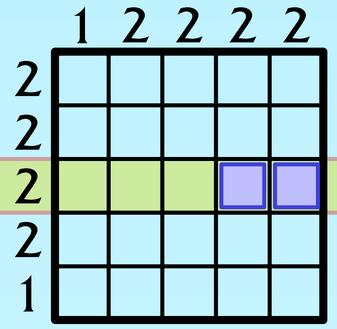

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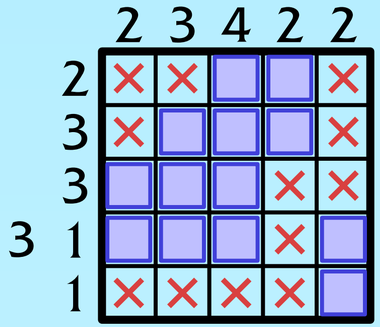

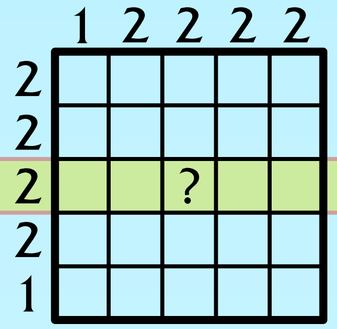

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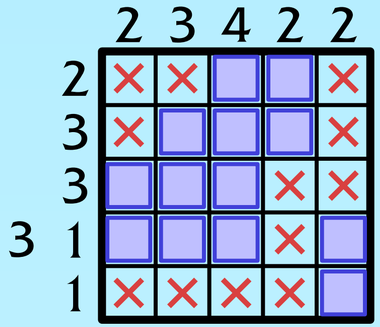

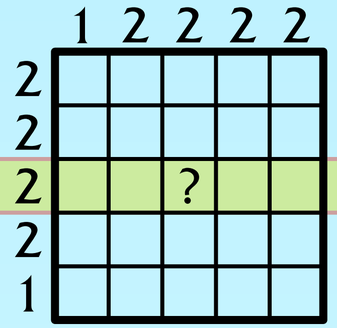

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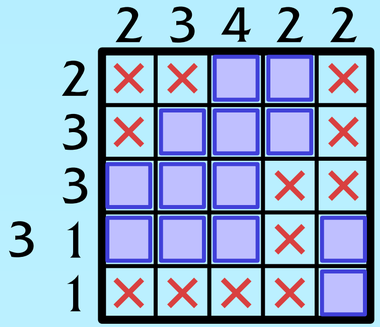
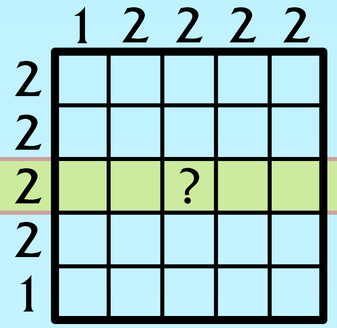
Solving nonograms is NP-hard, but solving simple nonograms is not.

[Batenburg & Kosters, ICGA Journal, 2012]





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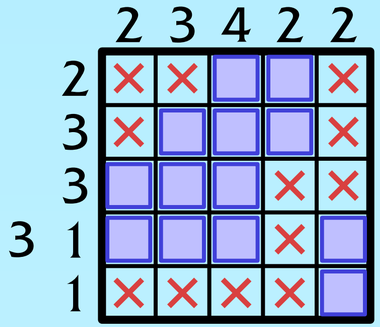

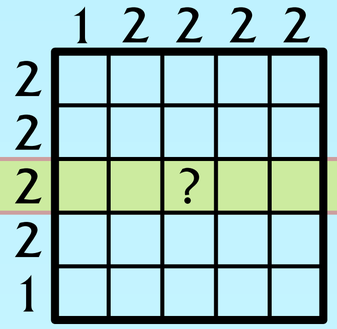

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More fine-grained measures of difficulty have also been studied.

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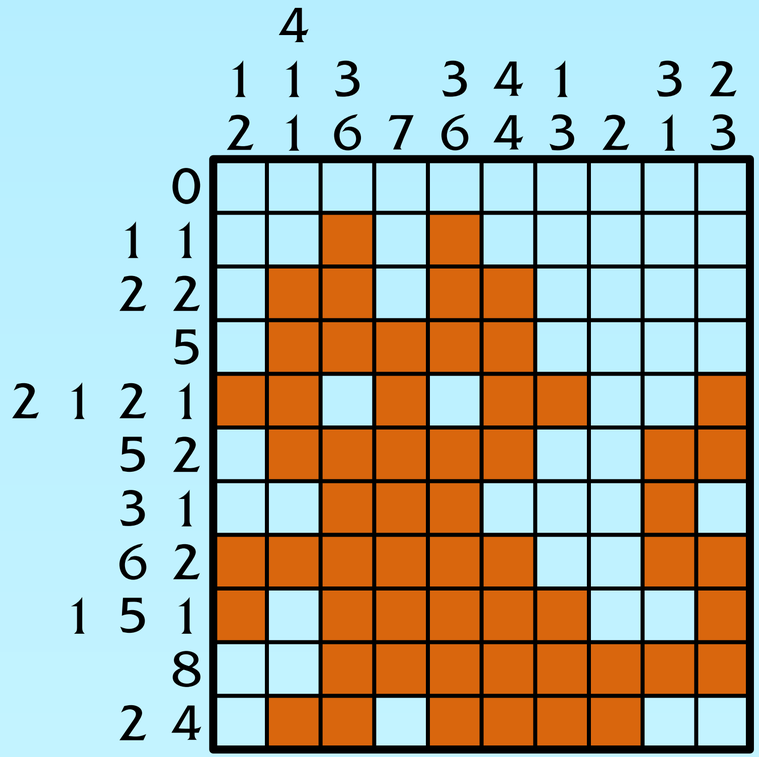
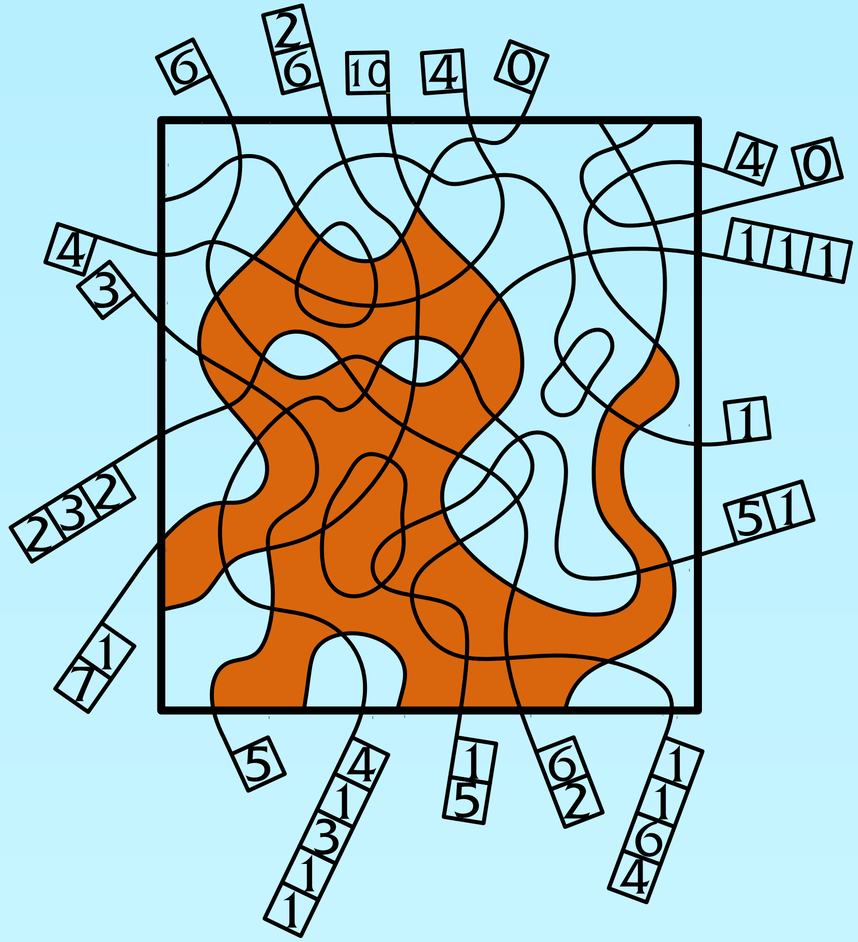


Curved nonograms are an adaptation of nonograms that work on curve arrangements rather than grids.



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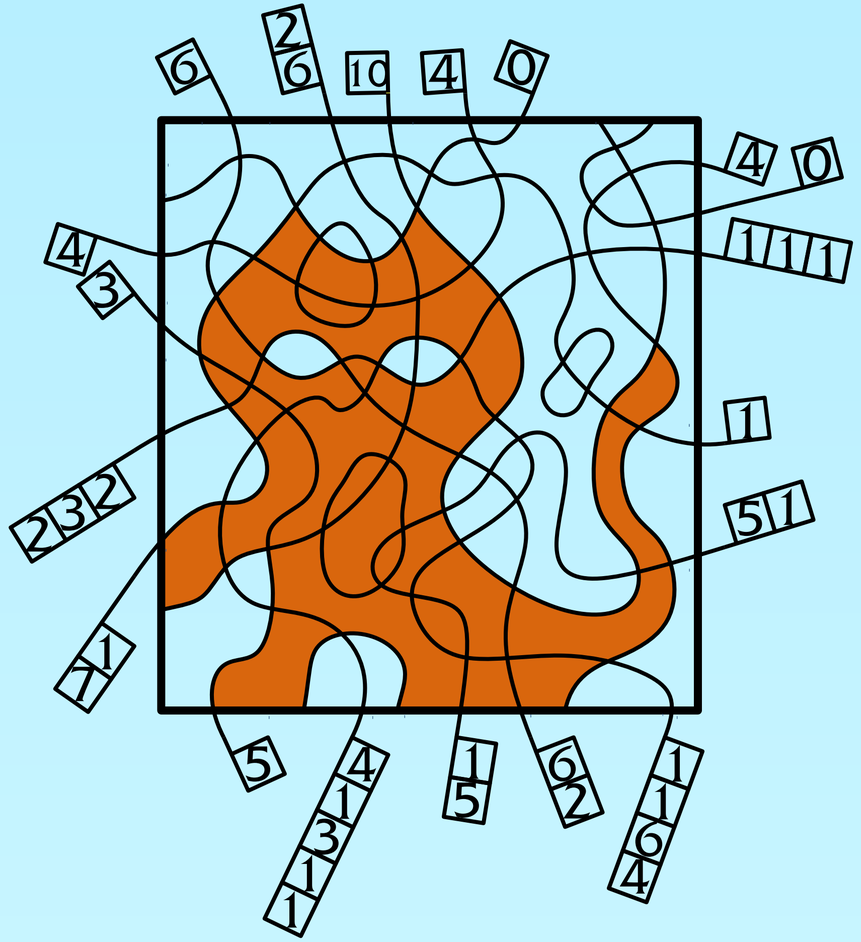

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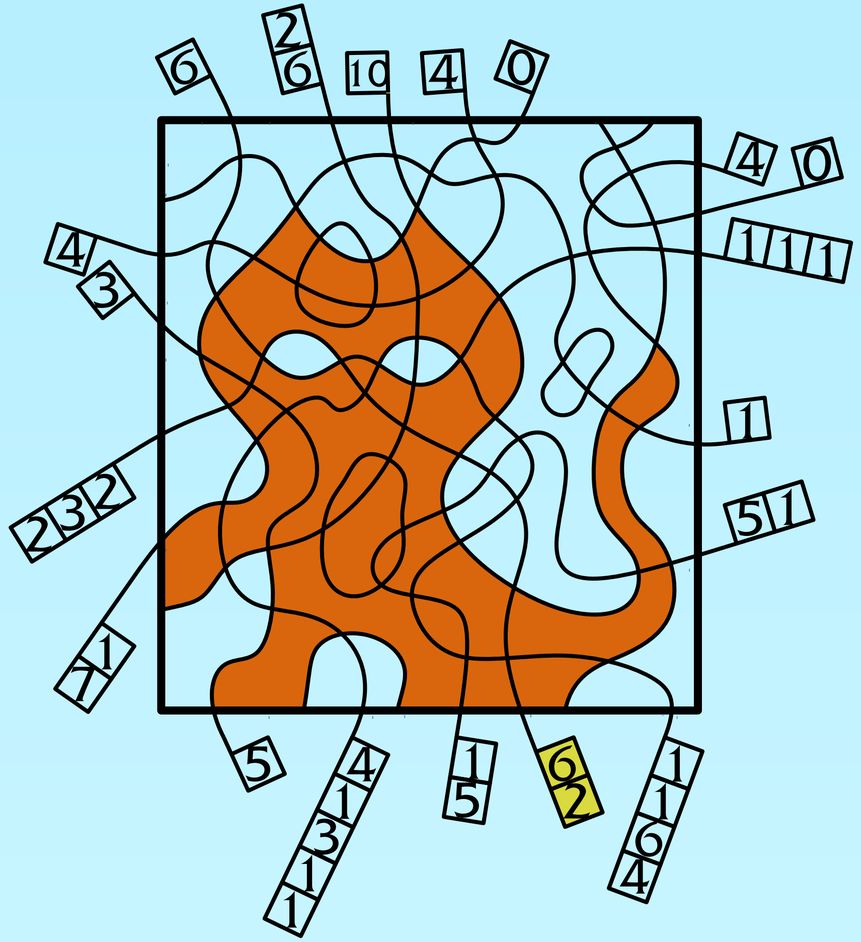

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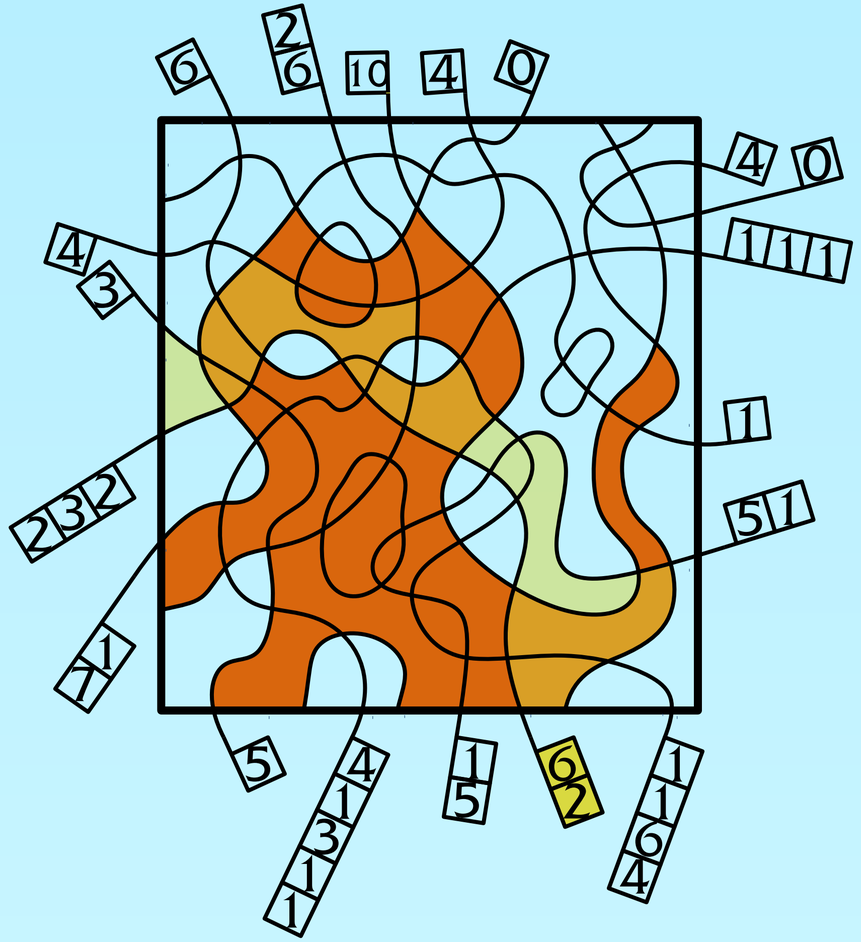

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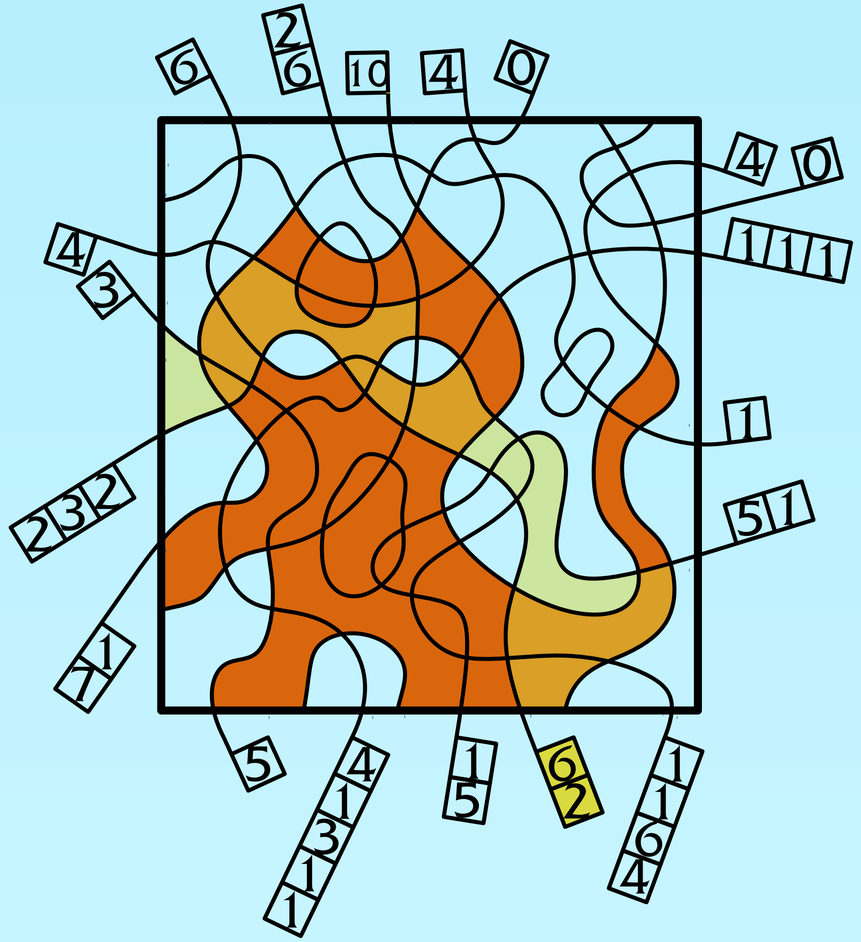

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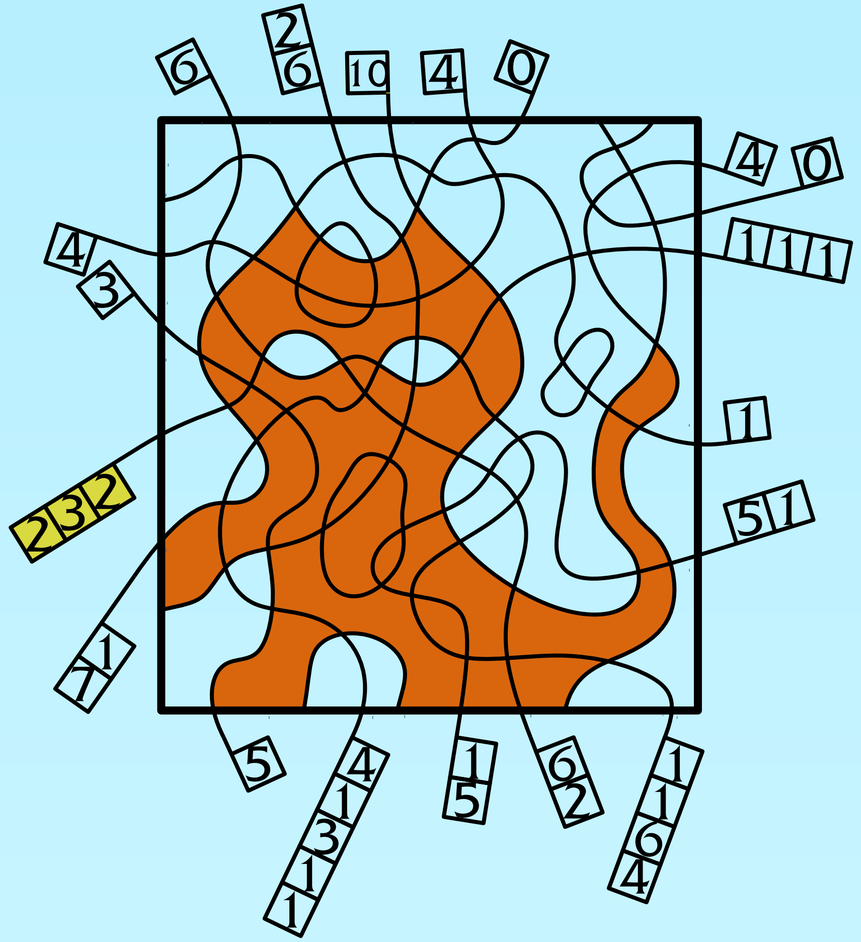
Curves have two sides with separate clues.



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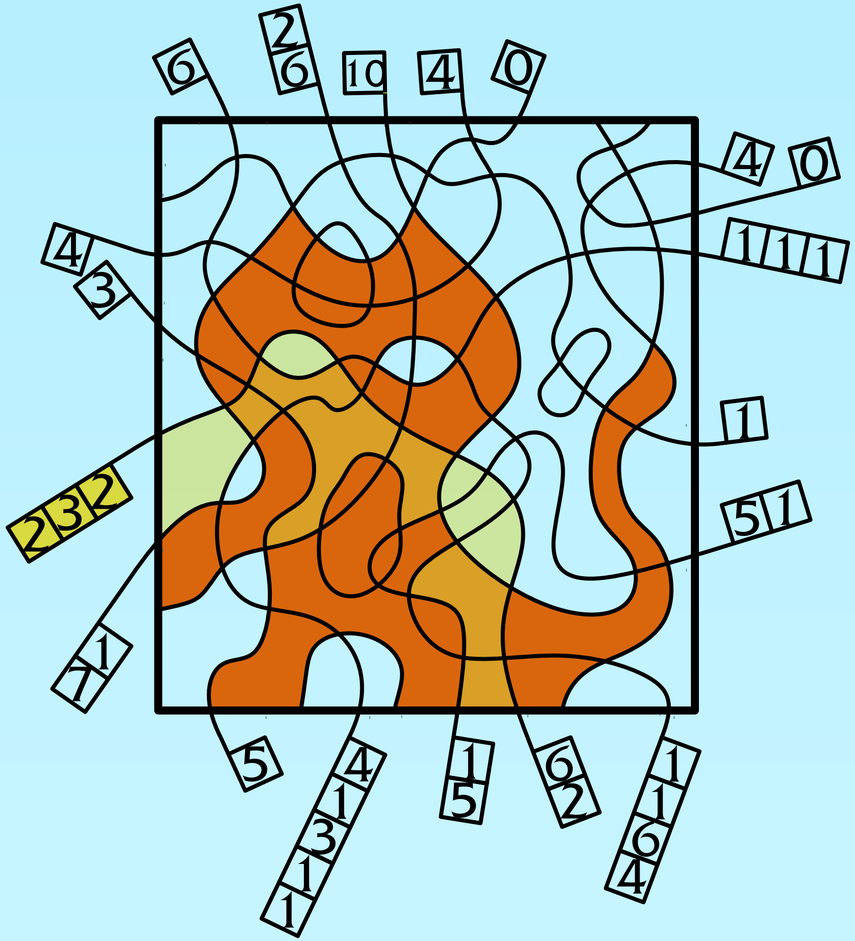

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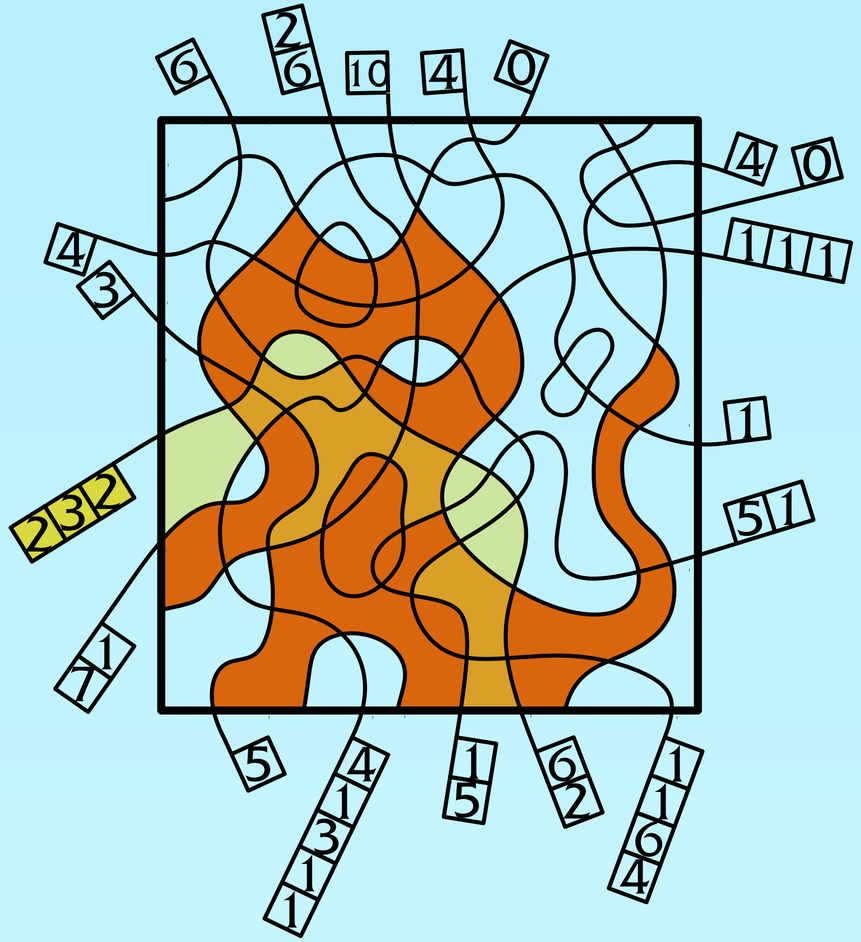

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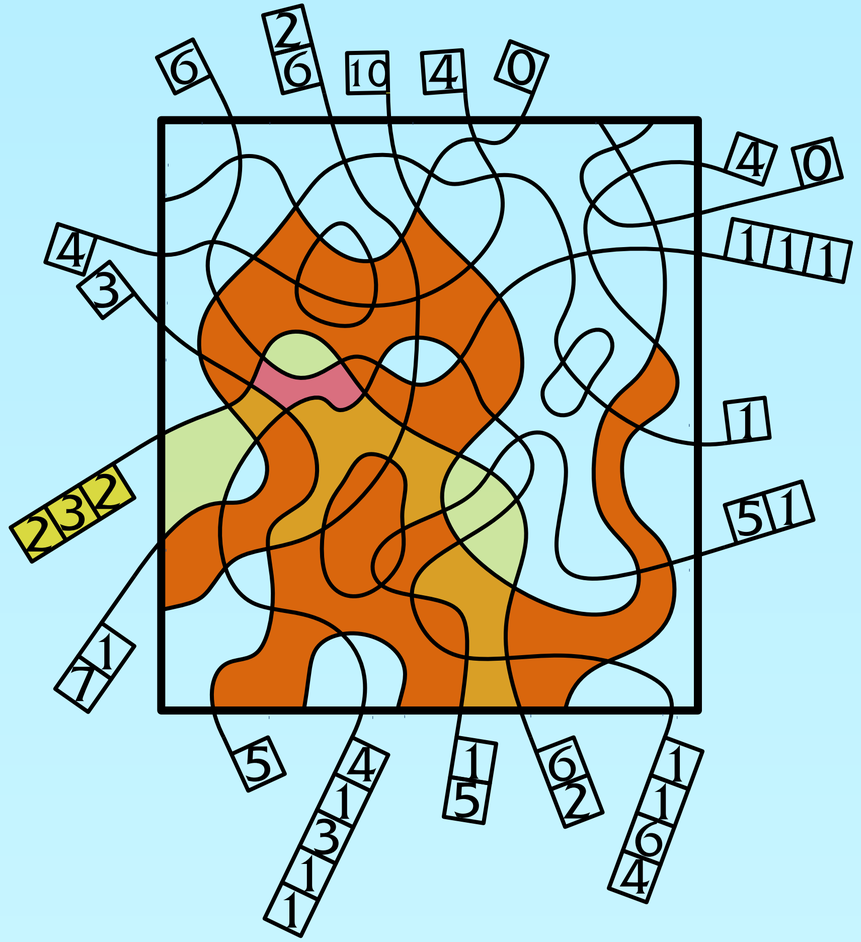
Cells can appear twice!



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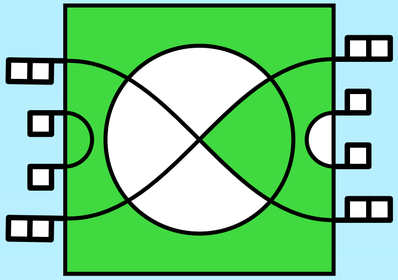
We can identify three levels of “conceptual difficulty”.



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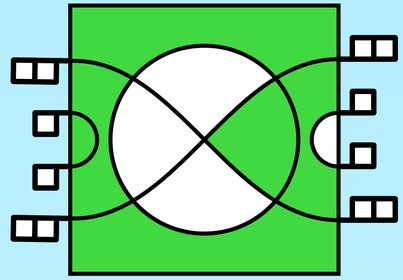
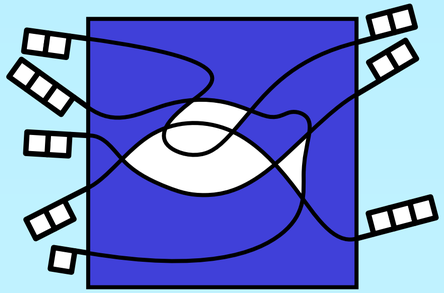
Basic nonograms do not have any duplicate cells adjacent to the same curve.





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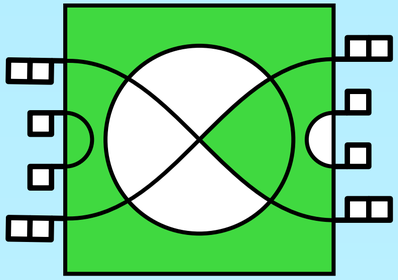

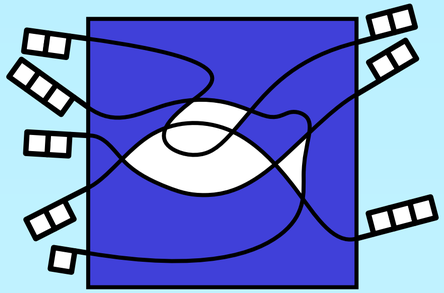

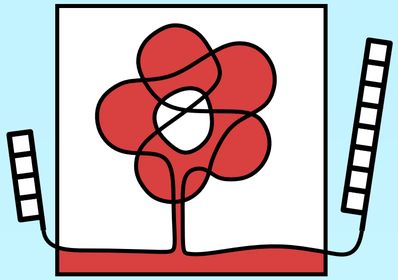

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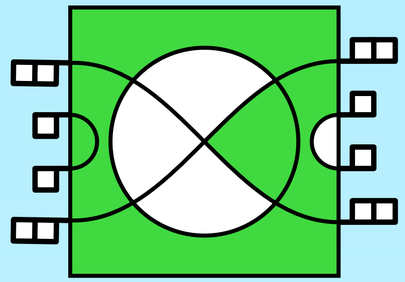
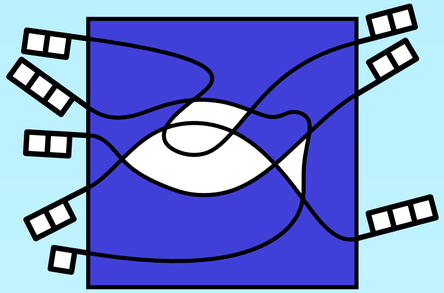
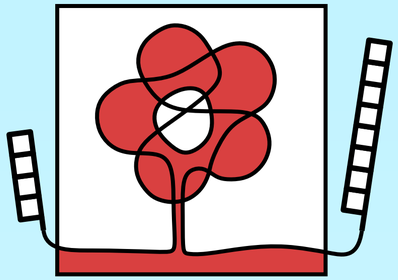
Advanced nonograms may have duplicate cells adjacent to the same side of a curve.

Expert nonograms have duplicate cells adjacent to opposite sides of a curve.







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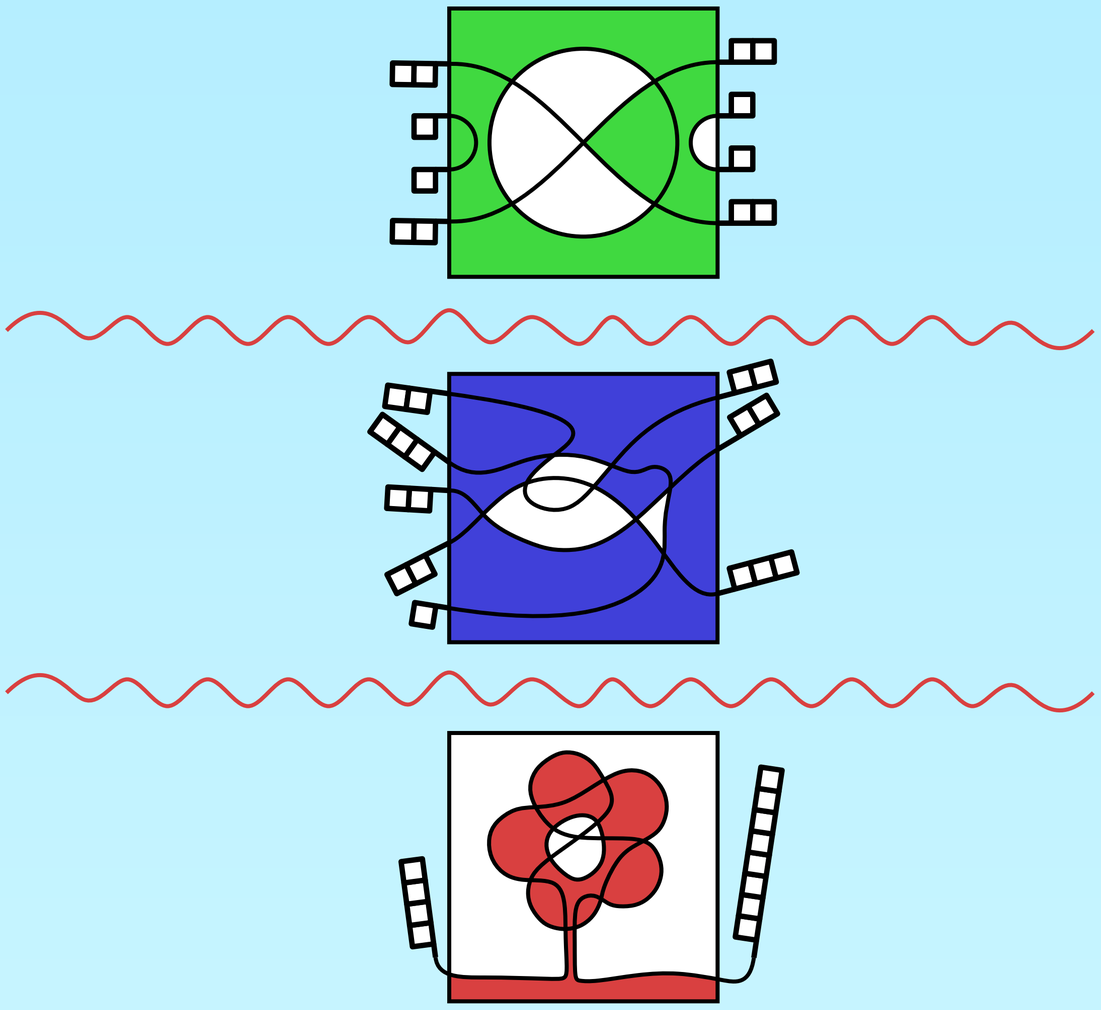
Advanced versus Expert:
presence of self-intersections



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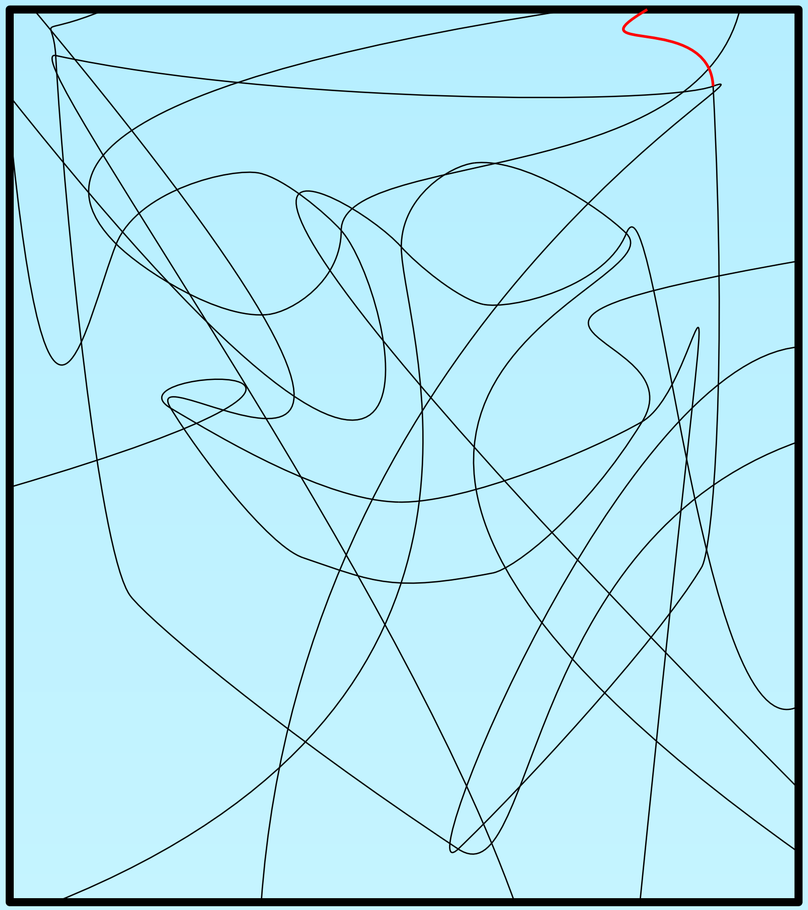
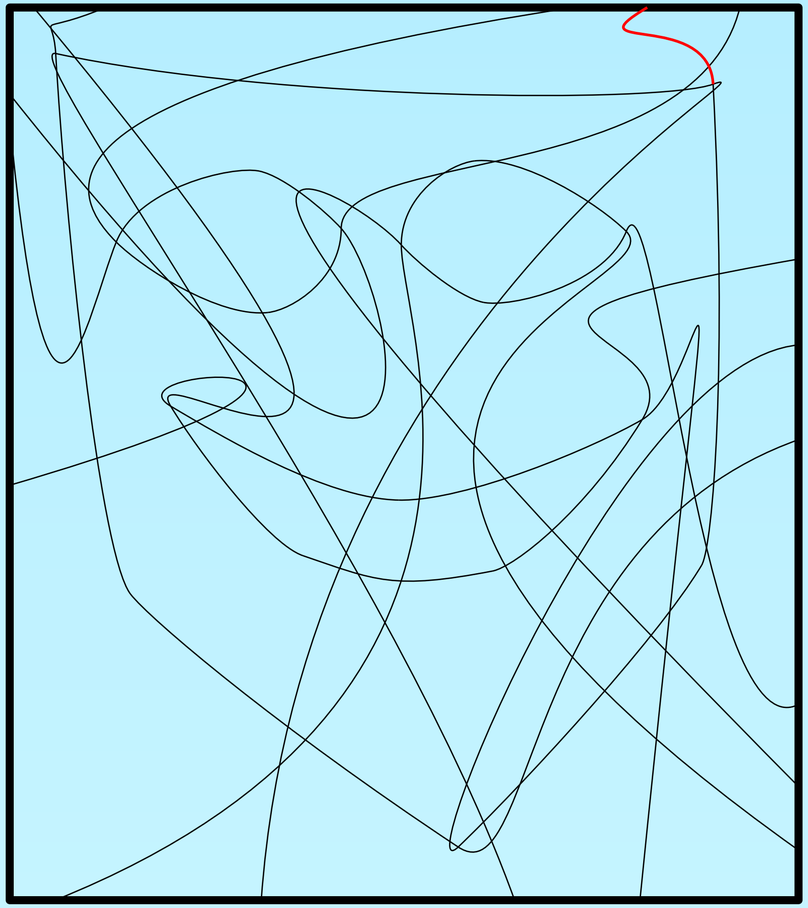
Basic versus Advanced:
presence of popular faces

Advanced versus Expert:
presence of self-intersections



One can generate nonograms from desired output pictures.

[van de Kerkhof, de Jong, Parment, L, Vaxman & van Kreveld, EuroGraphics 2019]



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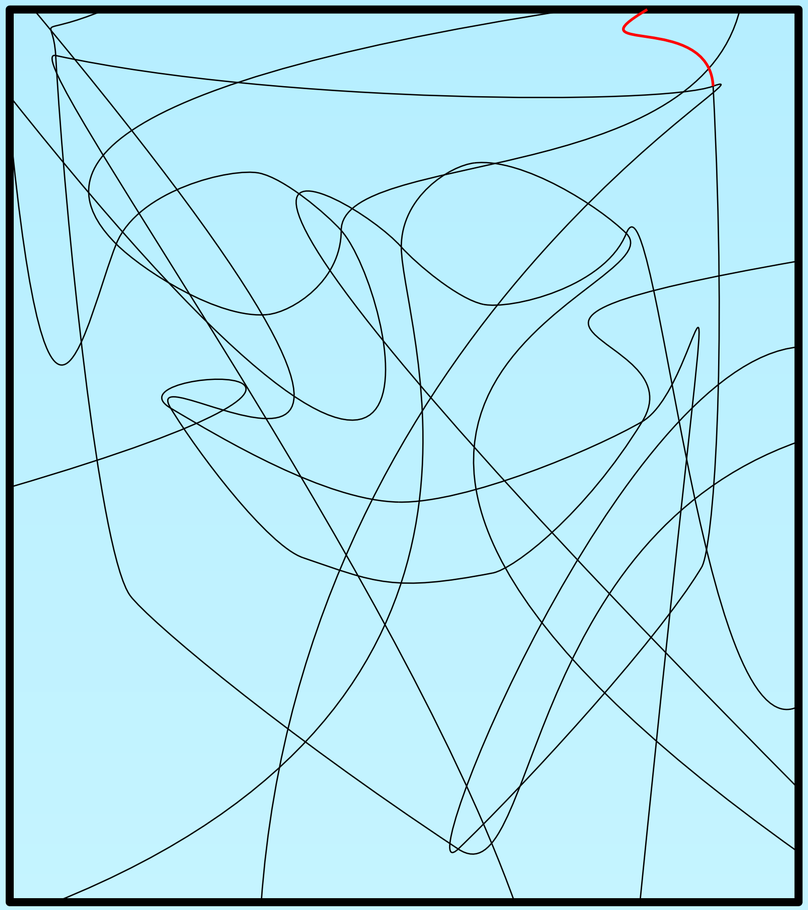

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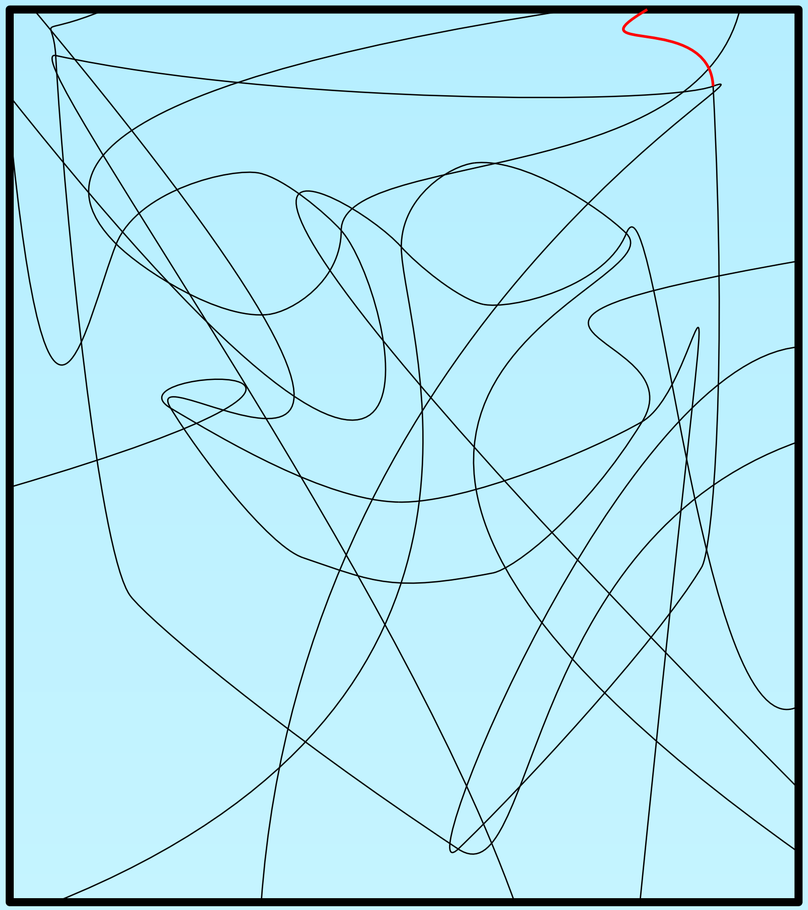

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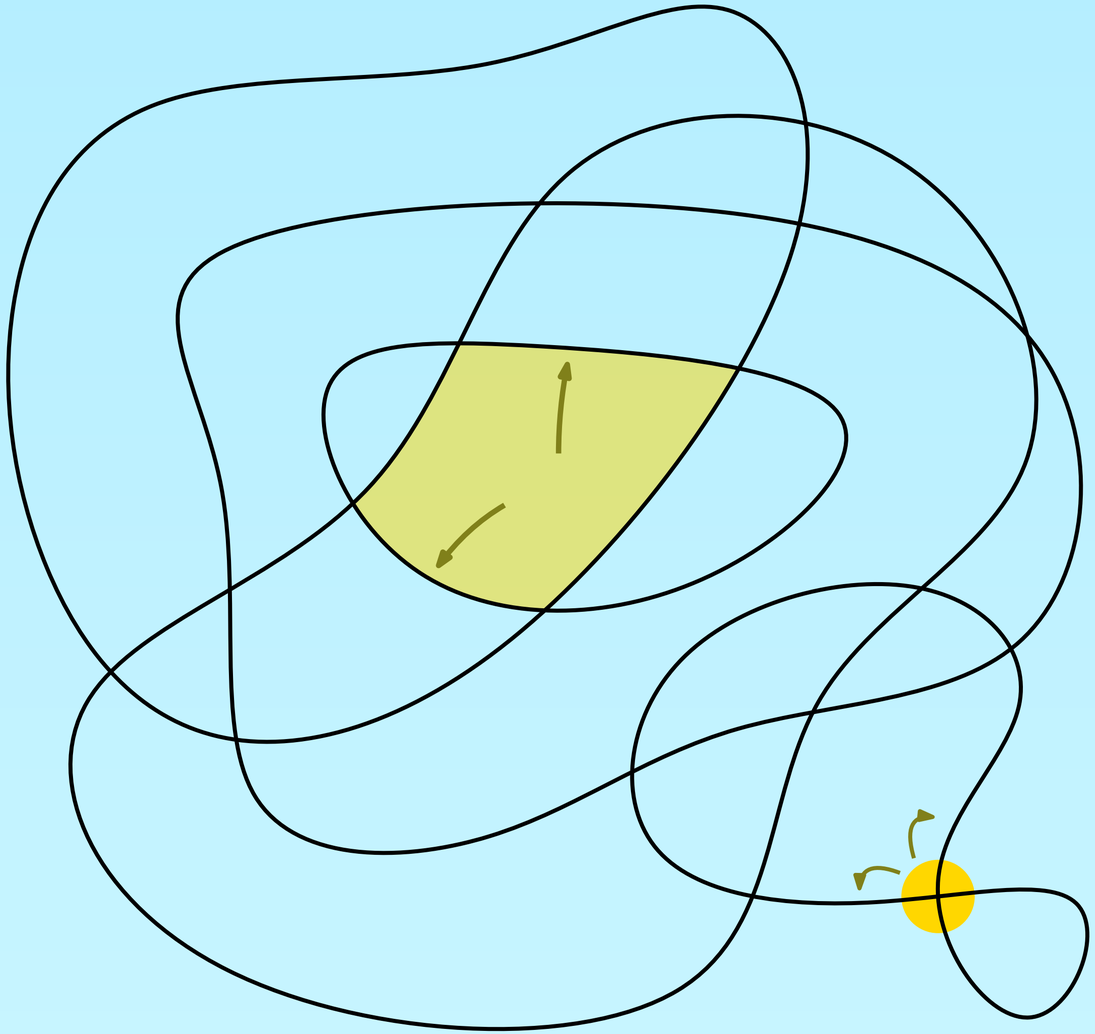
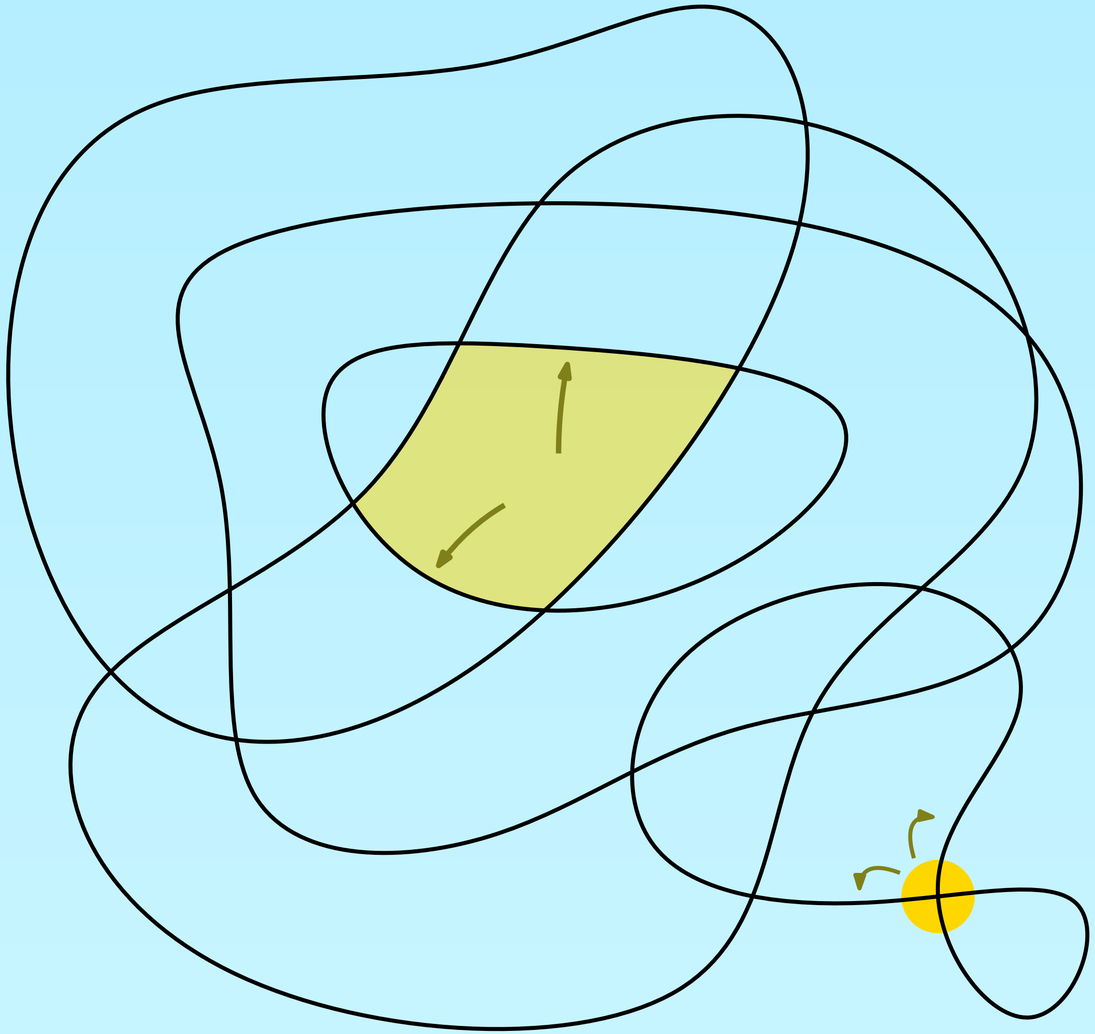
Can we turn expert puzzles into advanced or basic ones?

[van de Kerkhof, de Jong, Parment, L, Vaxman & van Kreveld, EuroGraphics 2019]

Part 2
WHAT?



So we want to get rid of self-intersections and/or popular faces in curve arrangements.



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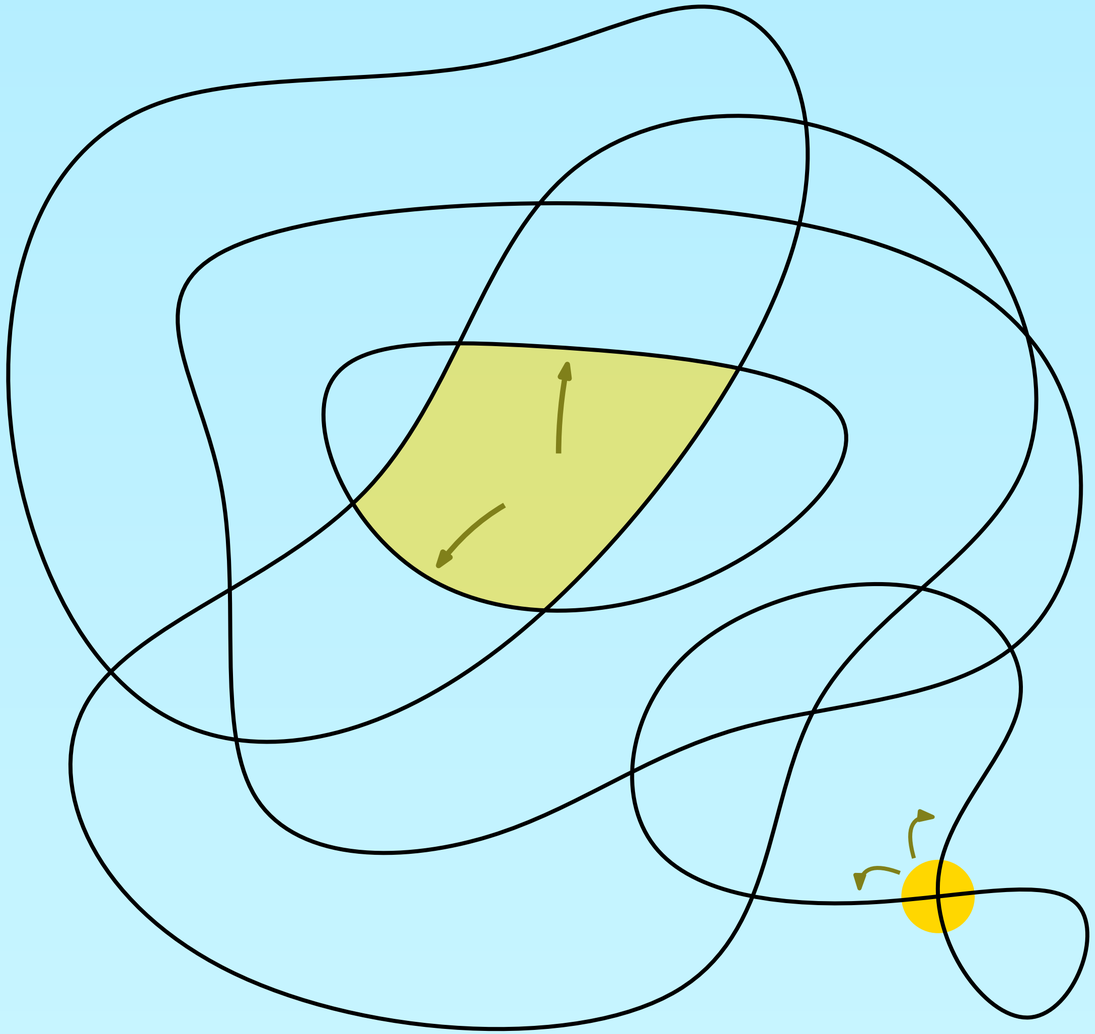

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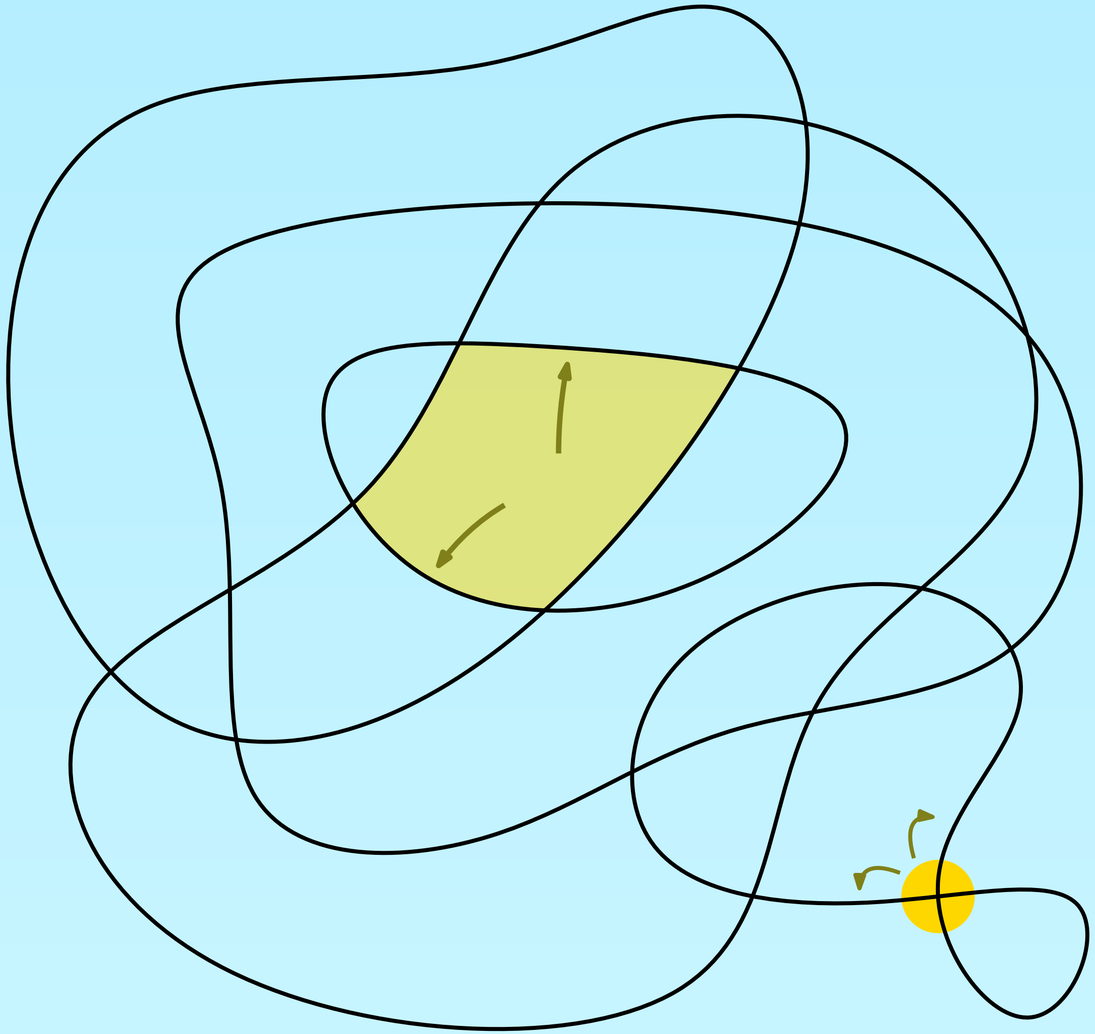

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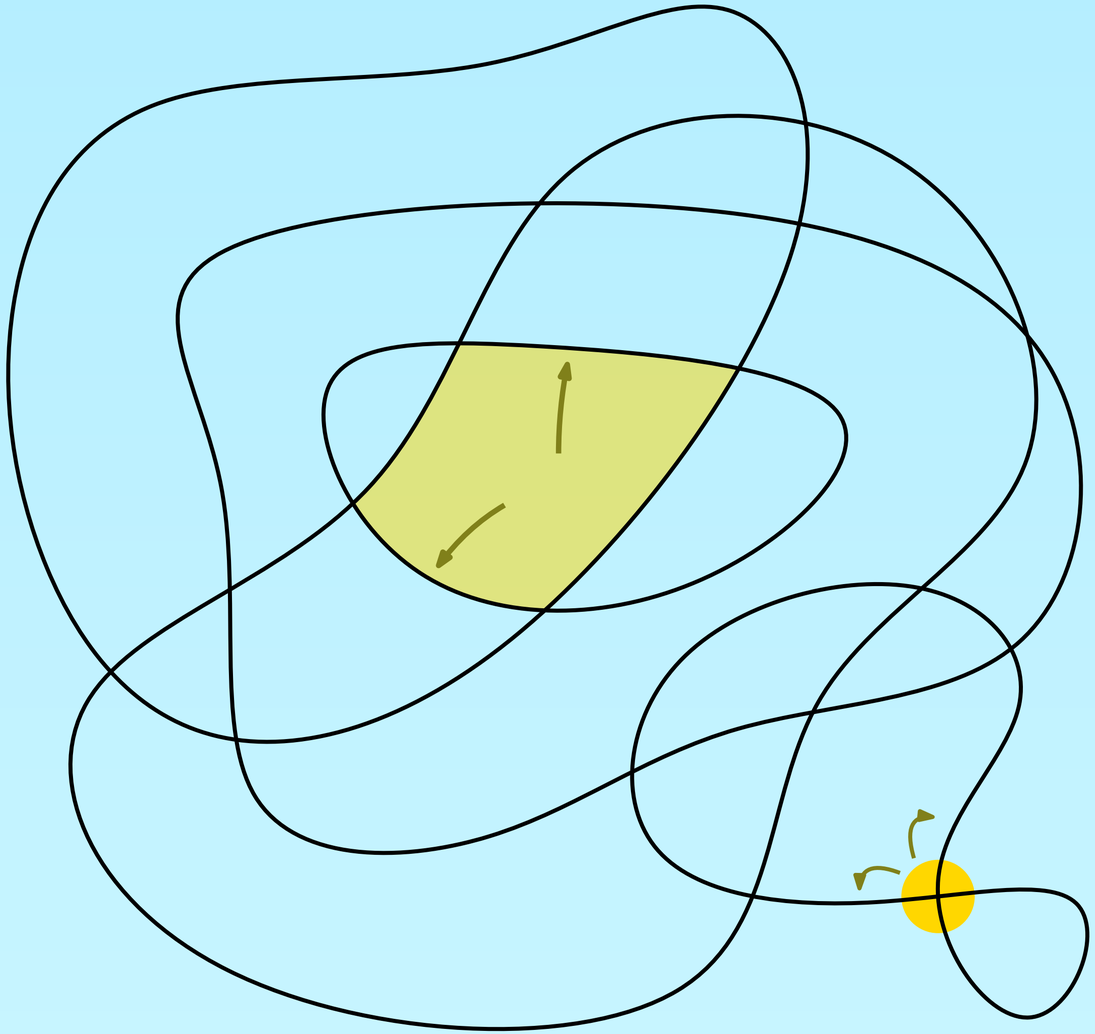
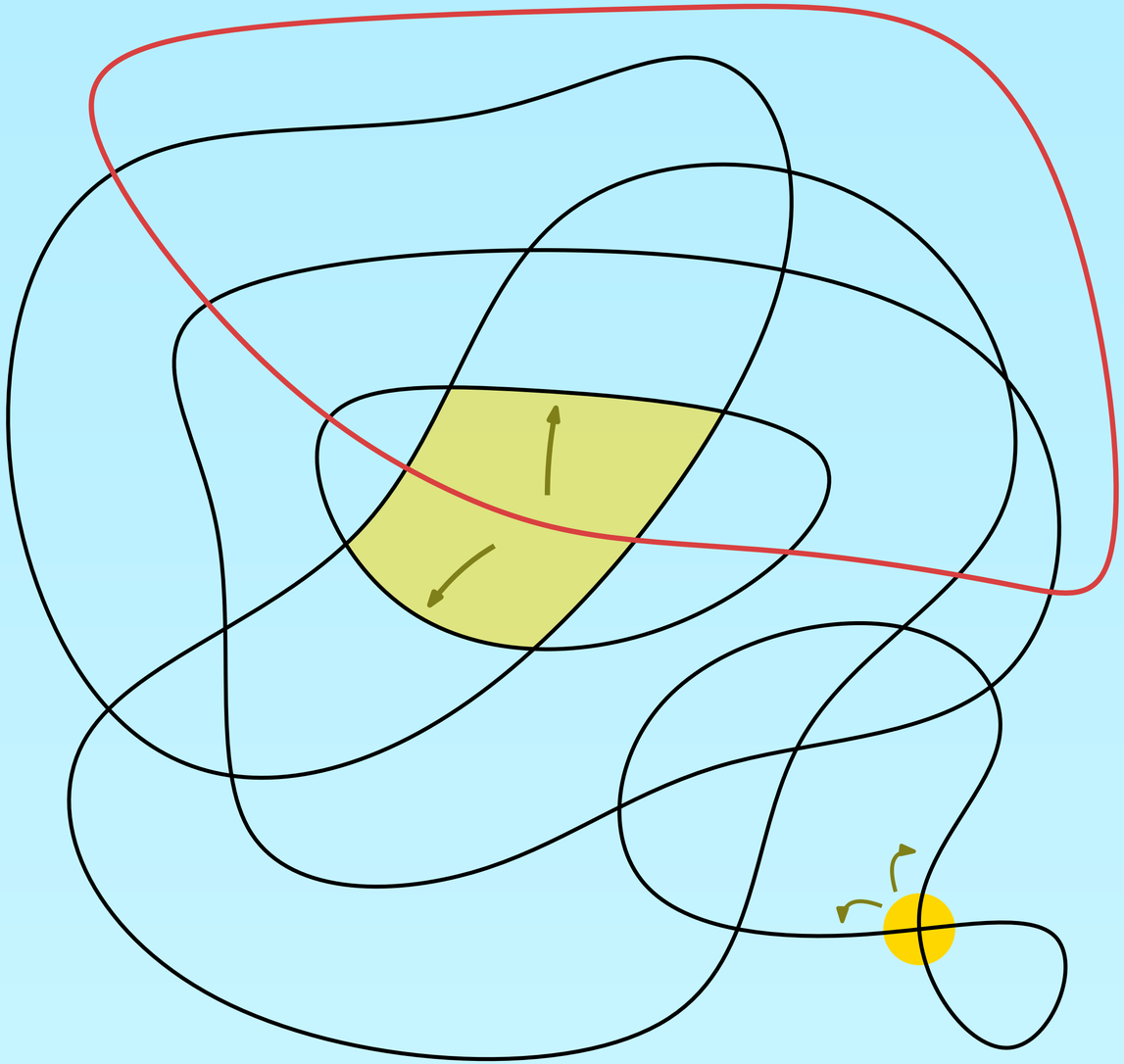
But how do we remove them?



De Nooijer et al. study how popular faces can be removed by adding more curves to the arrangement.



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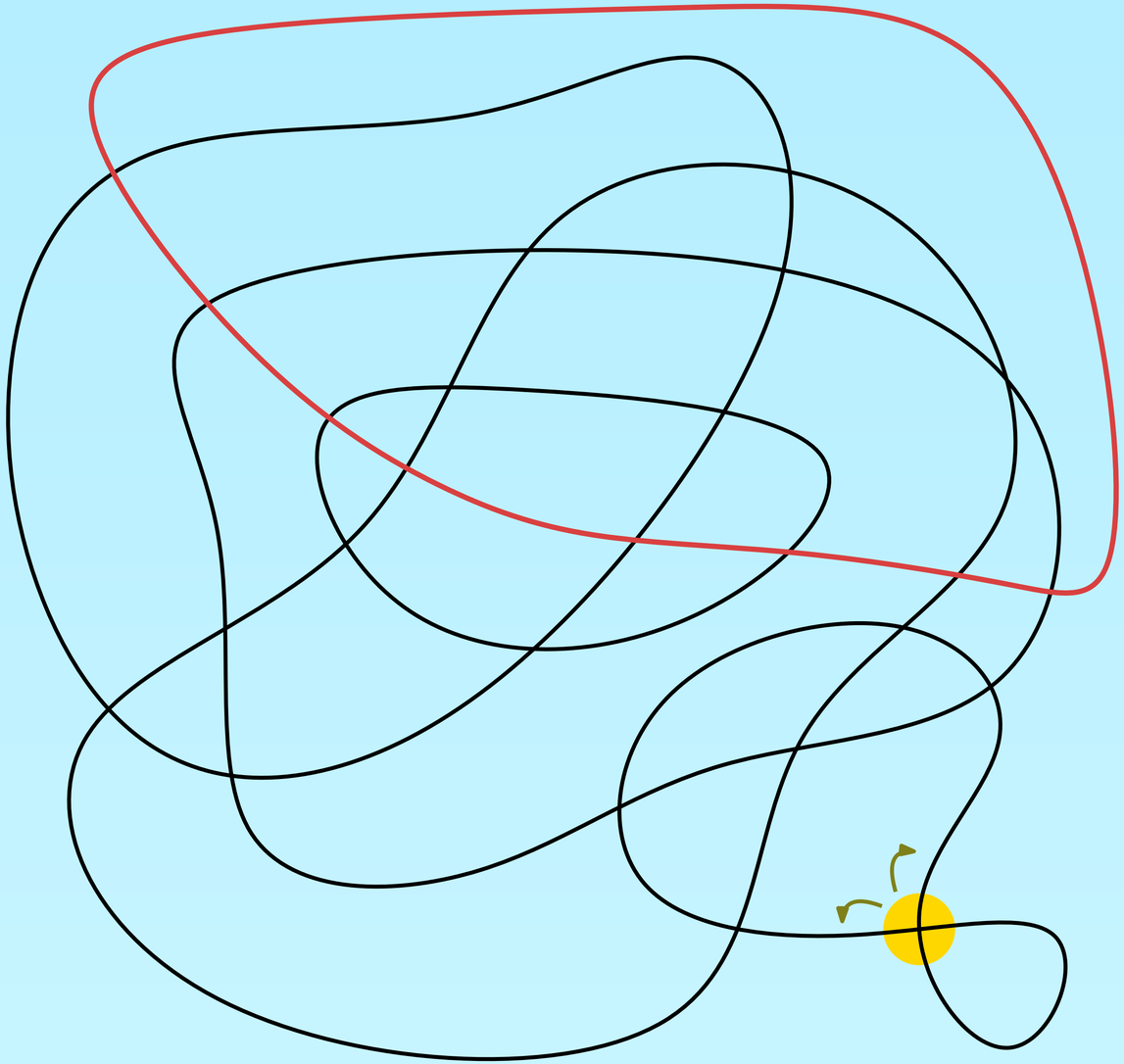

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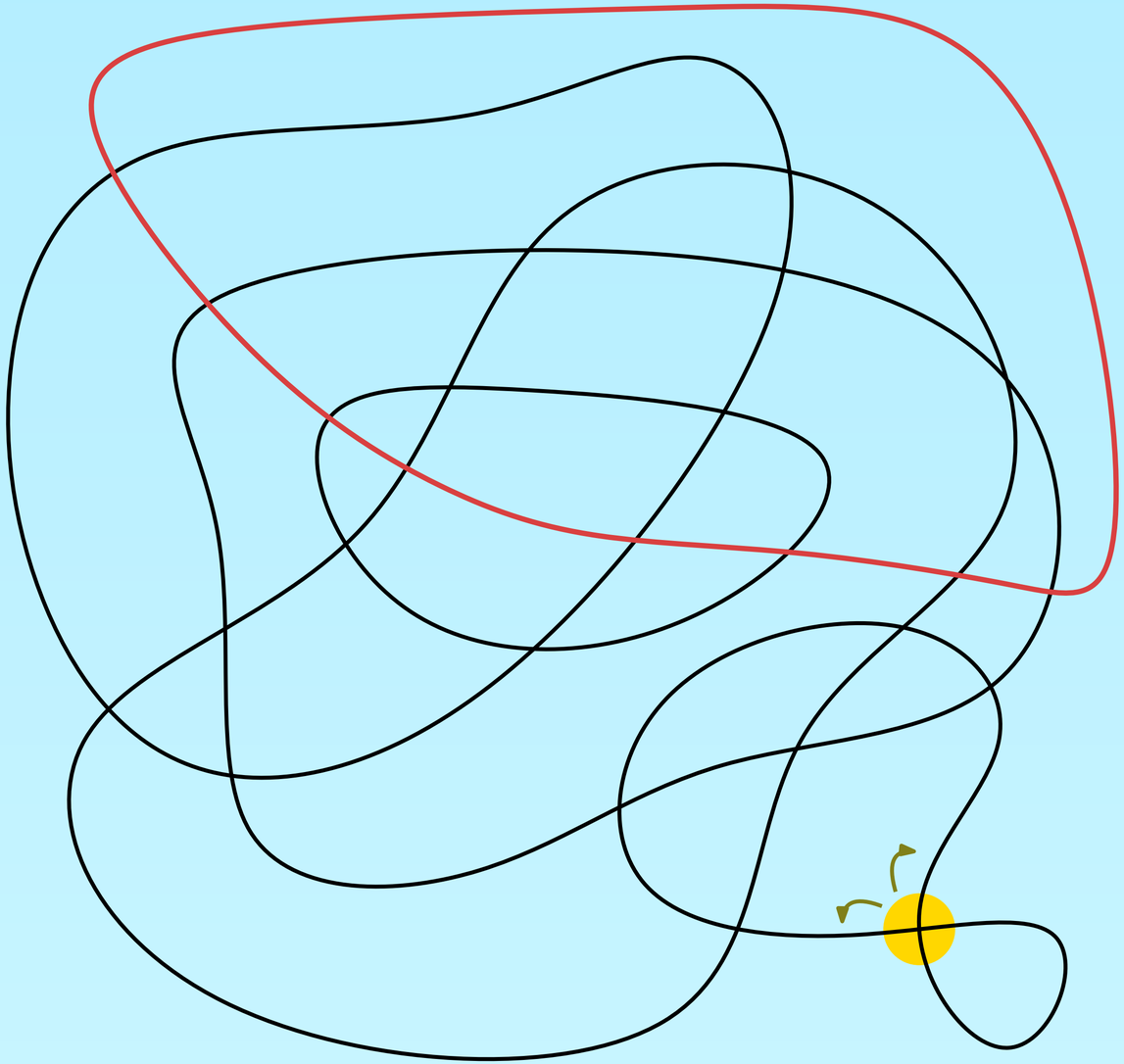

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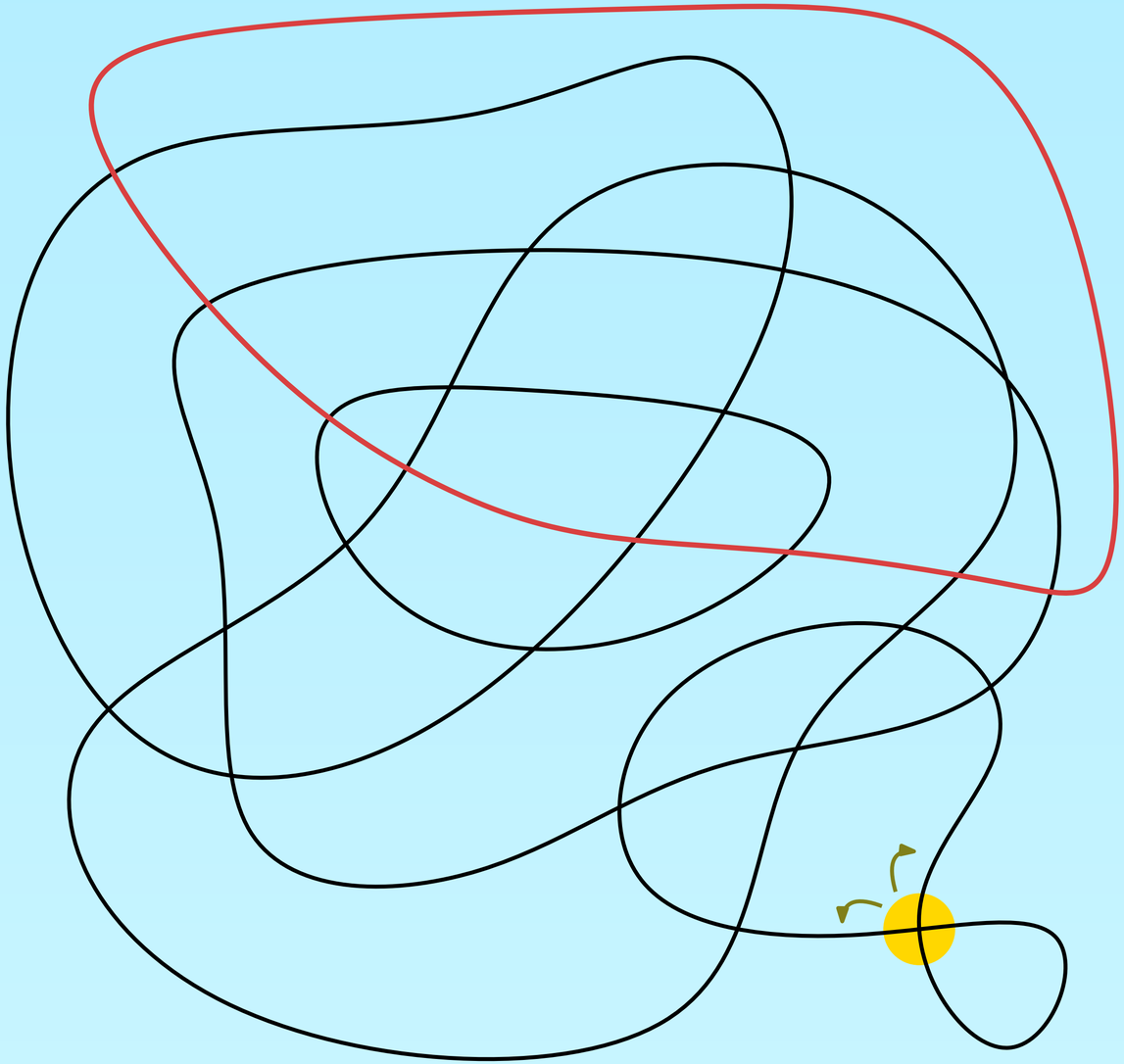

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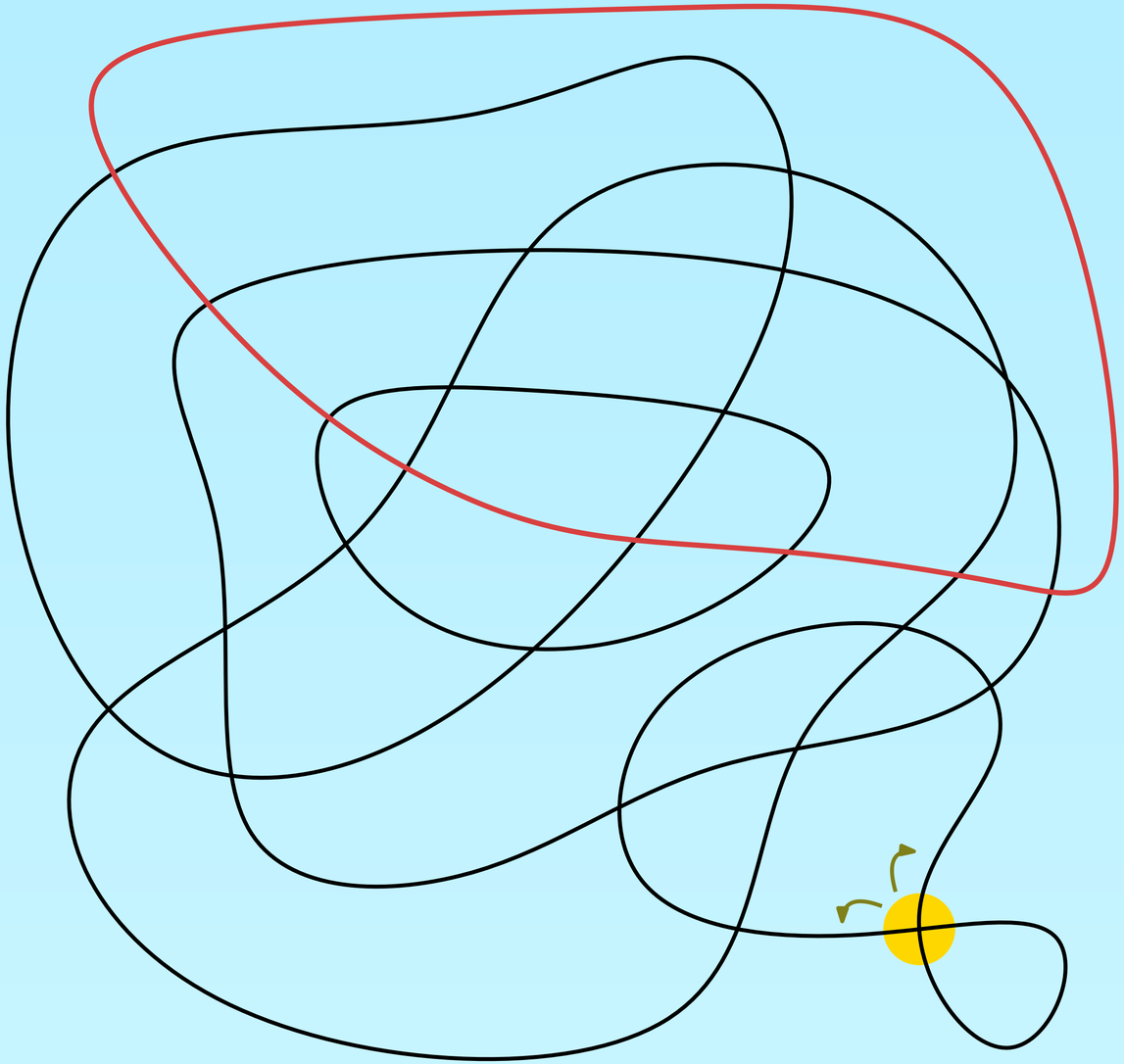

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Drawback: self-intersections cannot be removed this way.

Idea: change combinatorics, not geometry.

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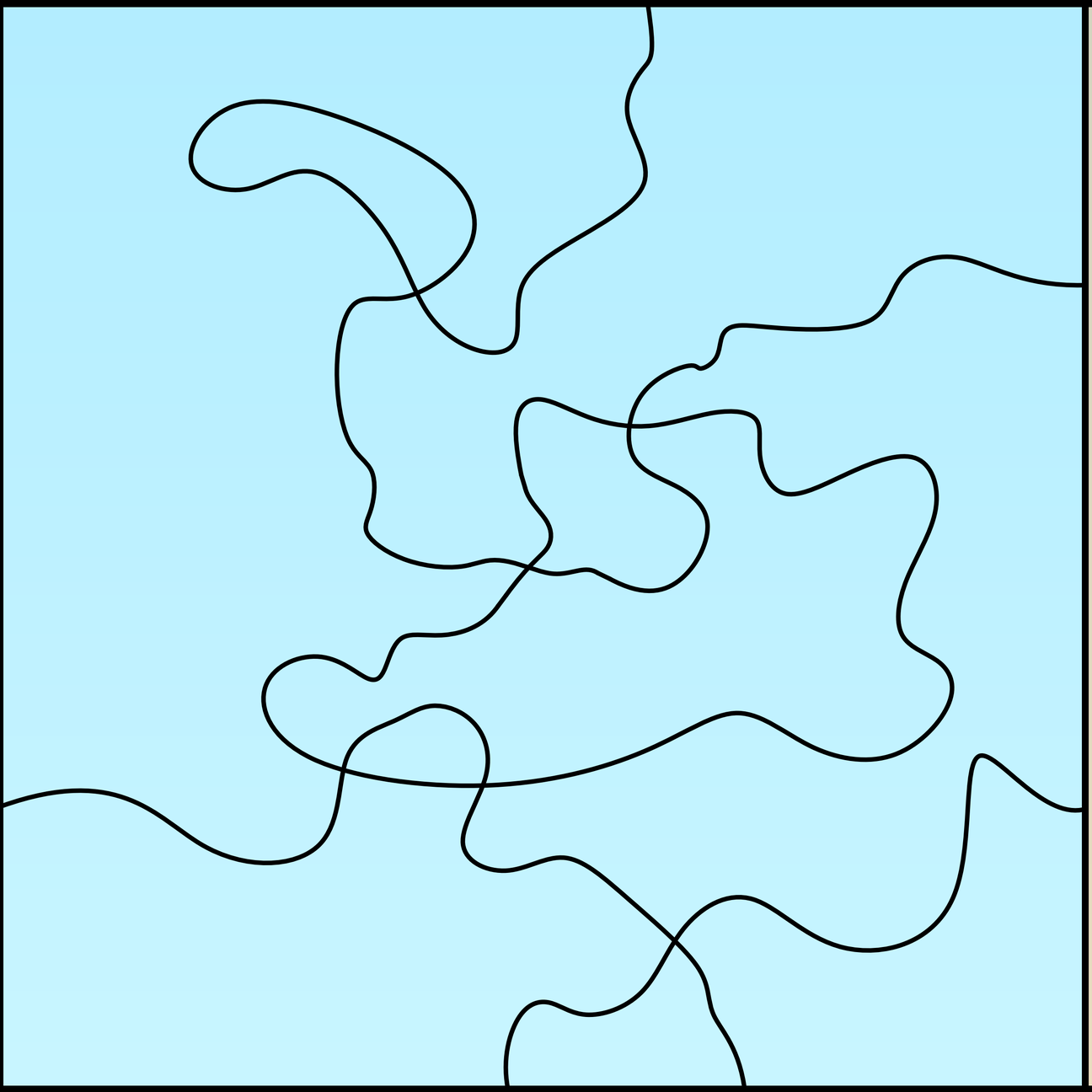
A switch is a disk of radius ε whose boundary intersects the curve arrangement exactly four times.



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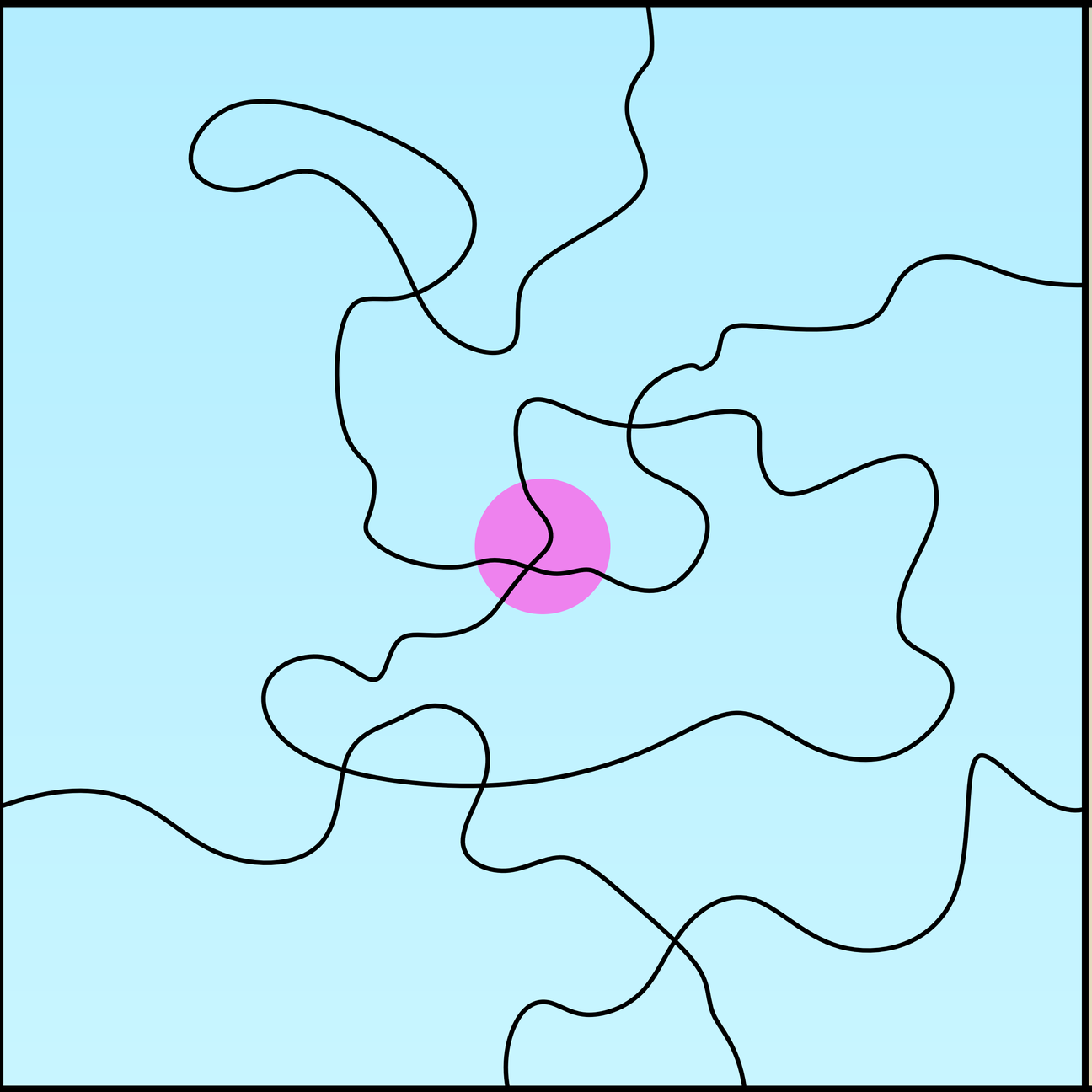

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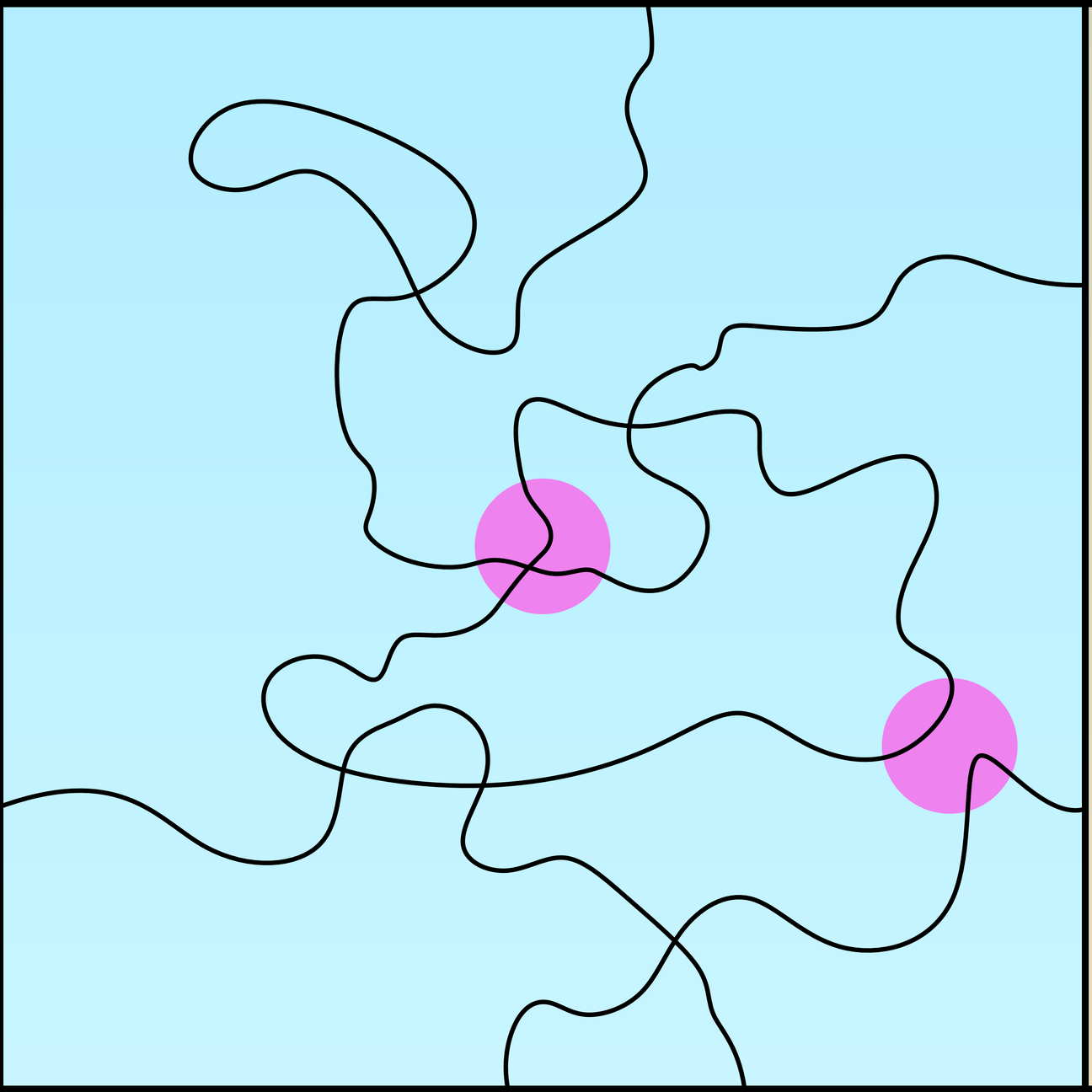

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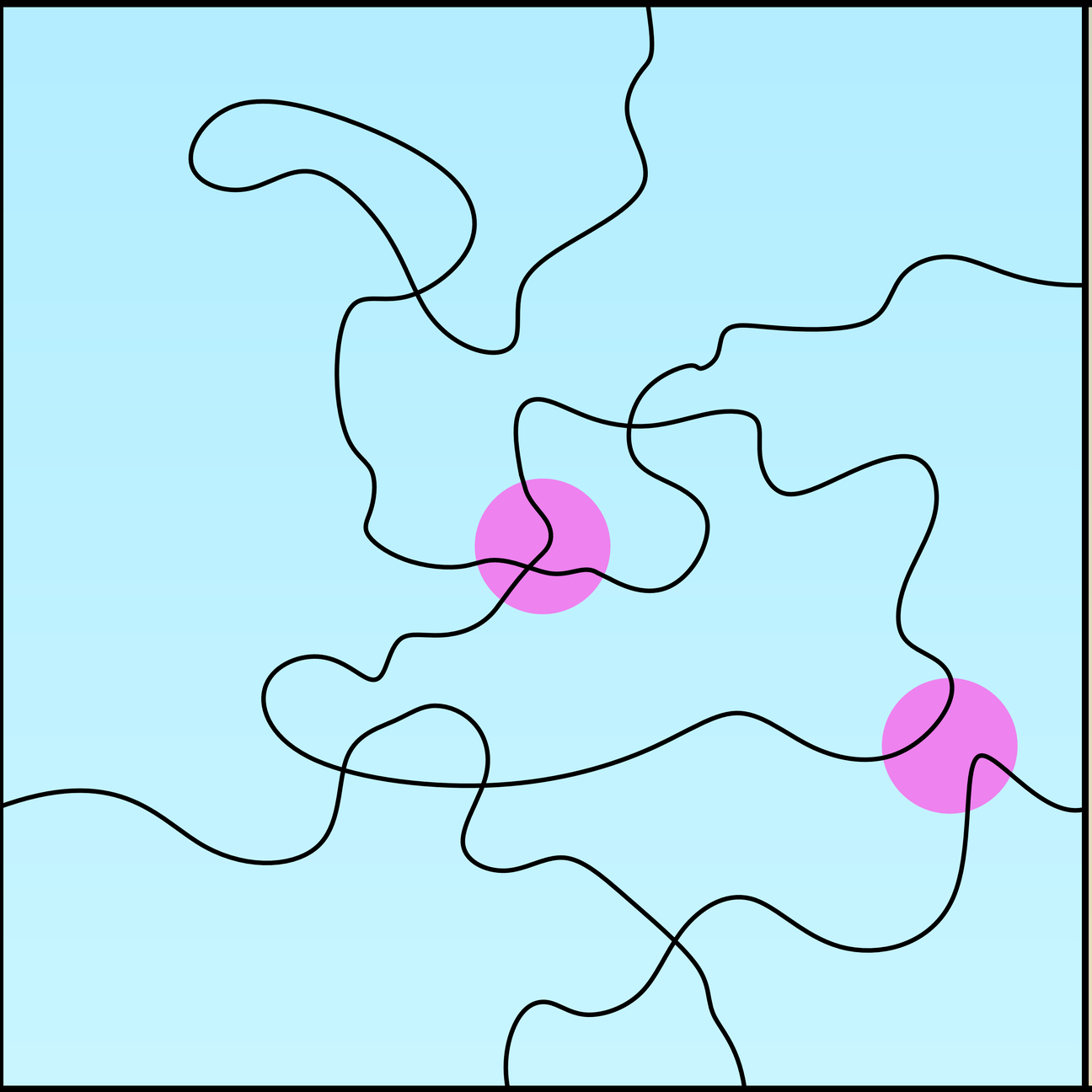

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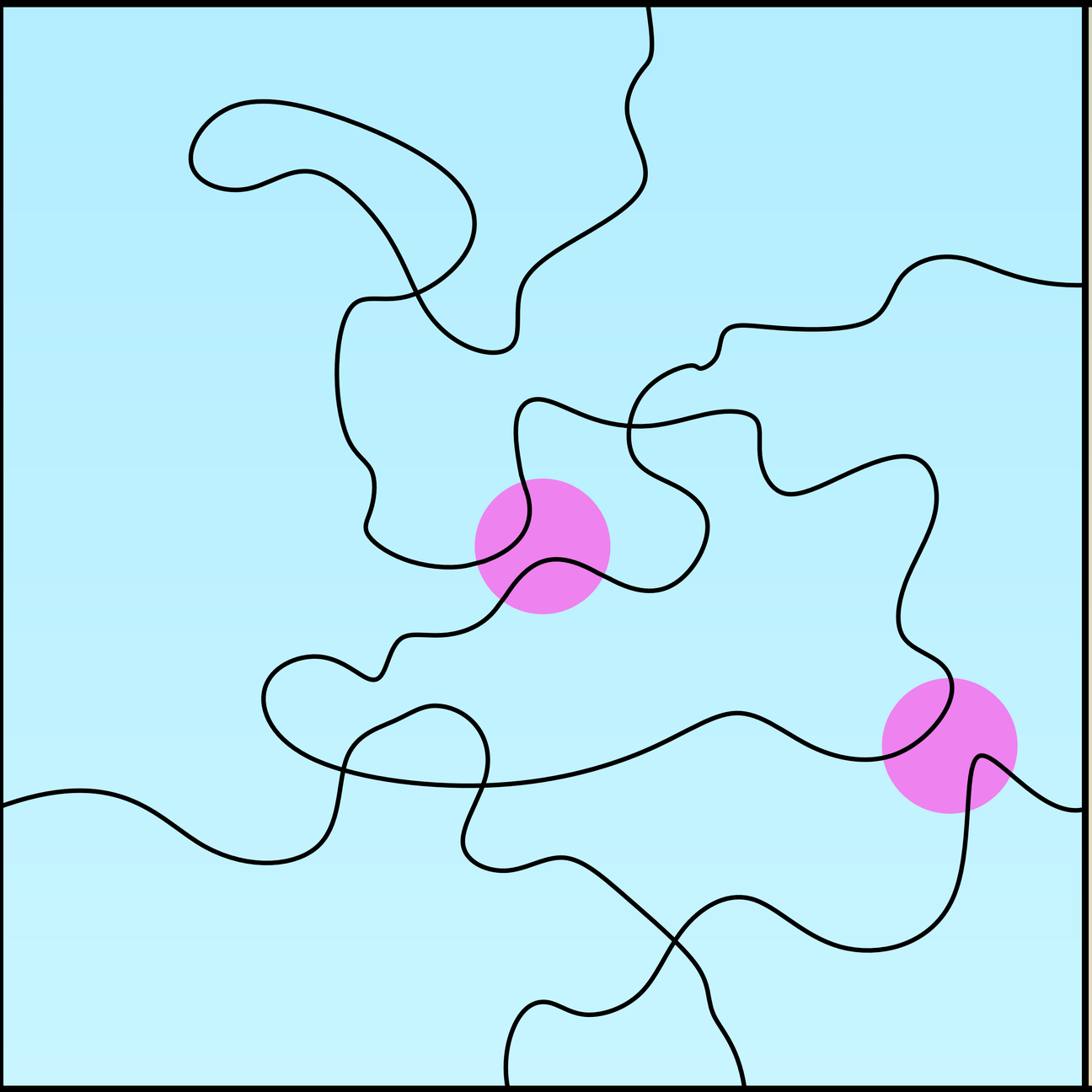
We can reroute the curves inside a switch in three ways.



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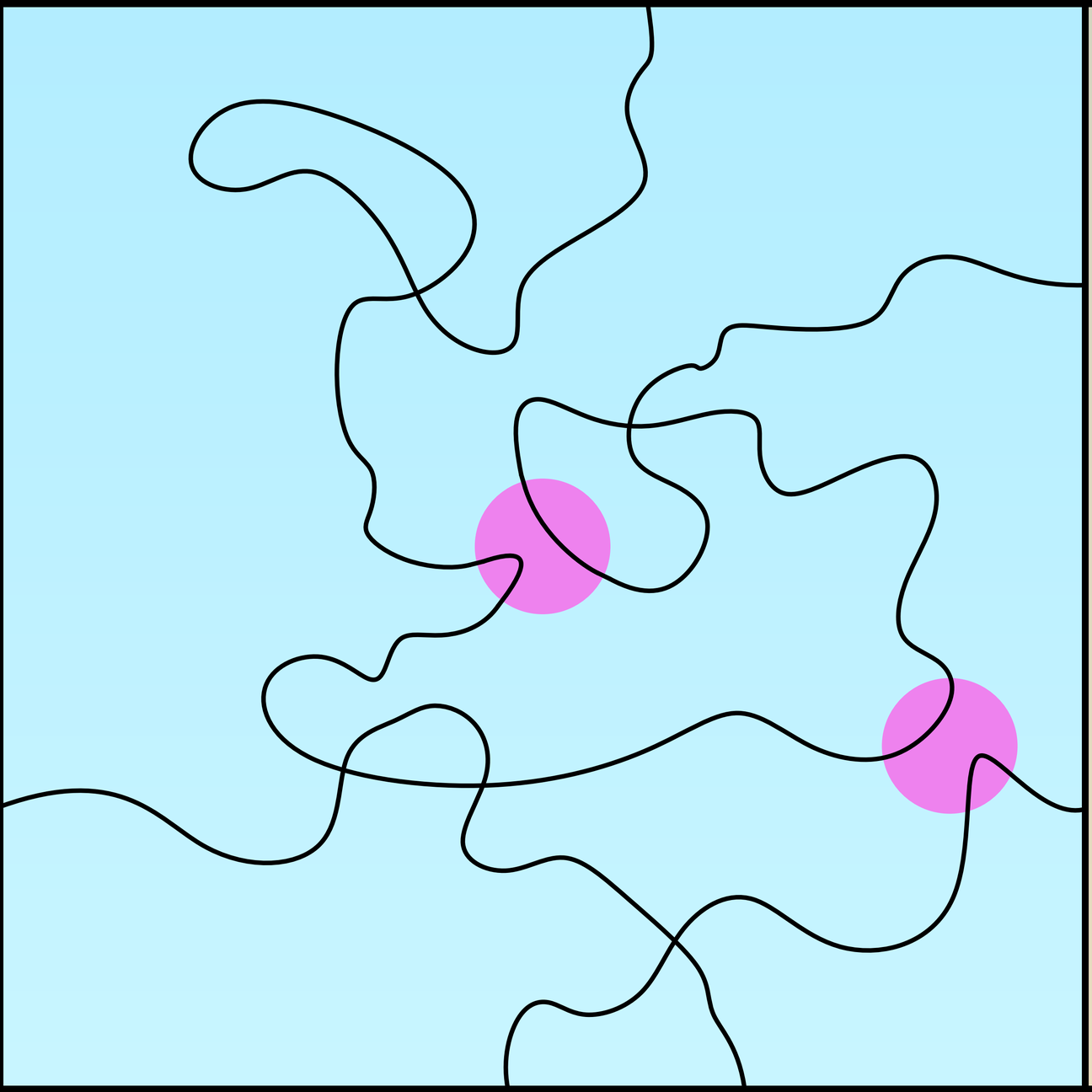

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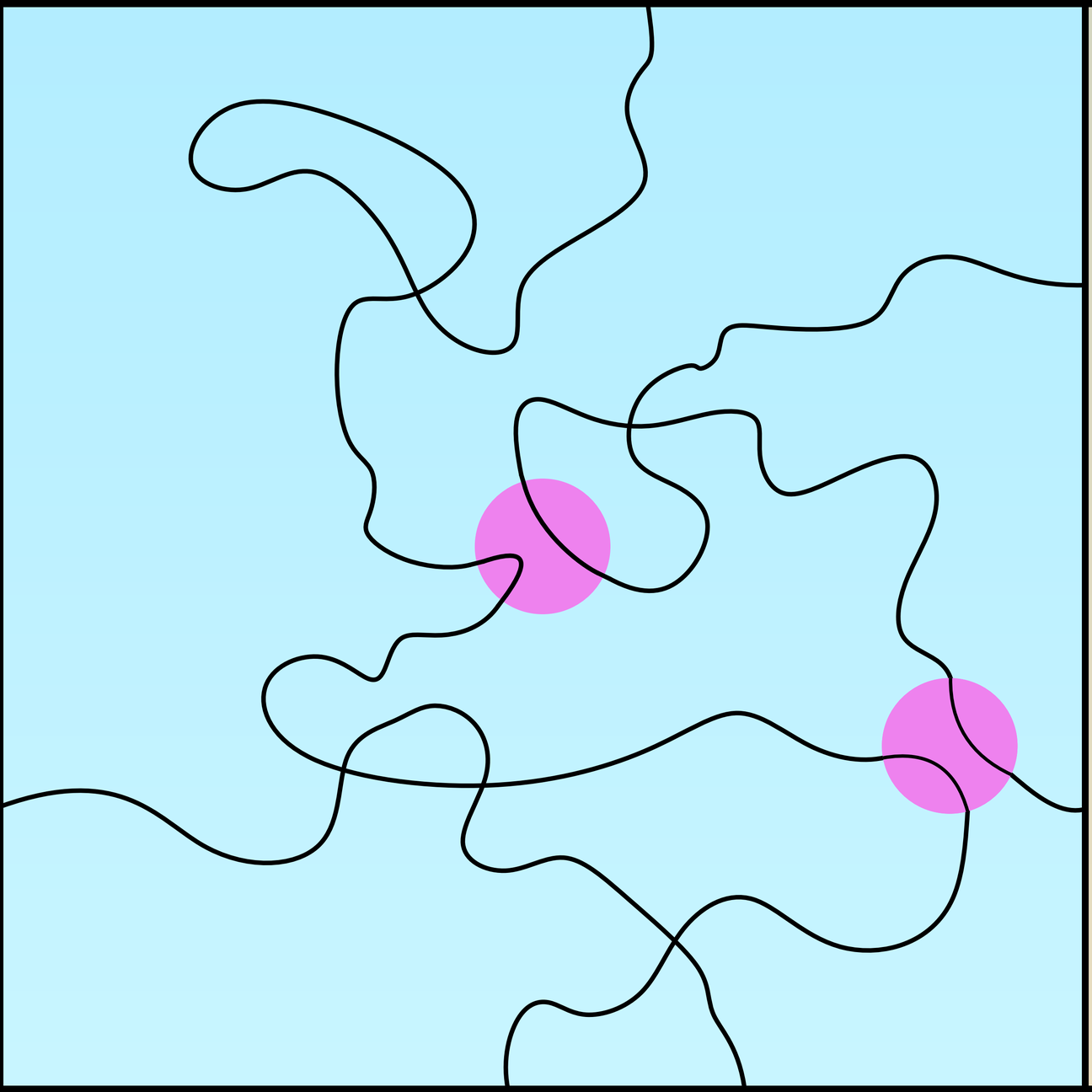

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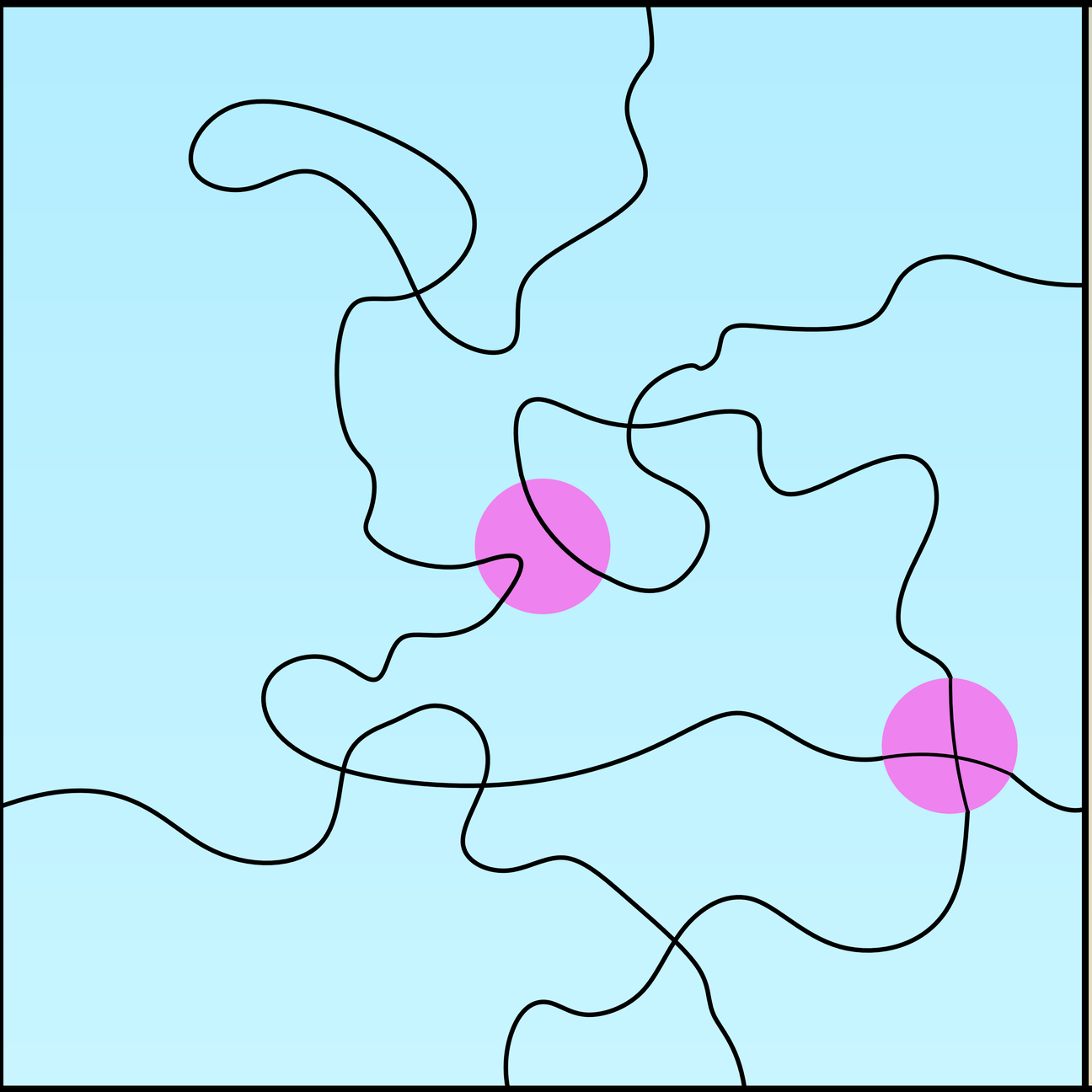

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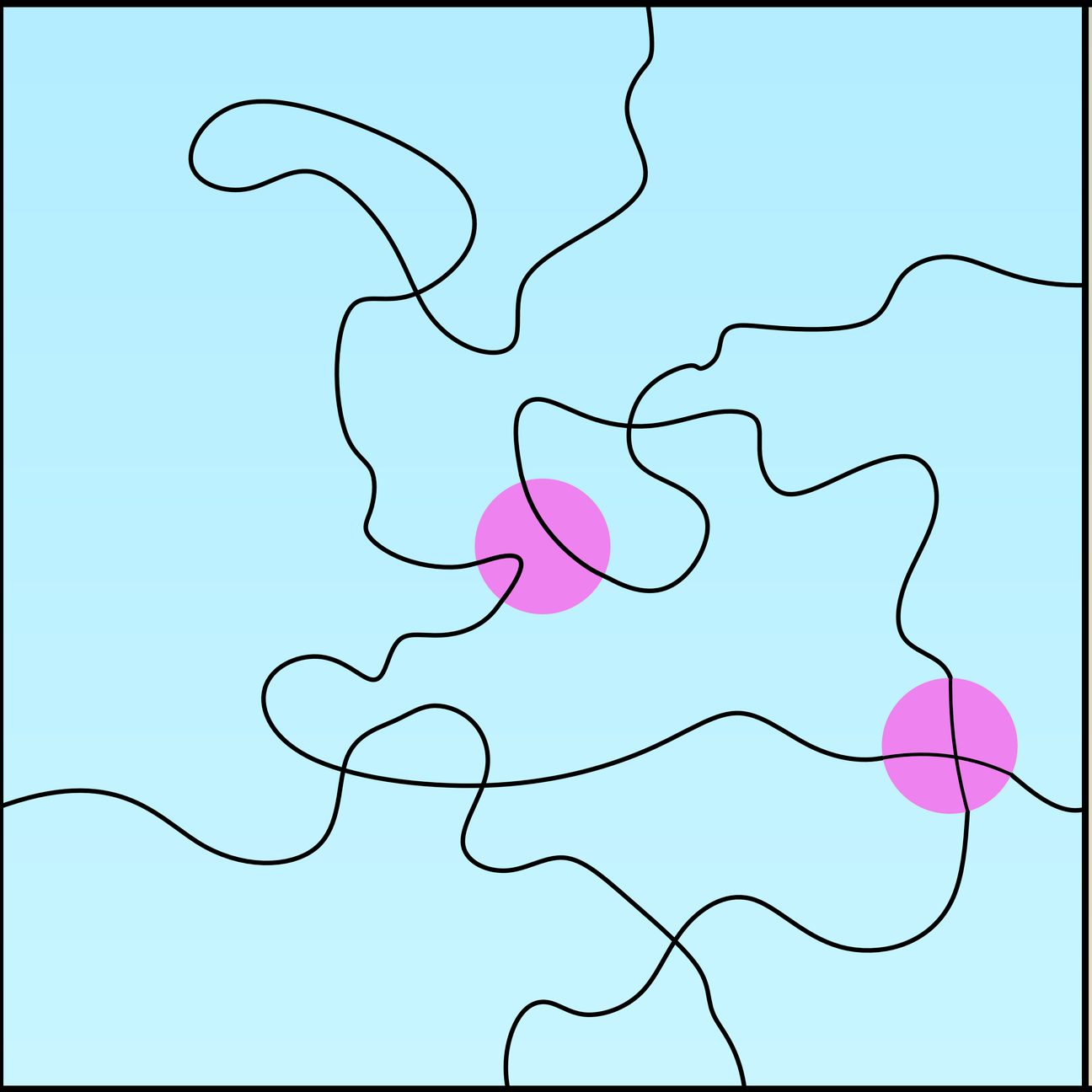

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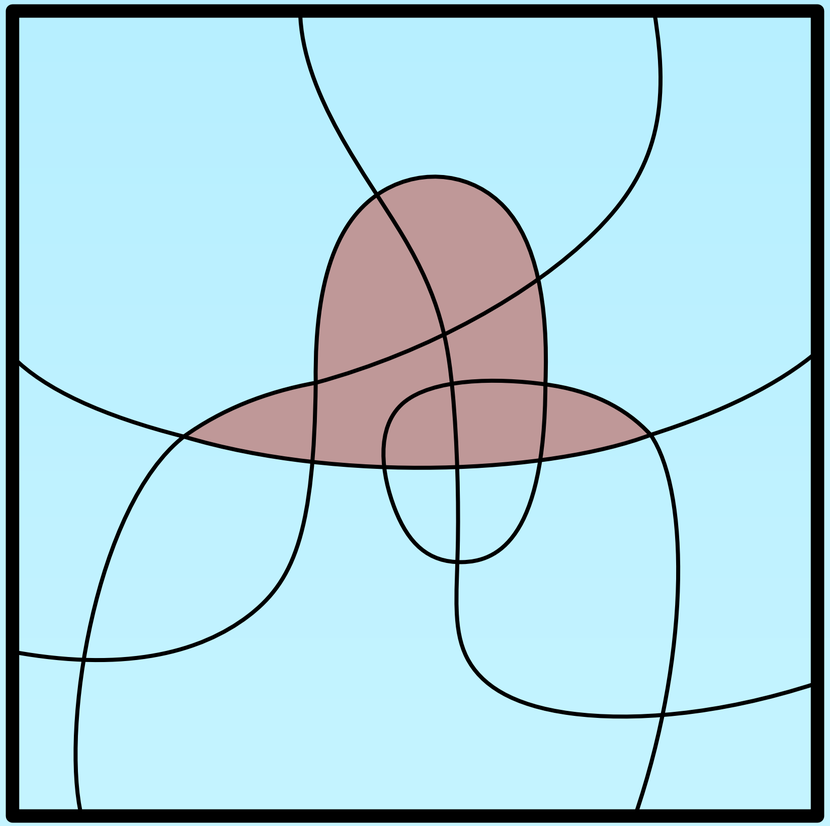
We can reroute the curves inside a switch in three ways.

Only limited impact on overall aesthetics, large impact on curve structure.



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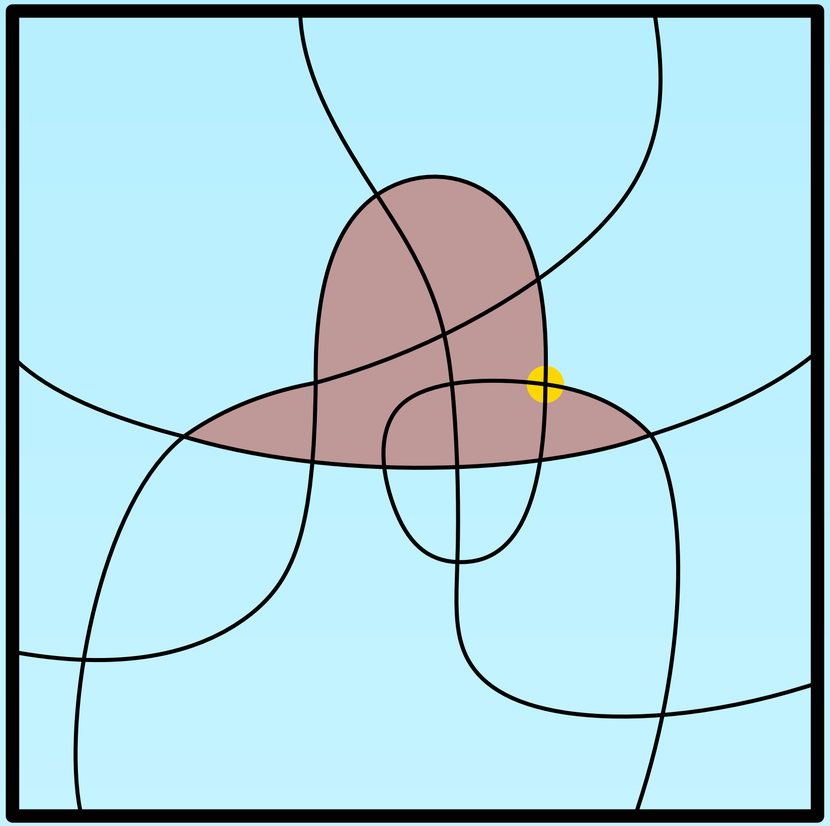

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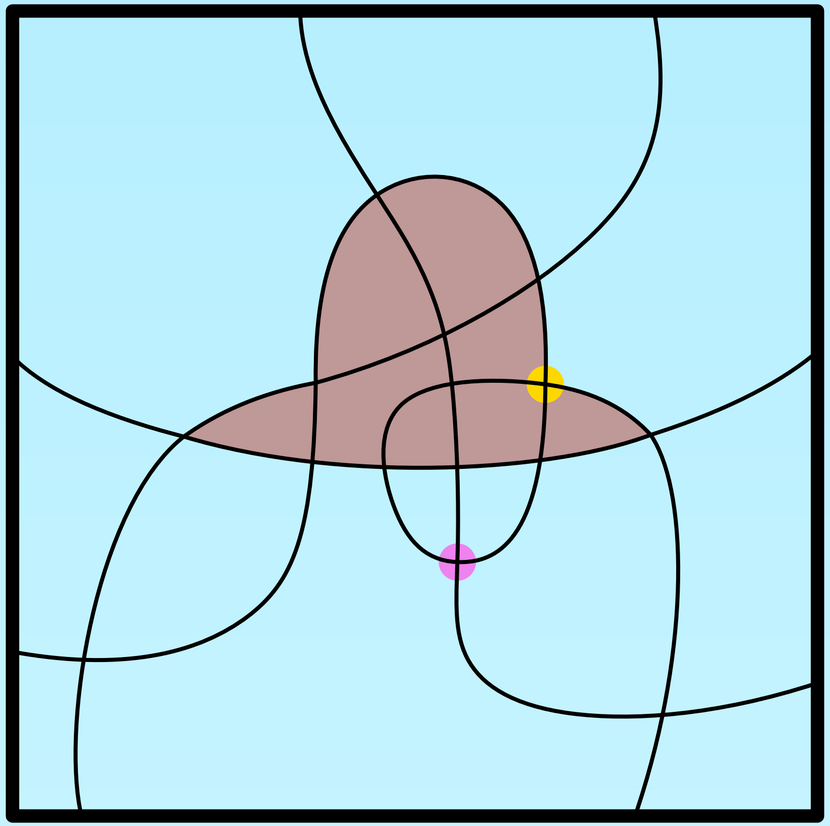

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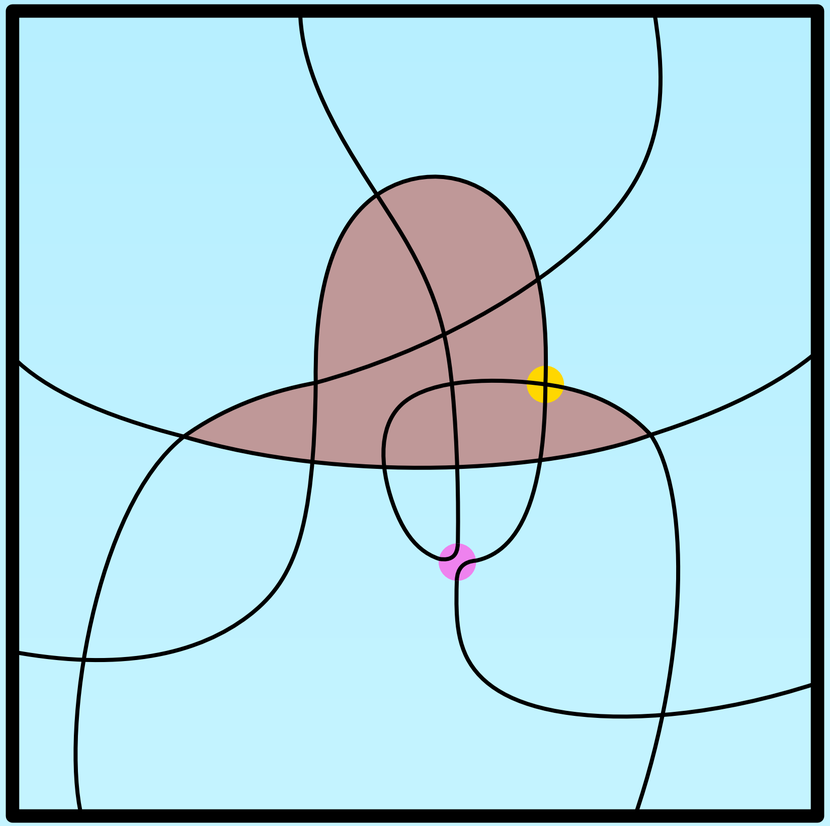

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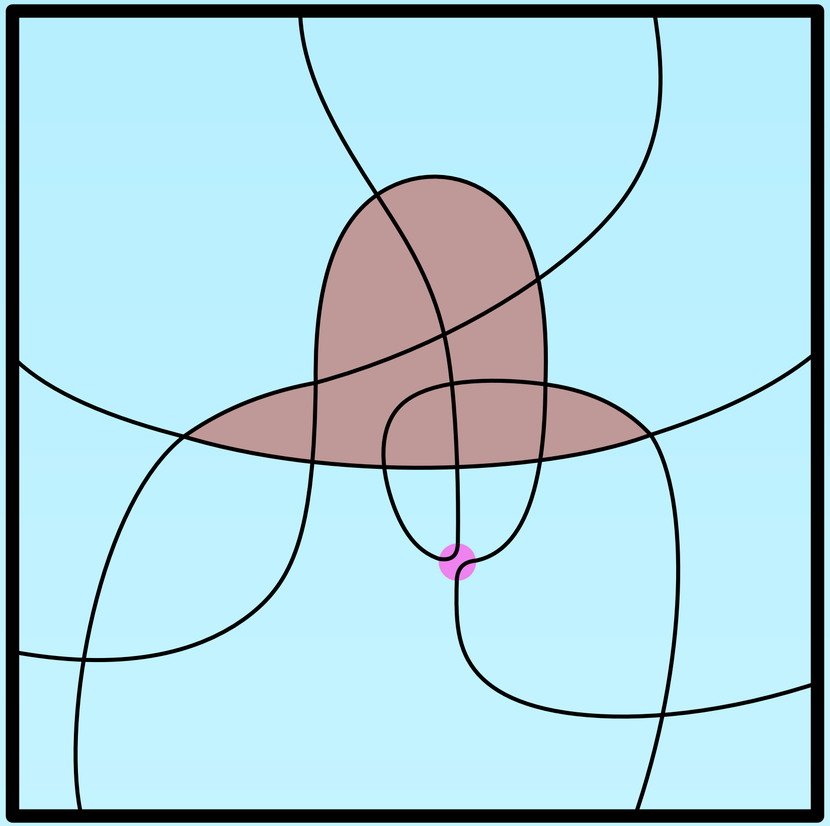

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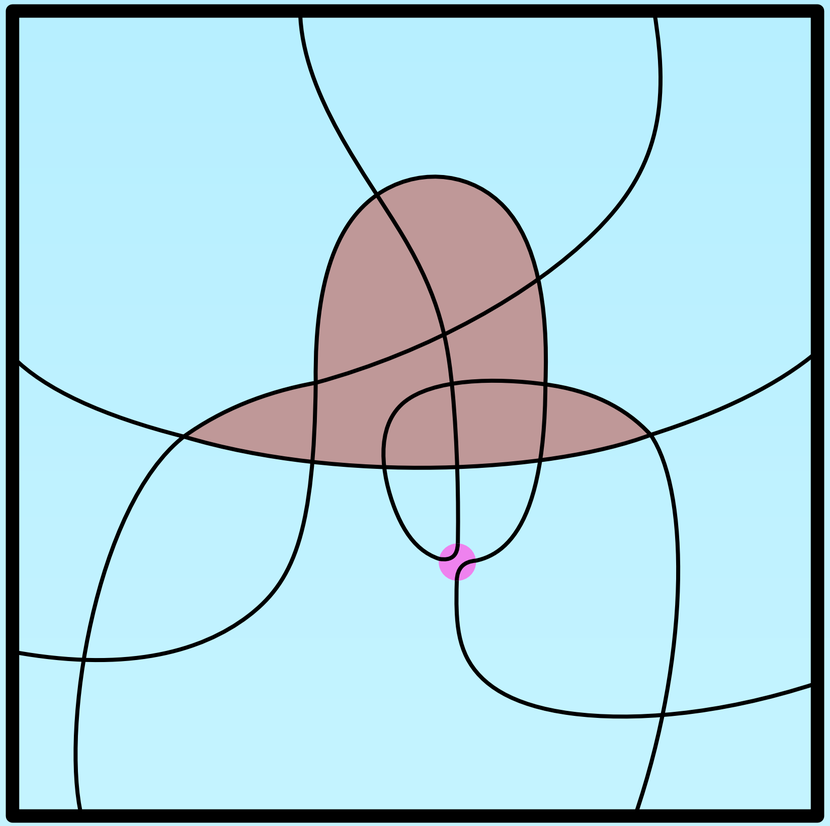

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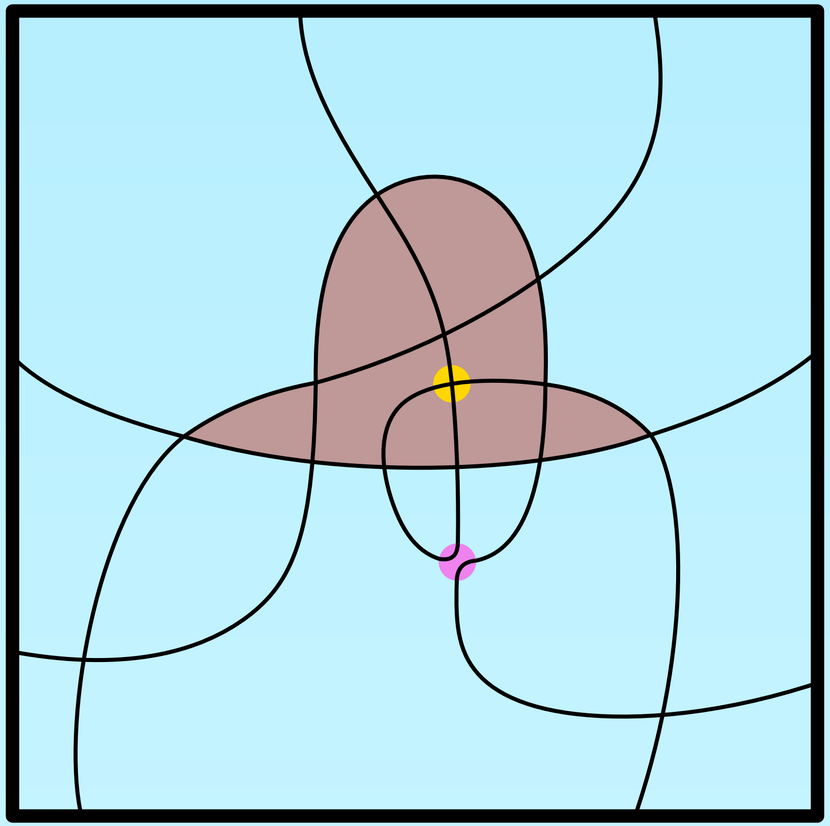

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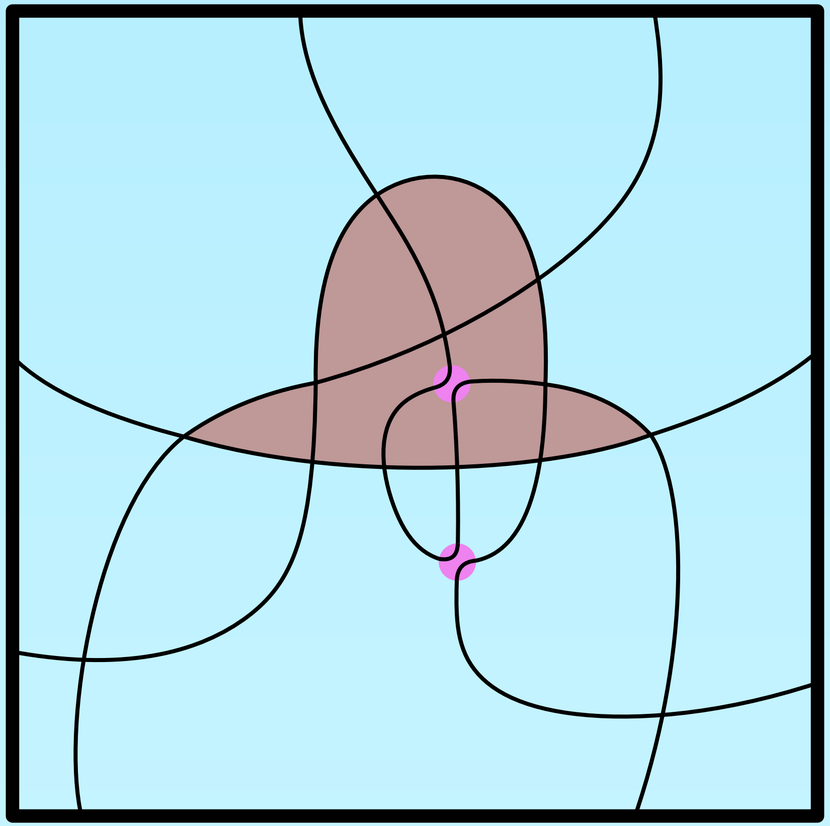

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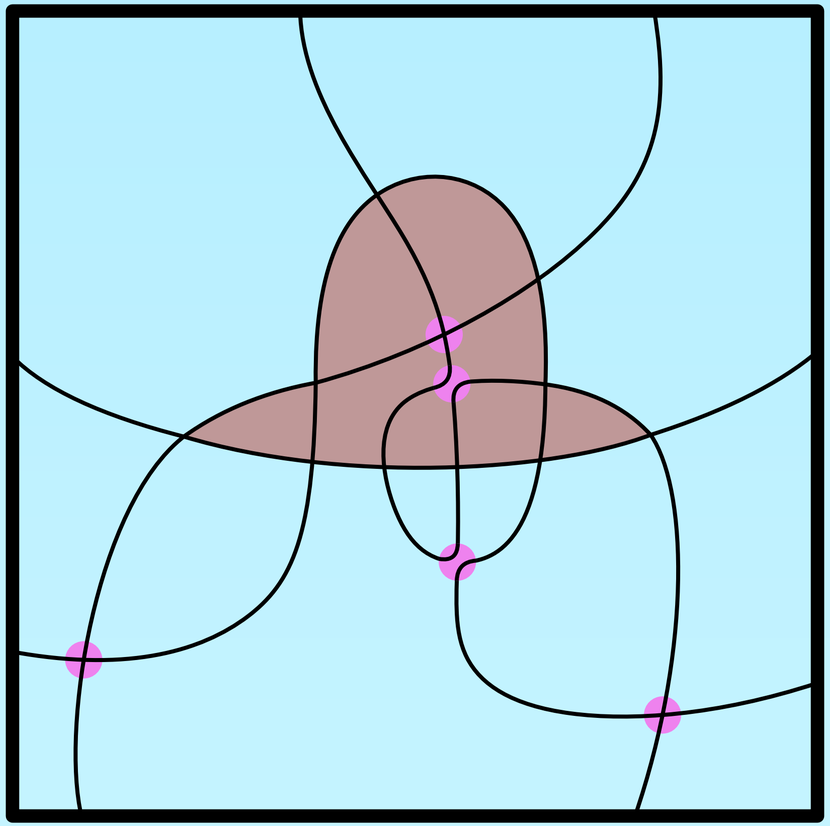

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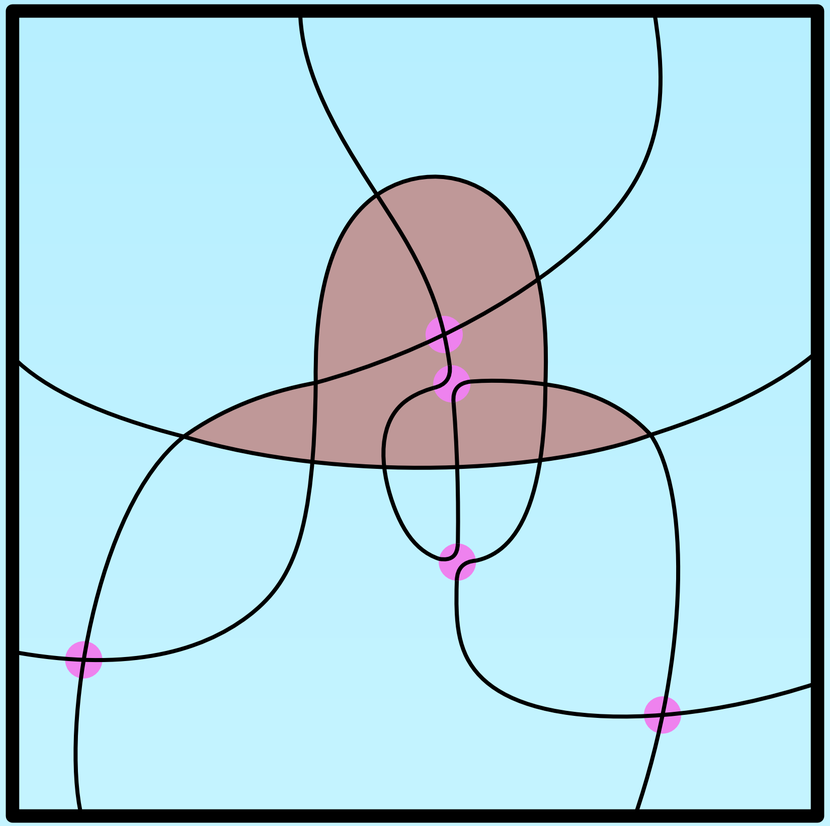

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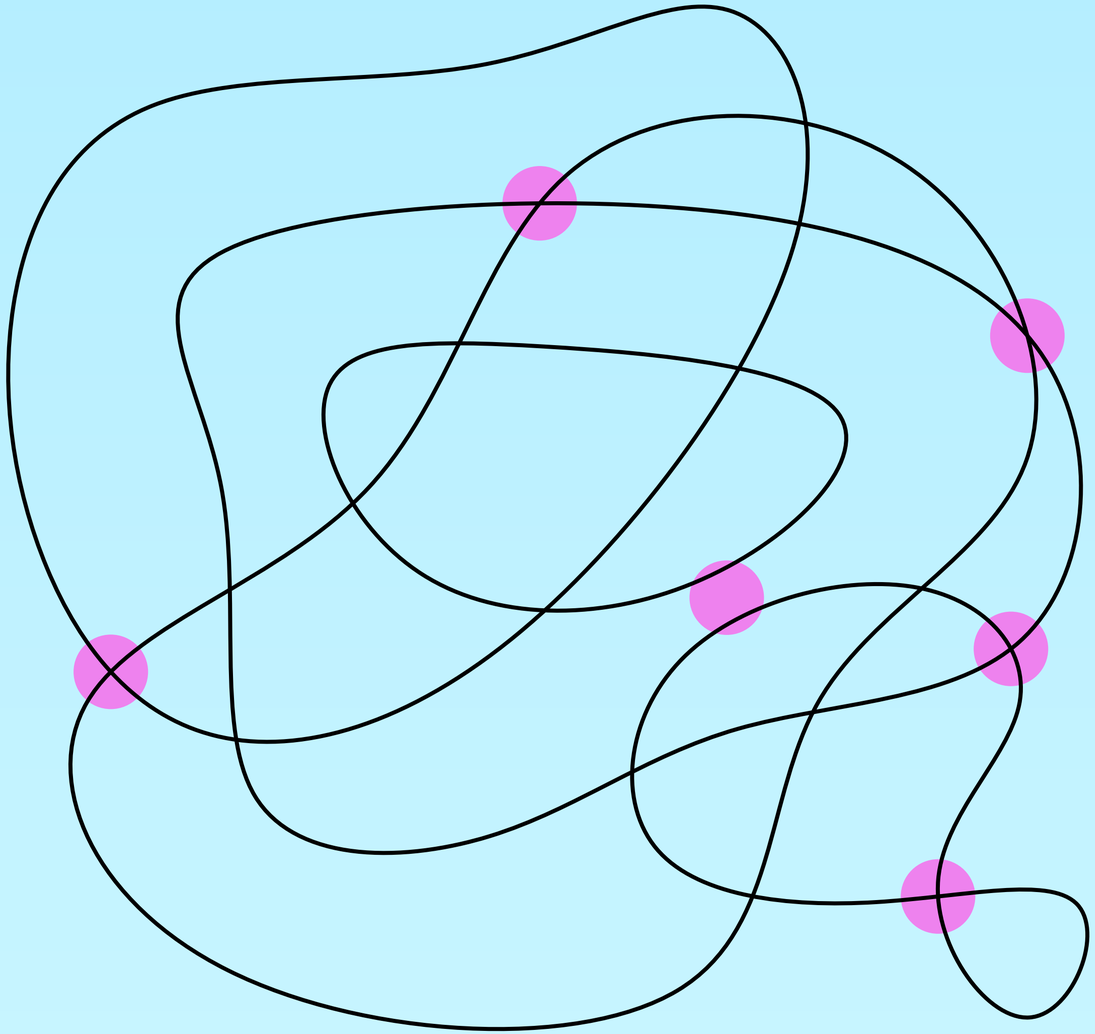
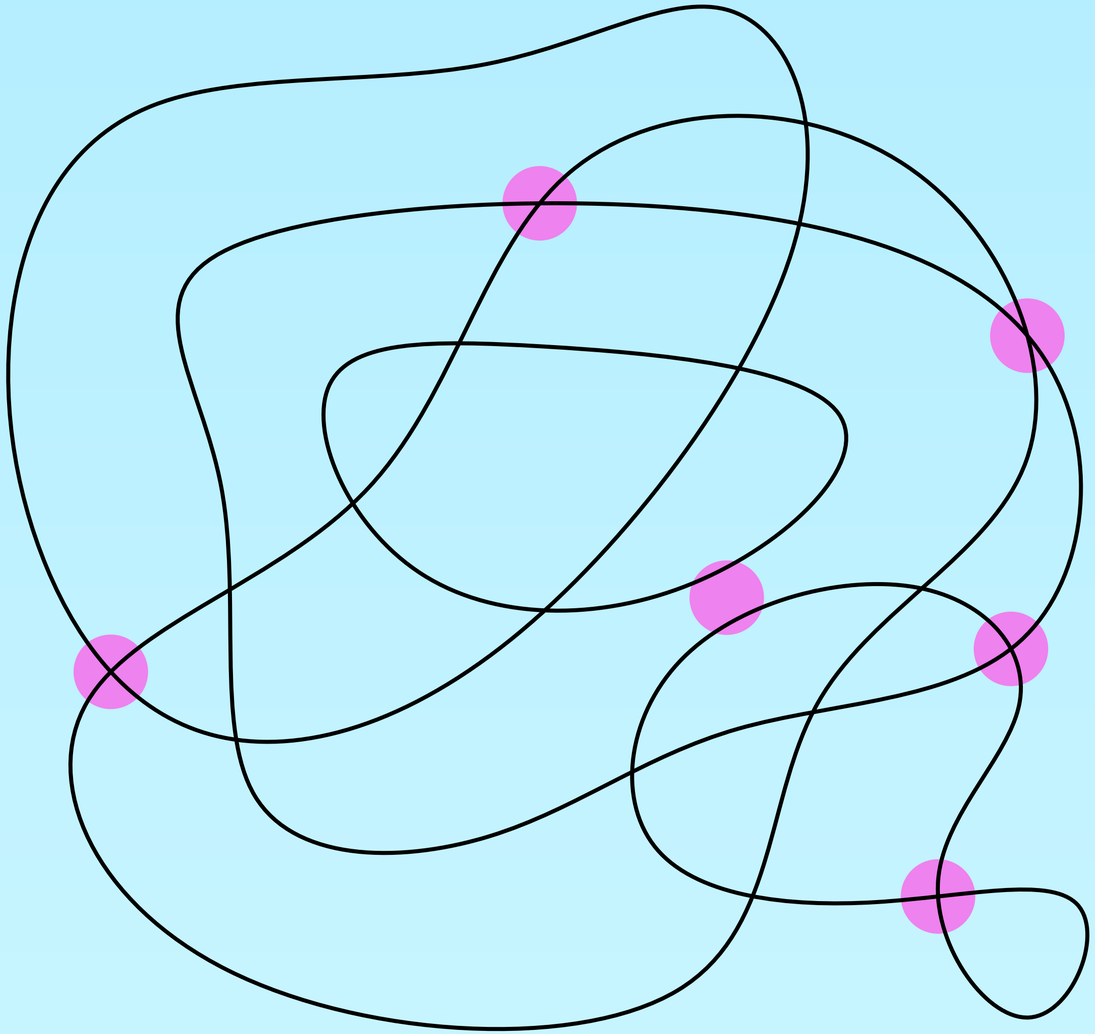
Not every switch is valid in the context of nonograms!

Also, the solvability of the puzzle may change.



A switchable arrangement is an arrangement of closed curves in the plane, together with a set of switches.



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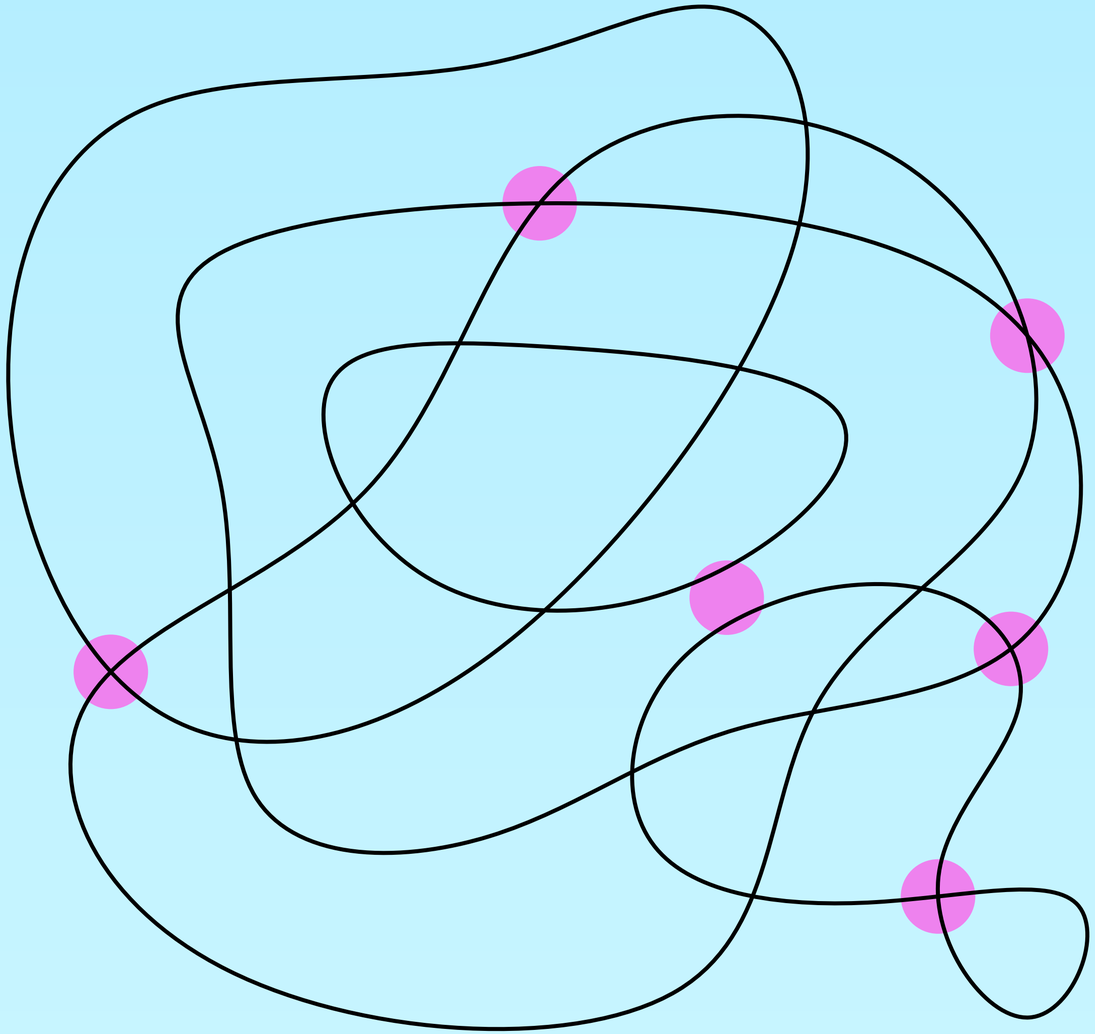

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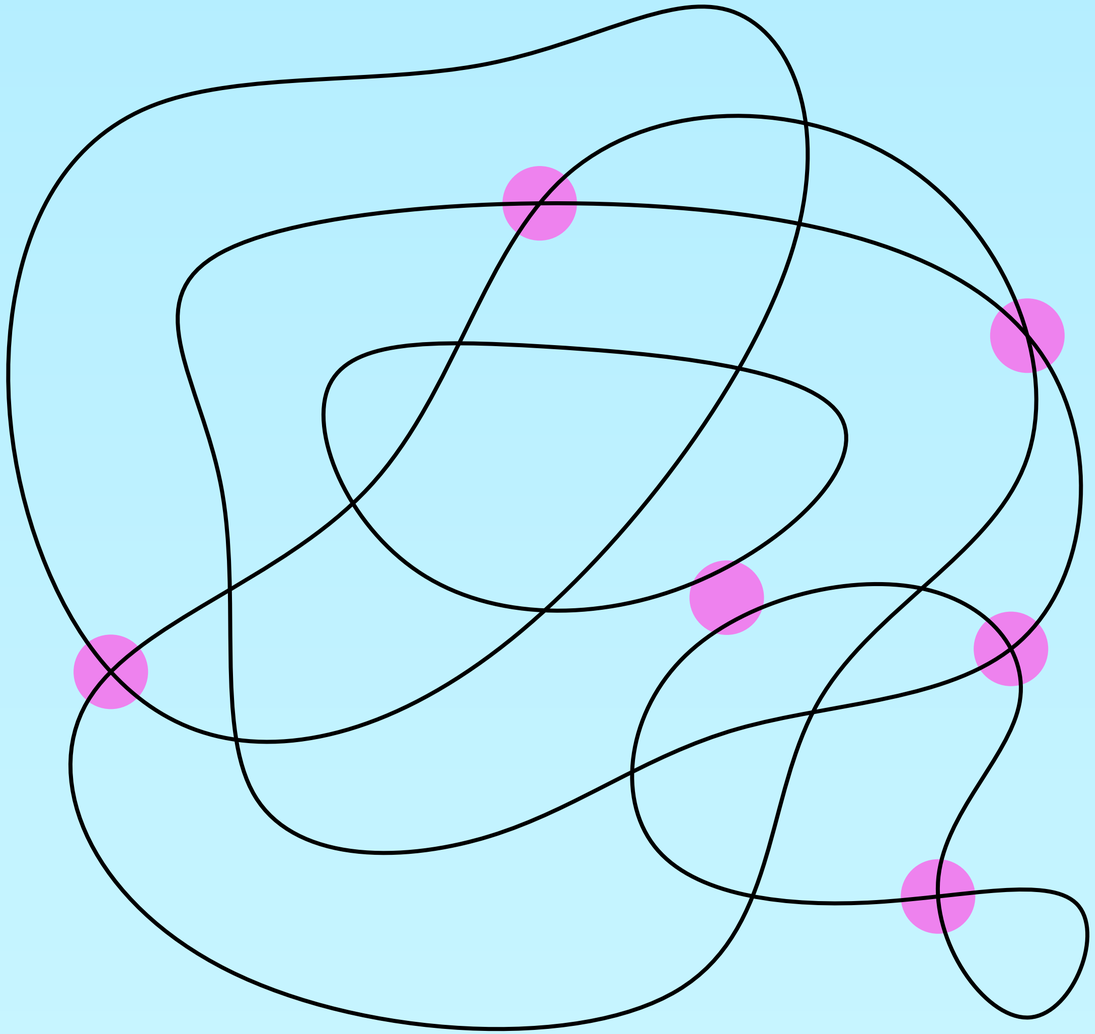

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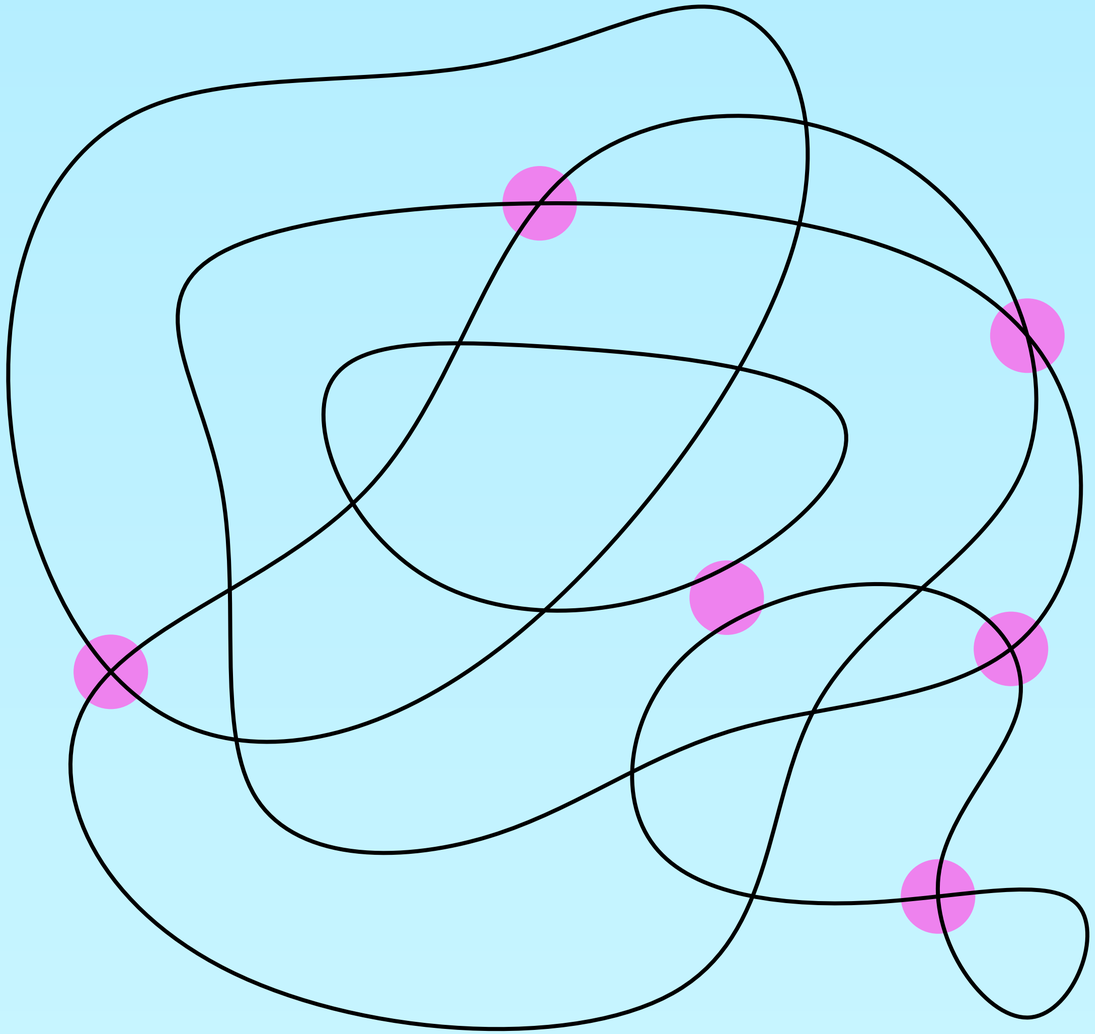
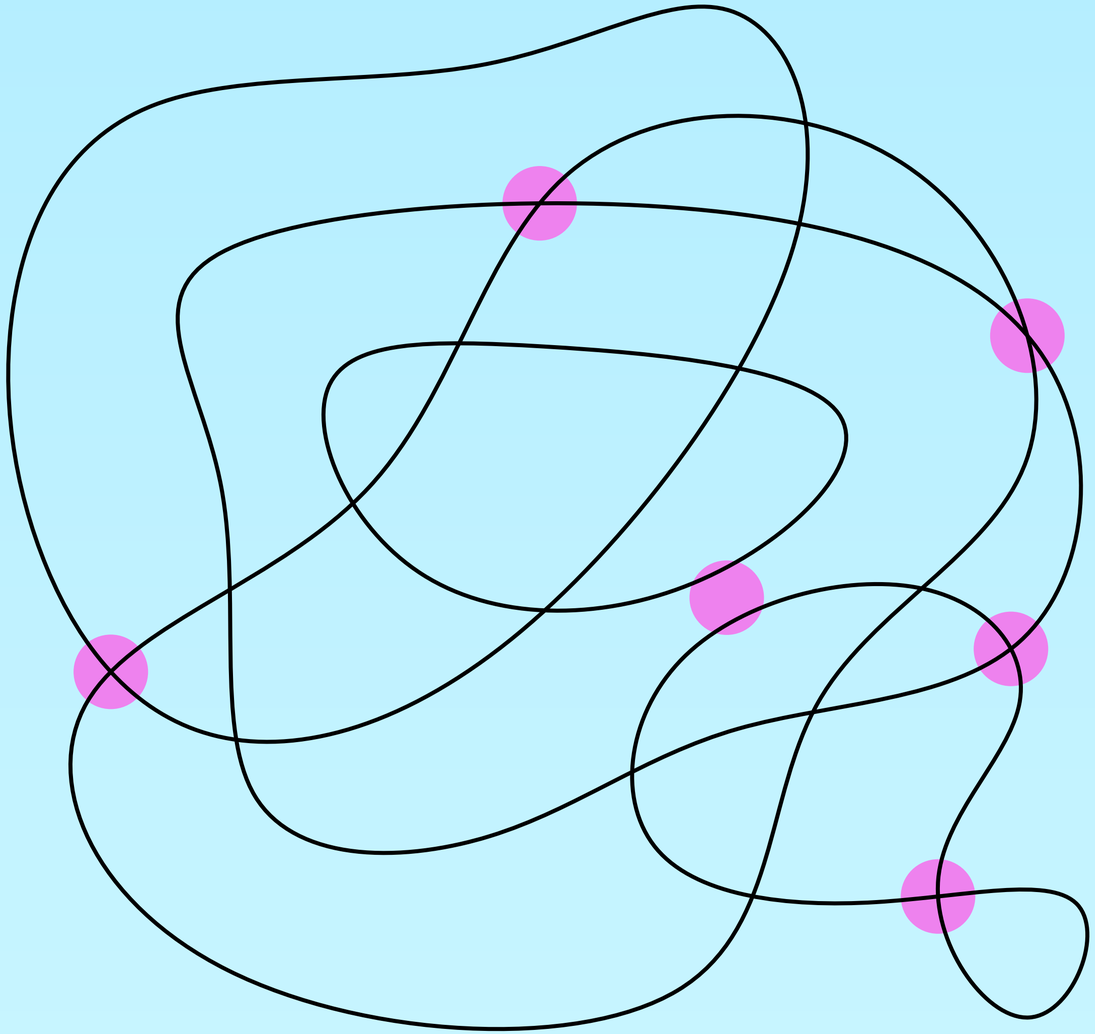
Now, we have a clean mathematical problem!



Problem statement.
Given a switchable curve arrangement, can we reconfigure the switches such that:



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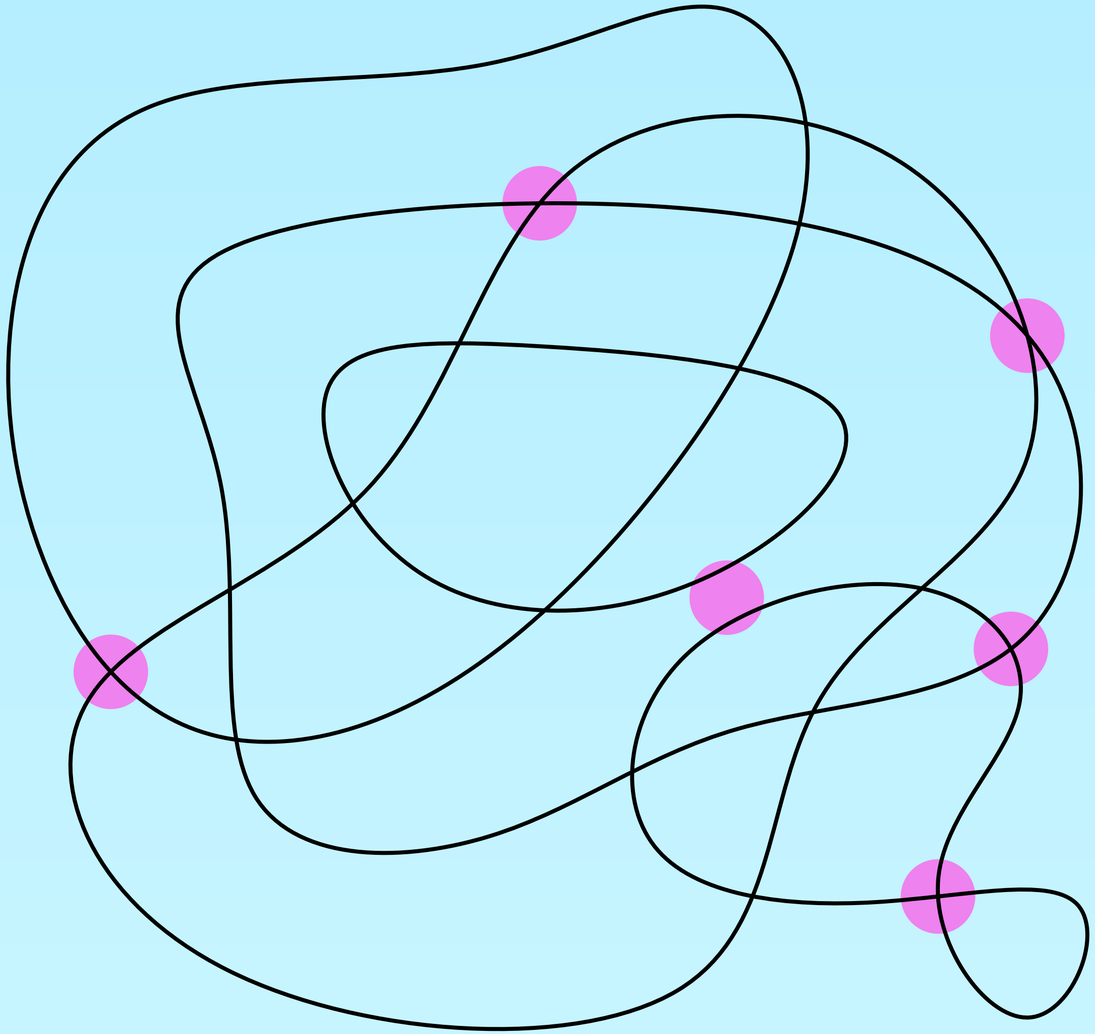
Given a switchable curve arrangement, can we reconfigure the switches such that:

- all self-intersections are removed?



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Given a switchable curve arrangement, can we reconfigure the switches such that:

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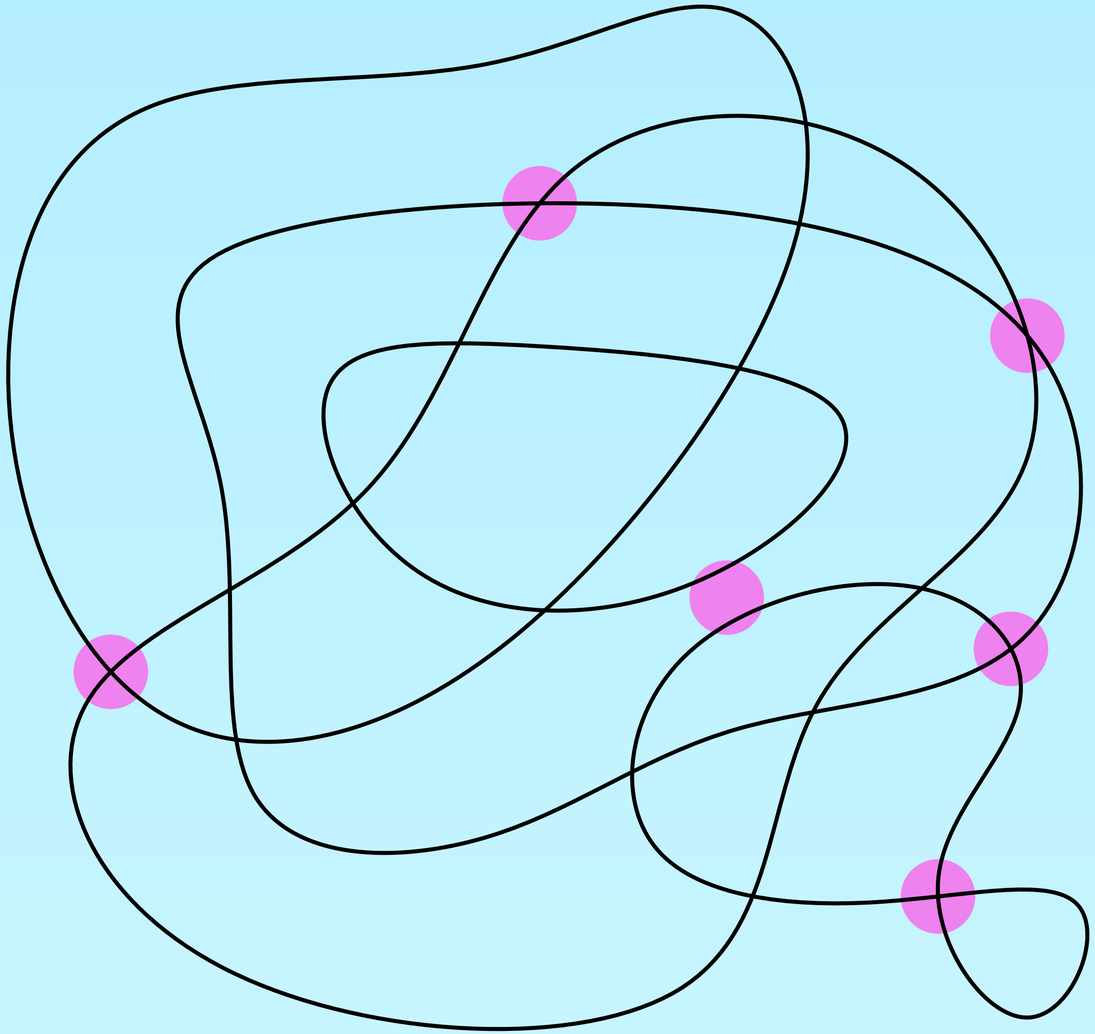
- all popular faces are removed?



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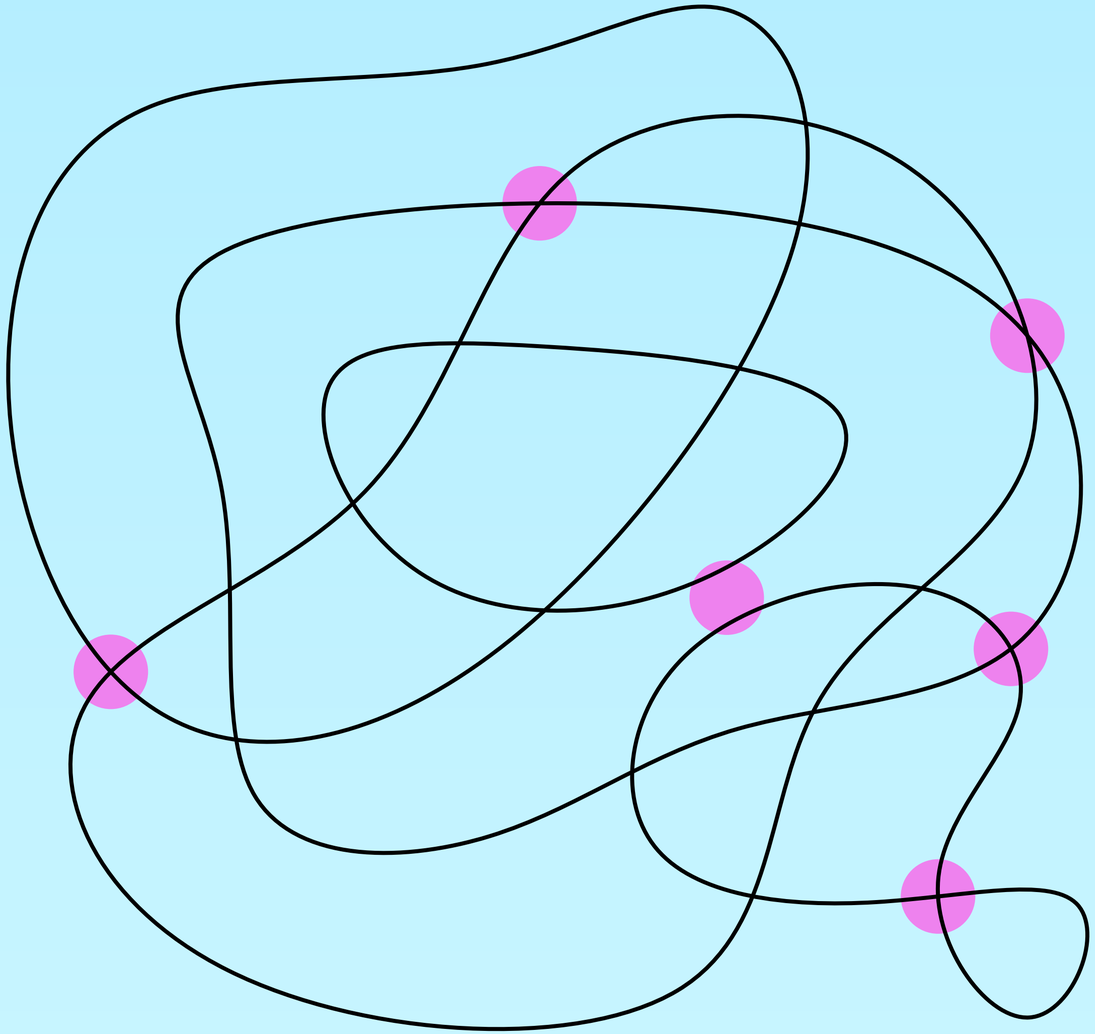
- we minimize the number of switch operations used?



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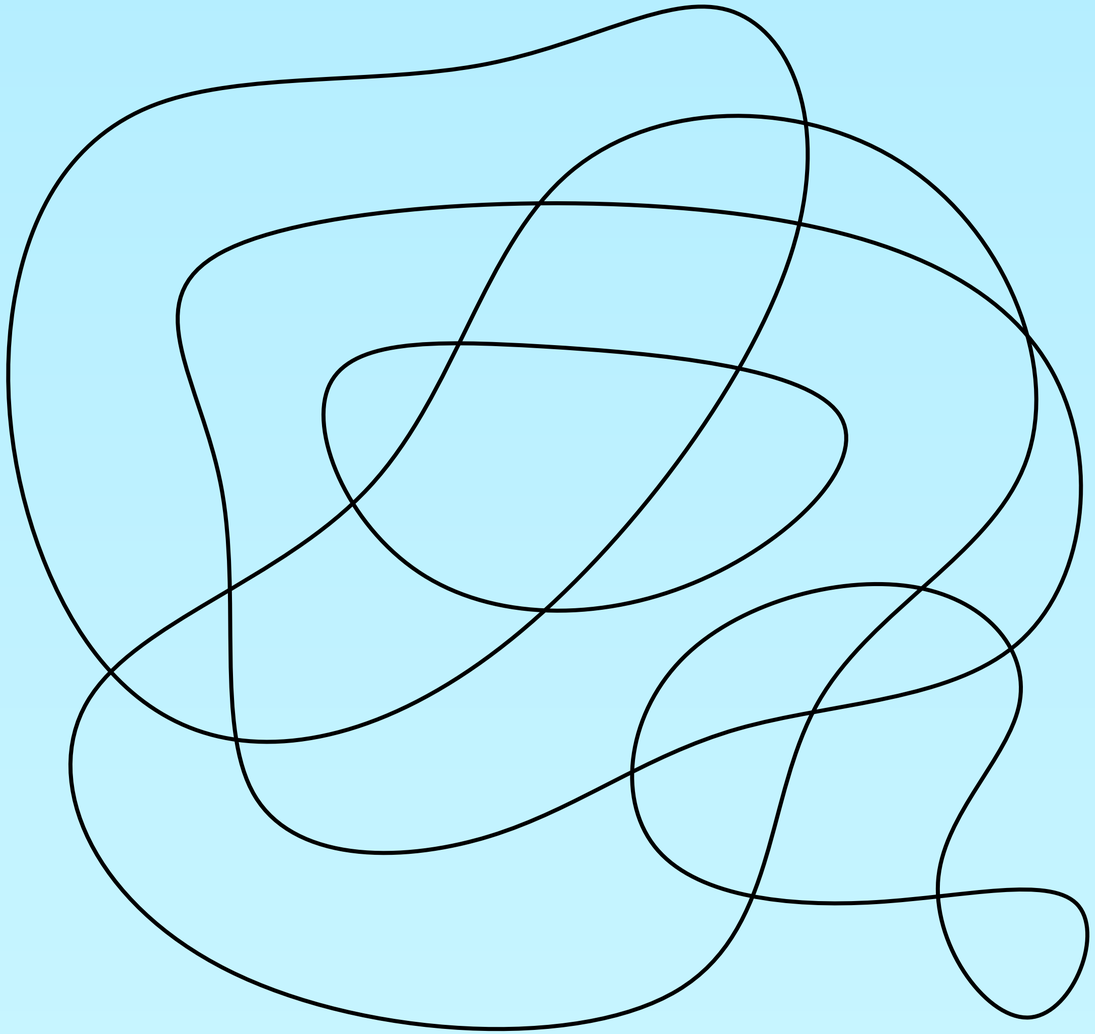
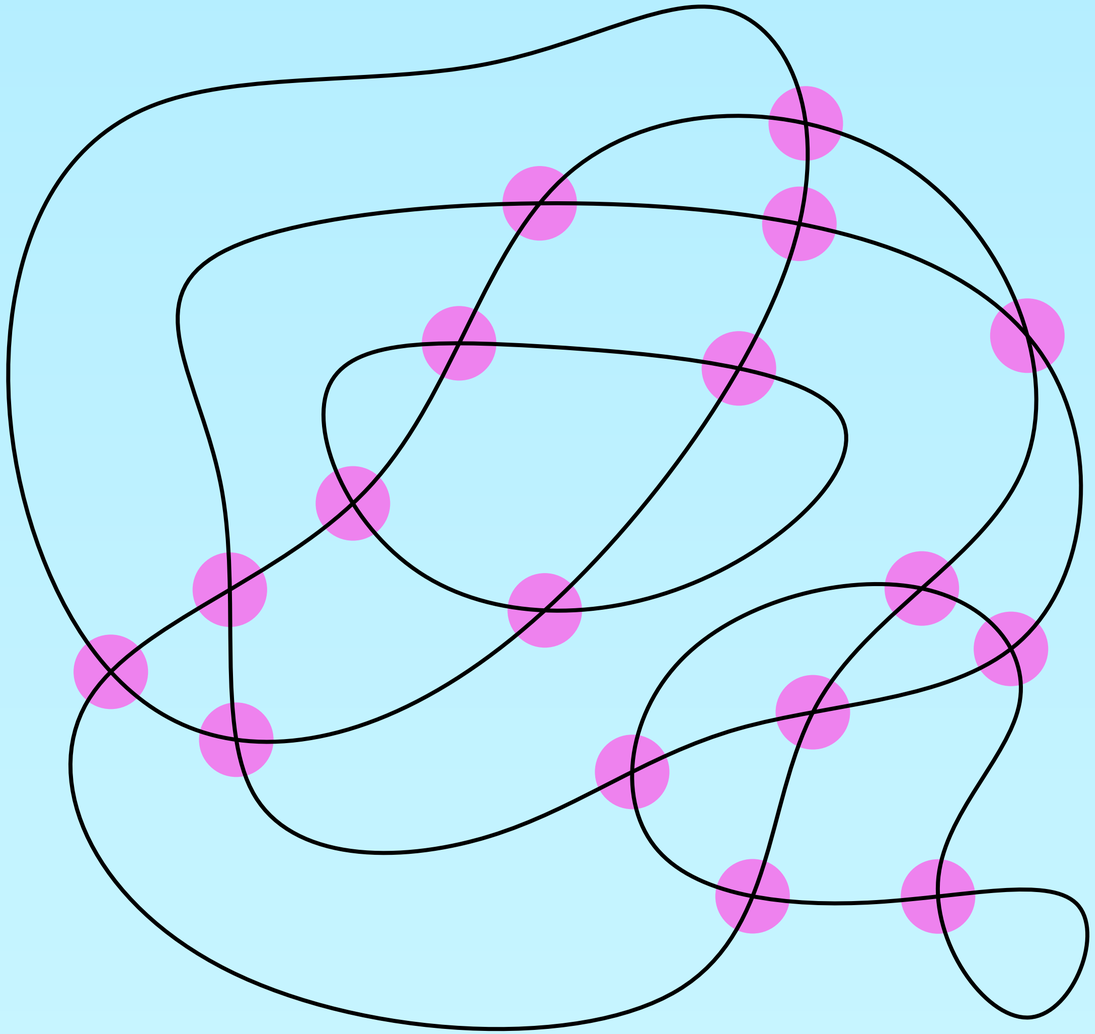
- we minimize the number of switch operations used?
- we minimize the number of self-intersections or popular faces in the output?



Observation.
When all vertices of the arrangement are switches, we can always remove all self-intersections and popular faces.



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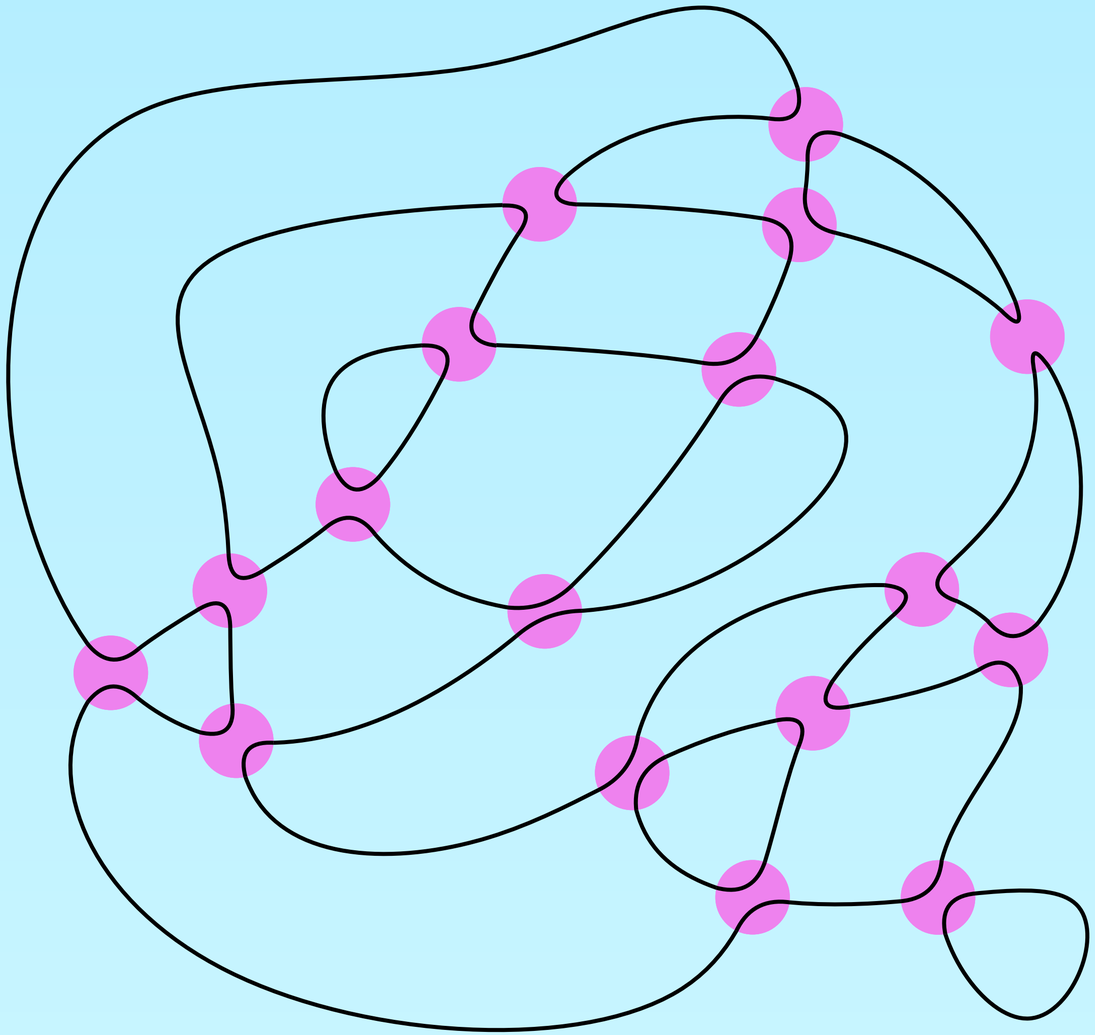

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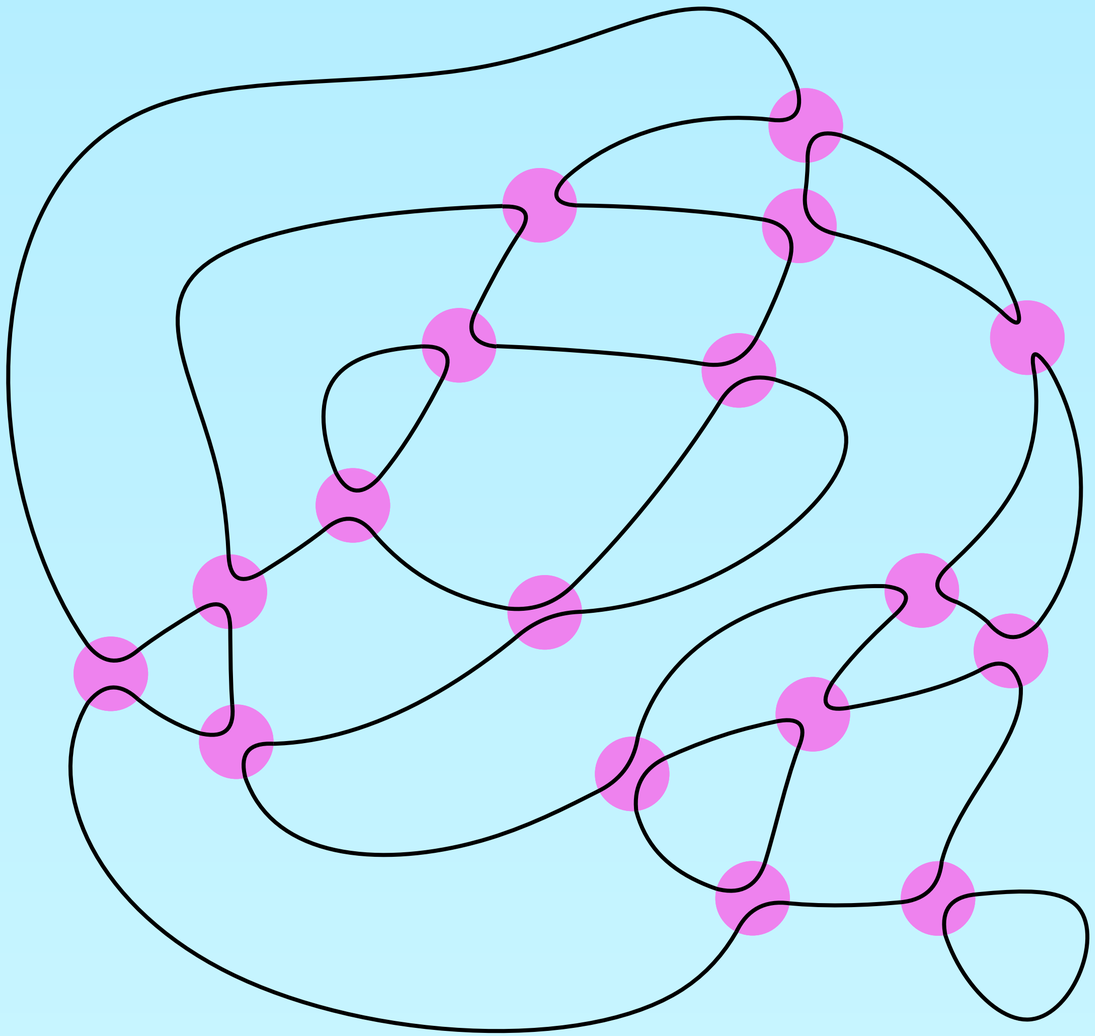

Observation.
When all vertices of the arrangement are switches, we can always remove all self-intersections and popular faces.

We probably want to allow as many switches as possible, but we want to switch as few of them as necessary.

Negative results:

Positive results:

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Testing whether it is possible to remove either all self-intersections or all popular faces from a switchable arrangement is NP-hard.

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When we have an arrangement with only k switches, removing all self-intersections of all popular faces is FPT in k.

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Still open for popular faces!

Part 3
HOW?

Hardness.

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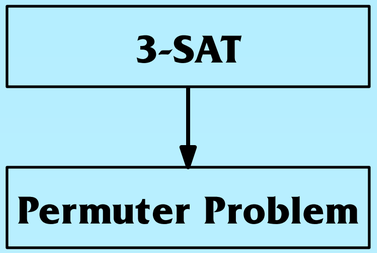
We have a series of reductions, all based on a common problem: the permuter problem.



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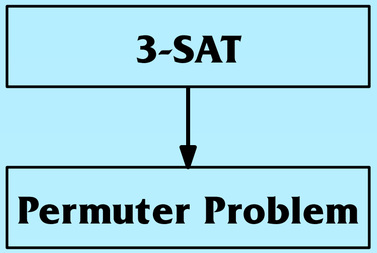

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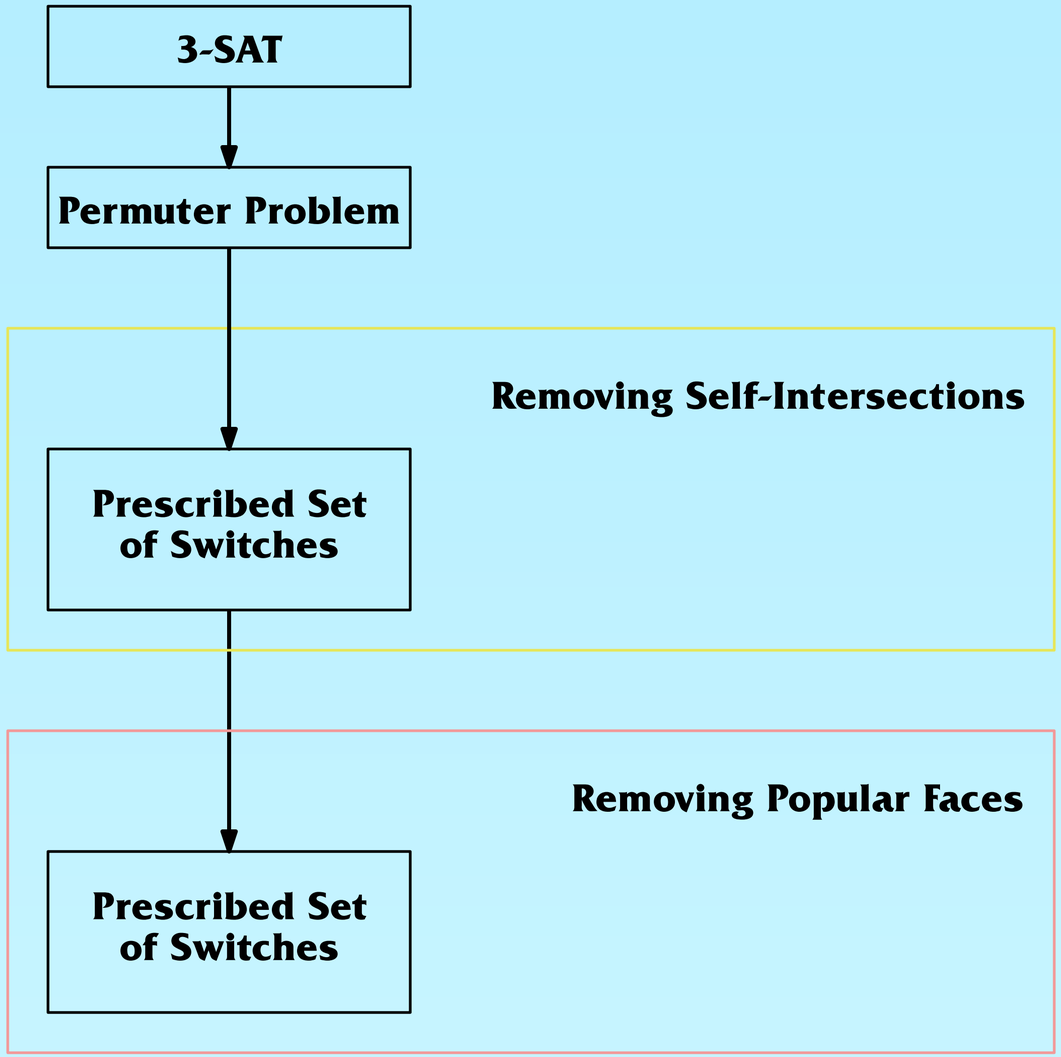
From there, we reduce first to removing self-intersections, and then to popular faces.



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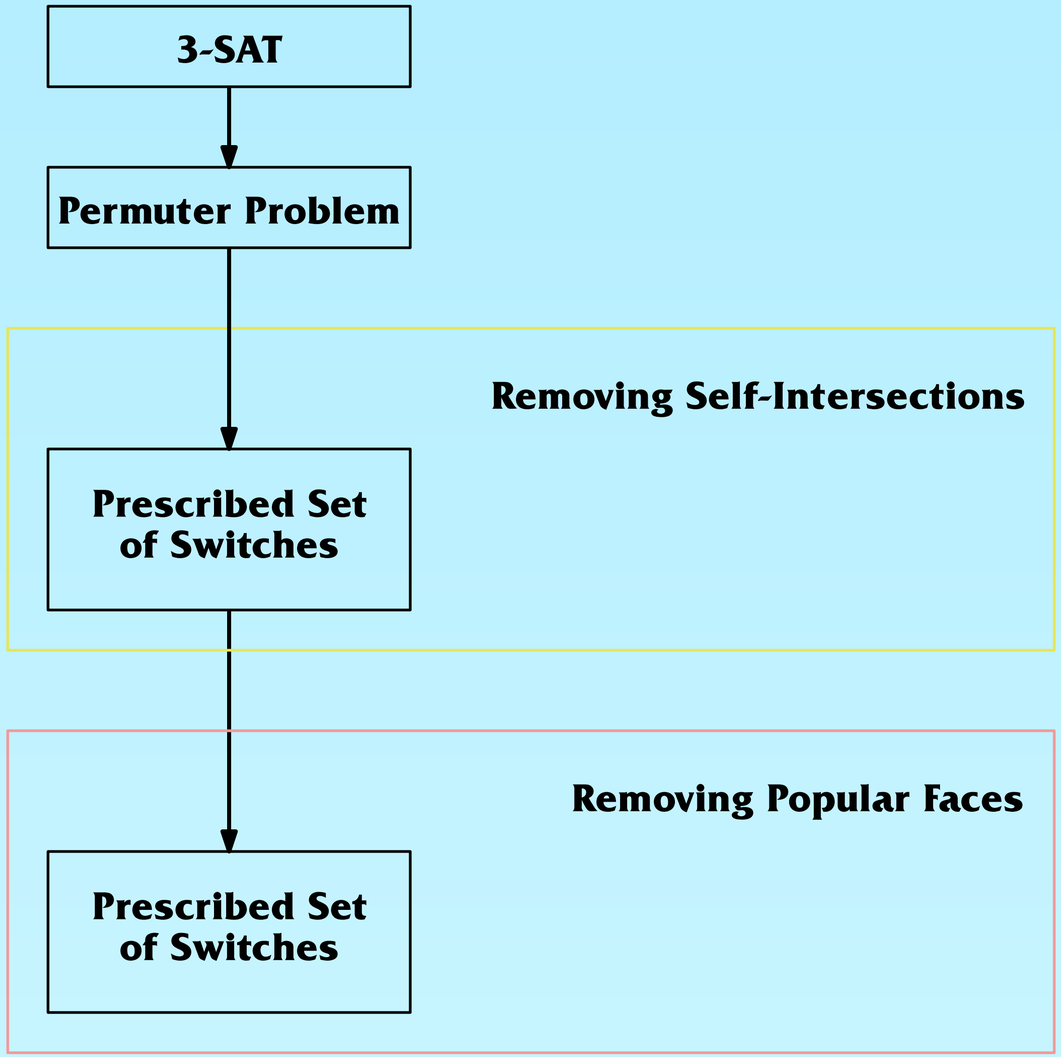

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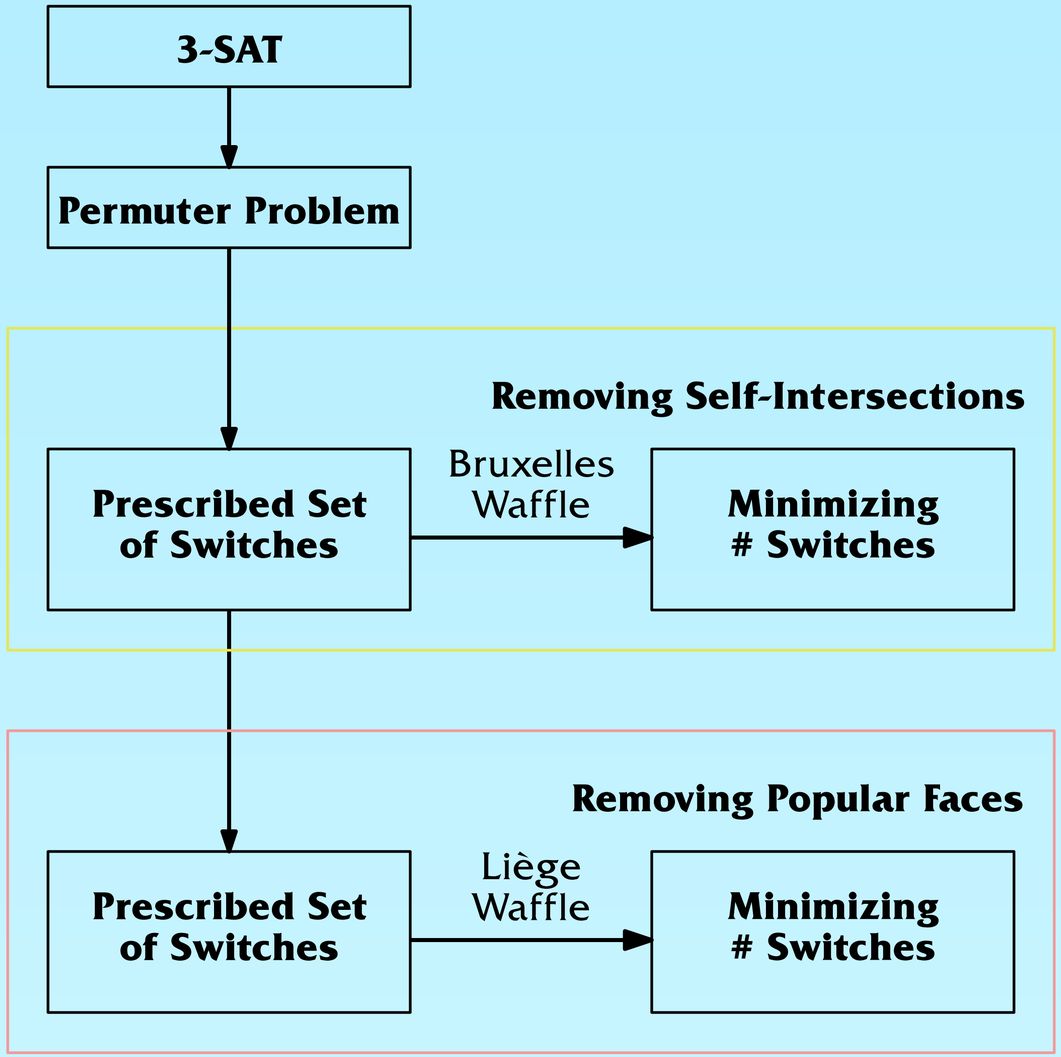

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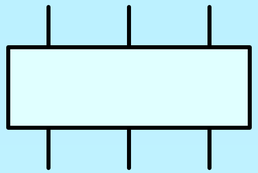

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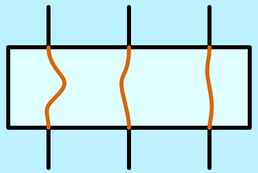

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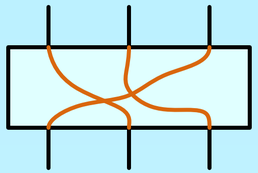

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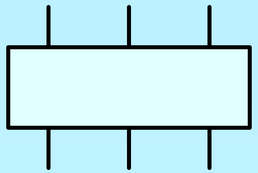

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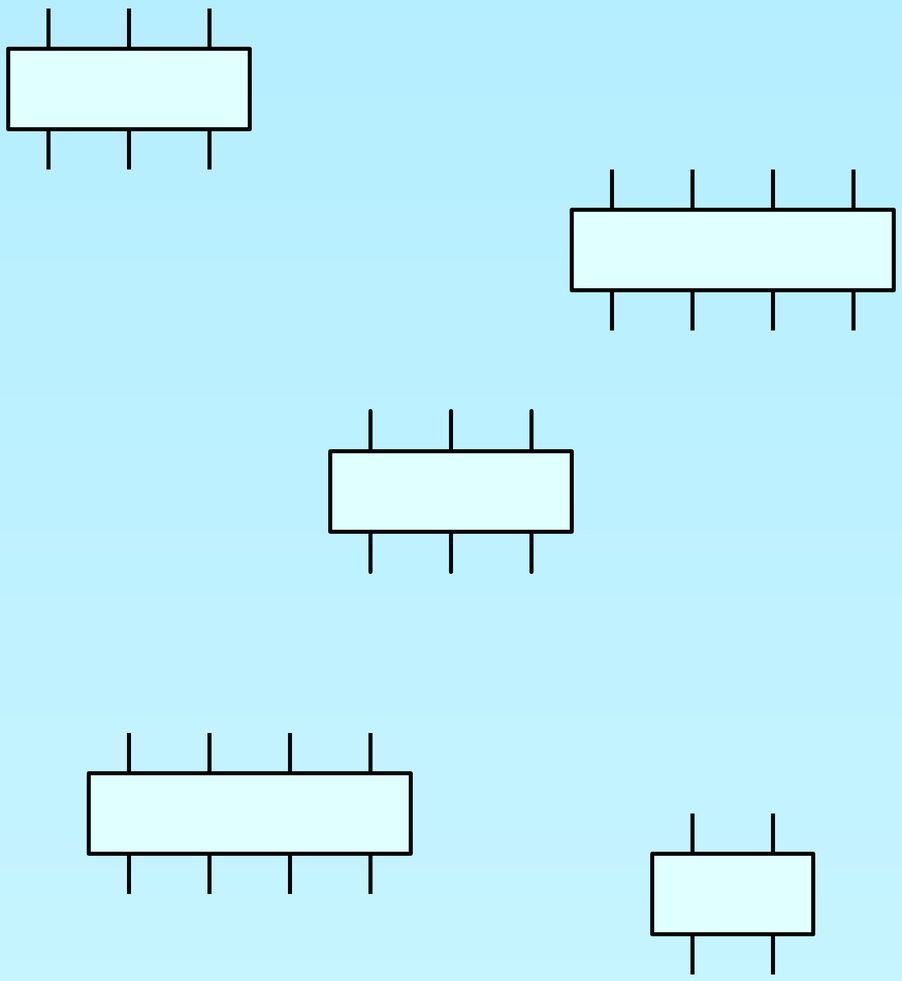
A permuter problem instance is a collection of permuters, whose connectors are linked with paths on the outside.



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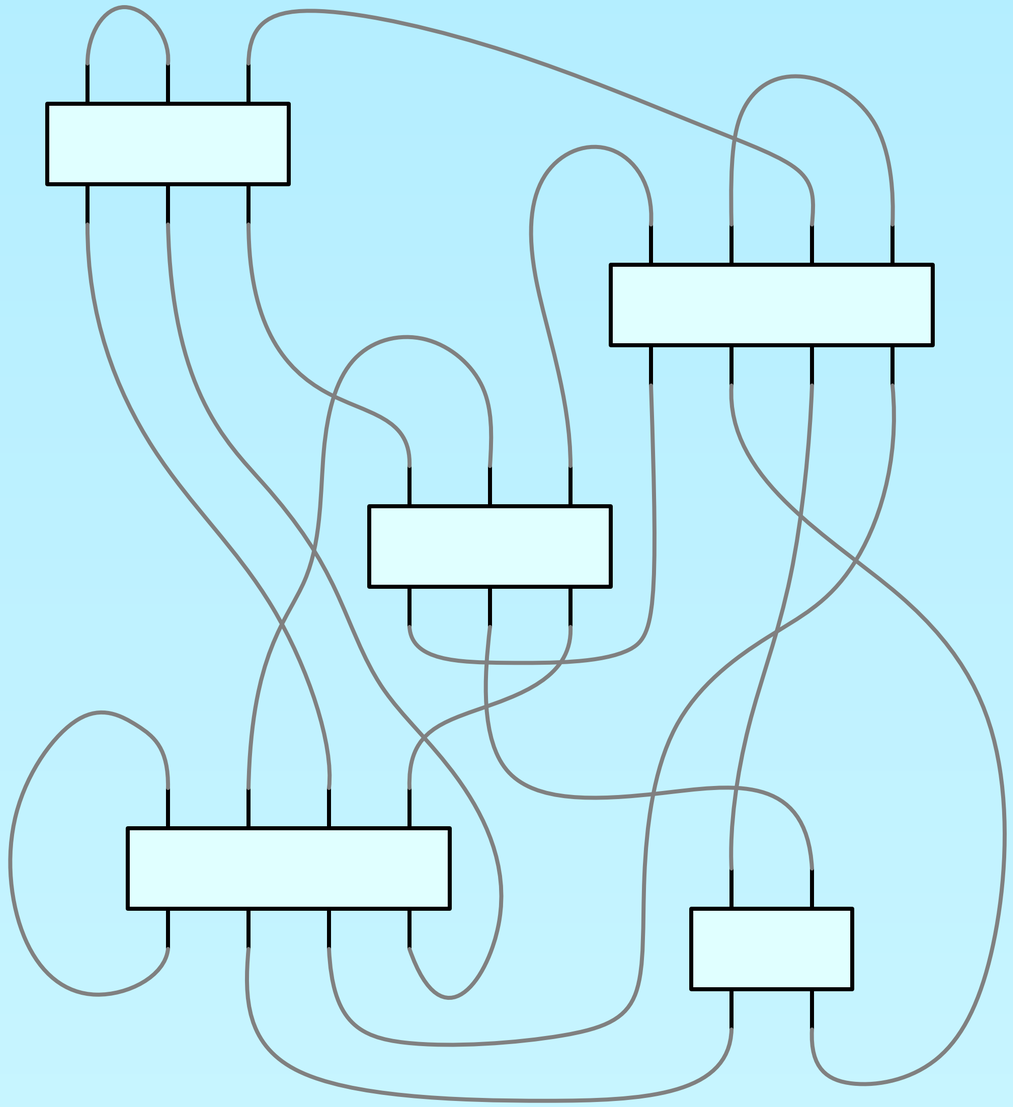

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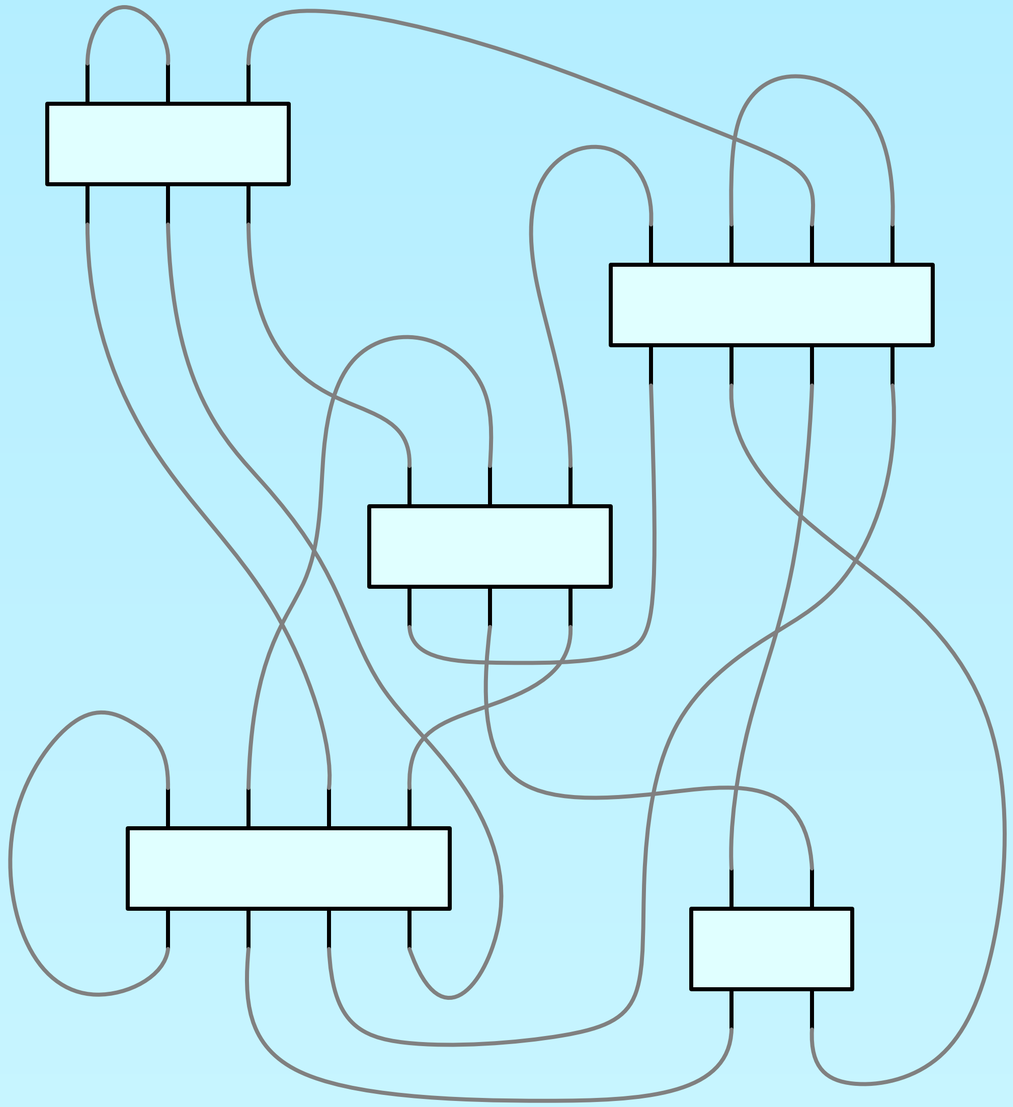

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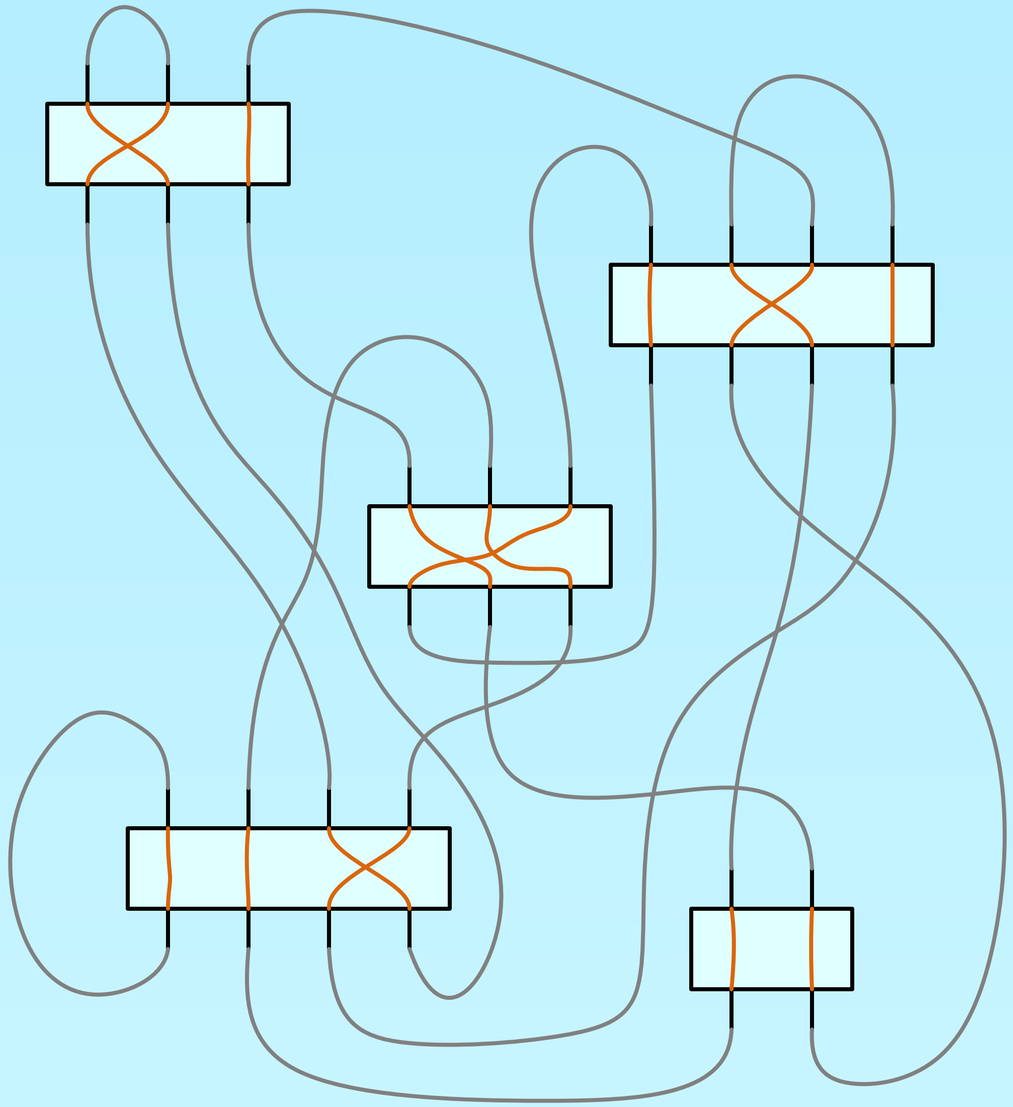

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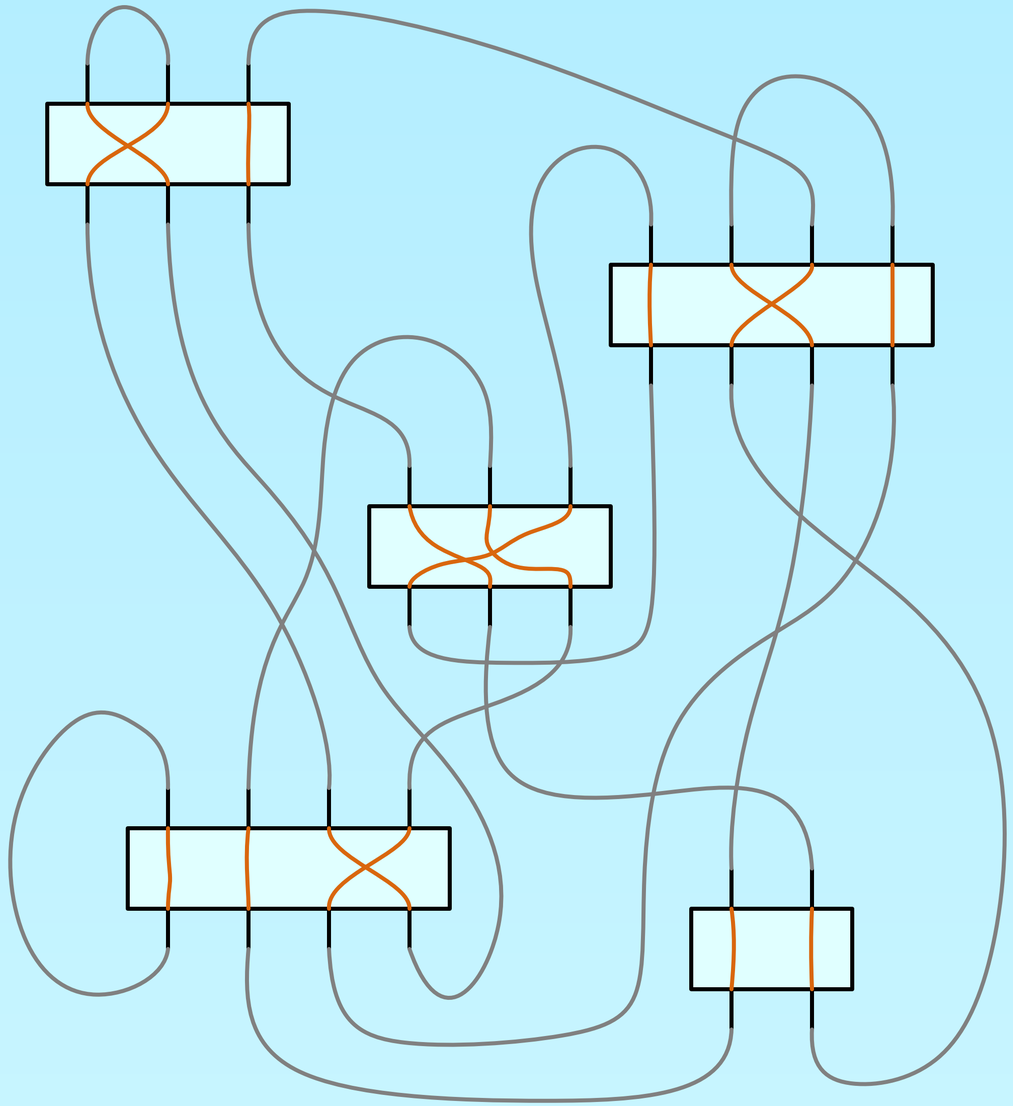

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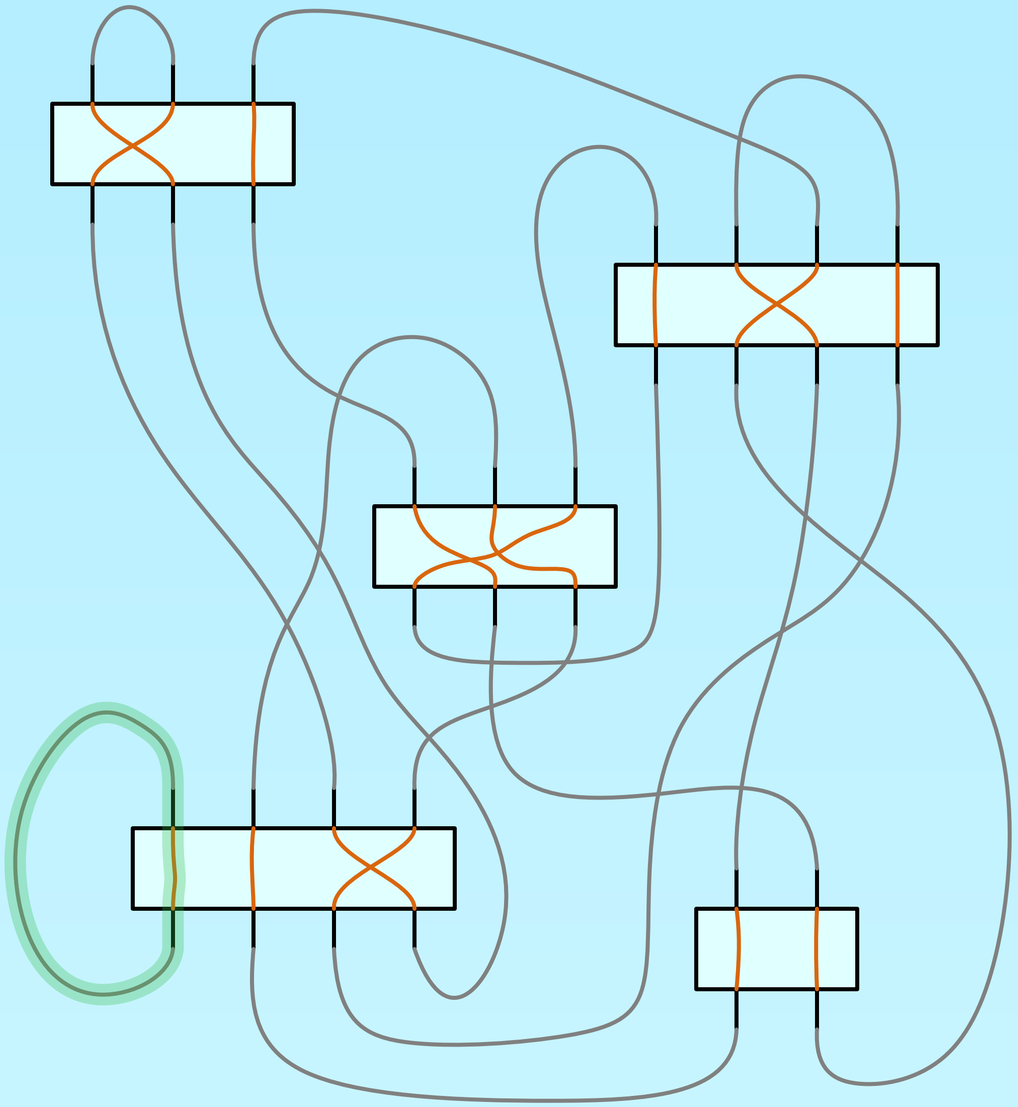

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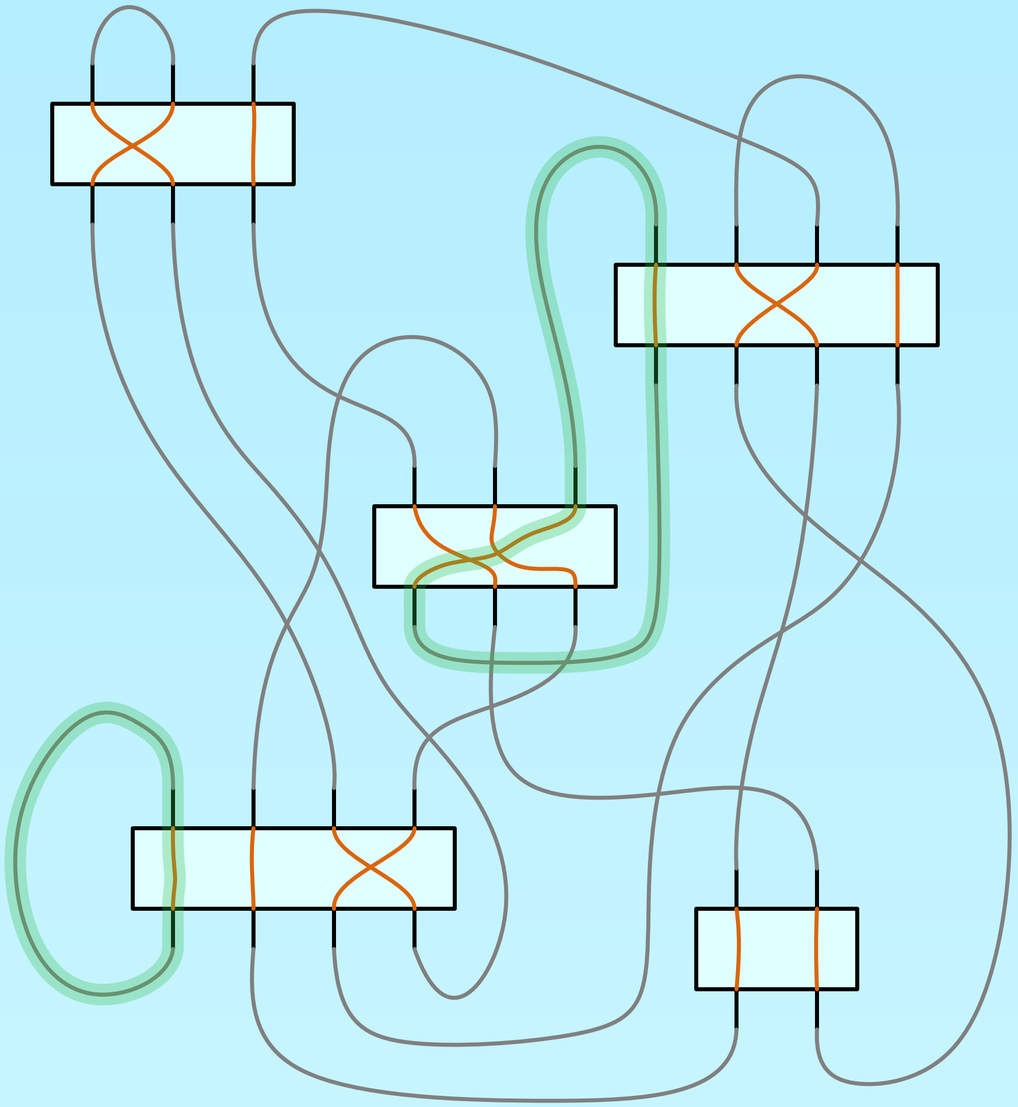

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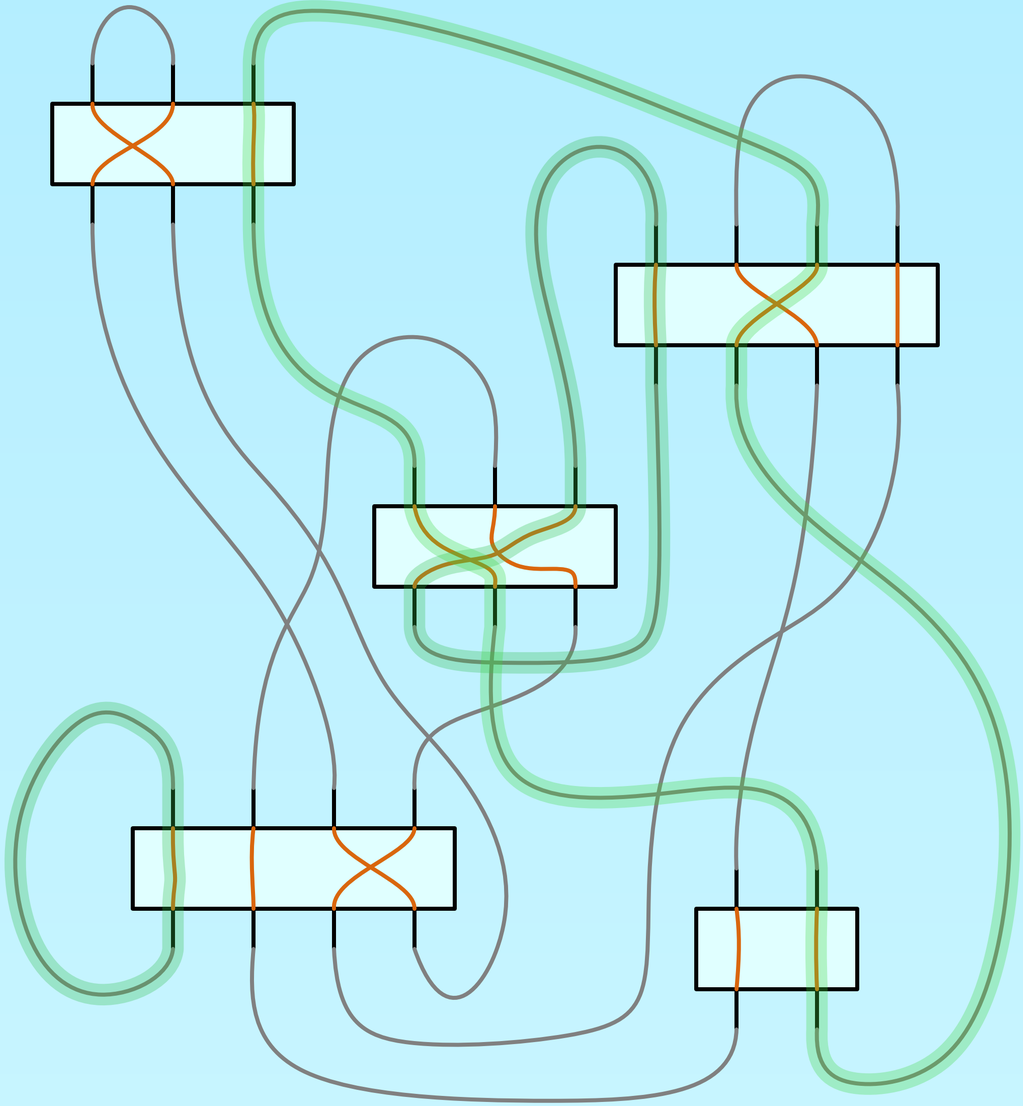

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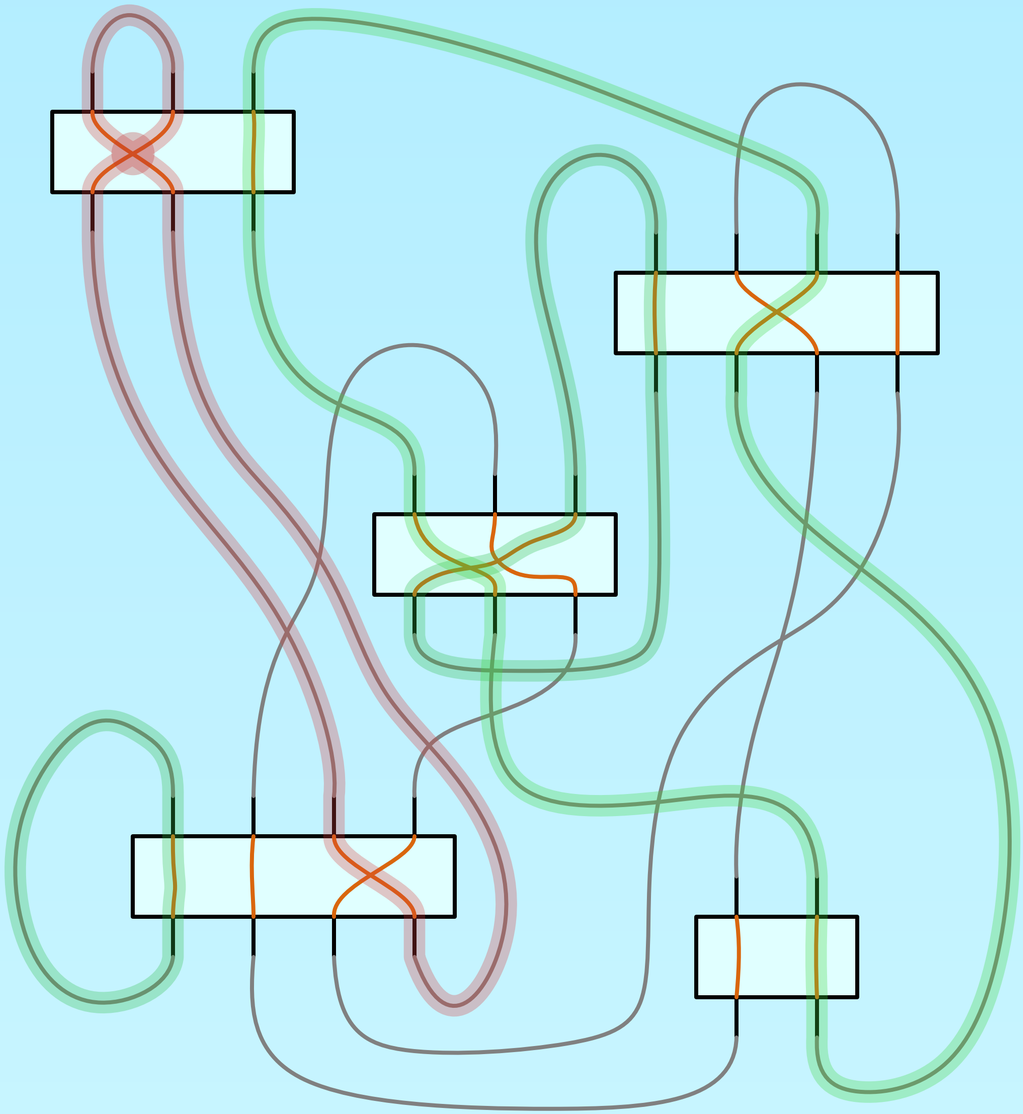

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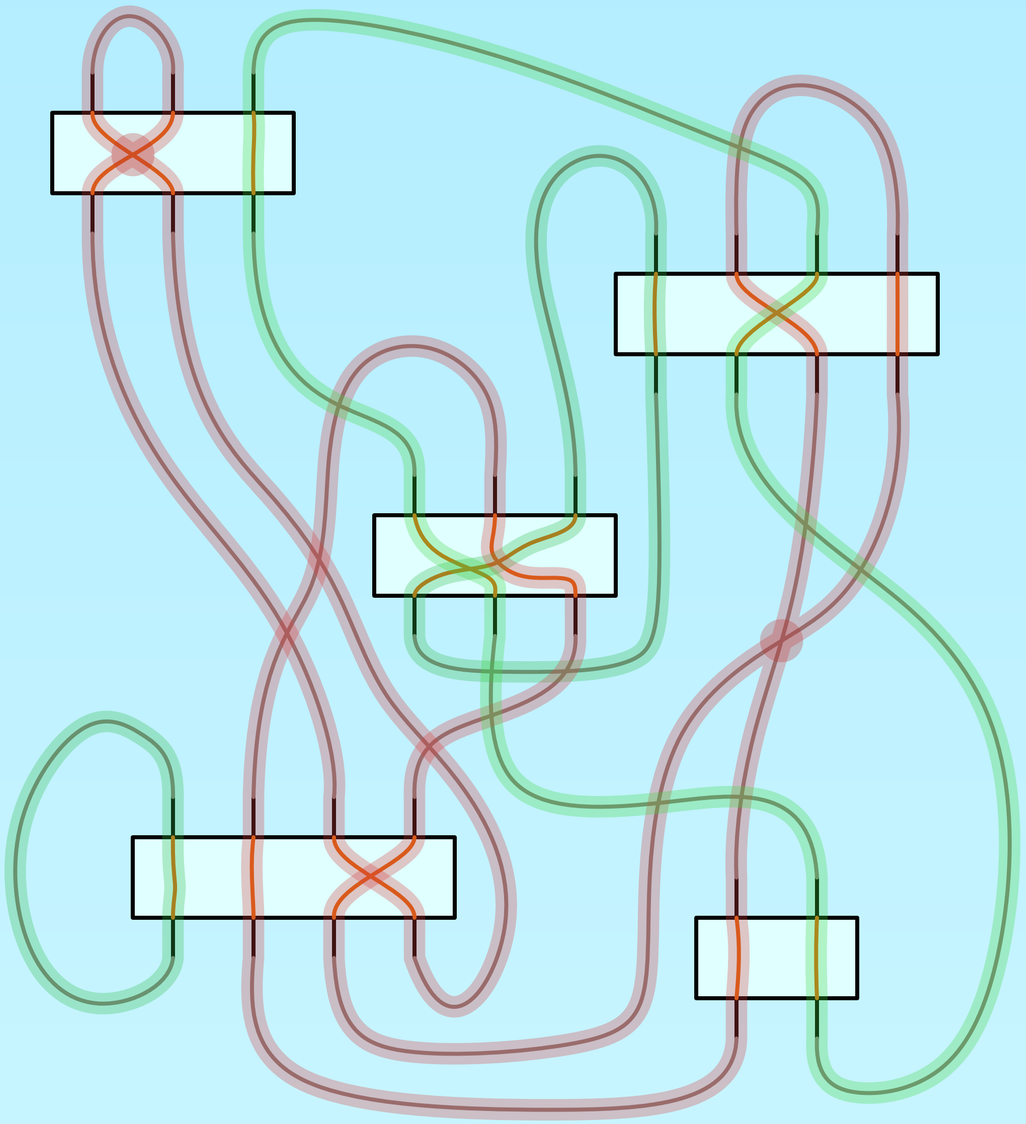

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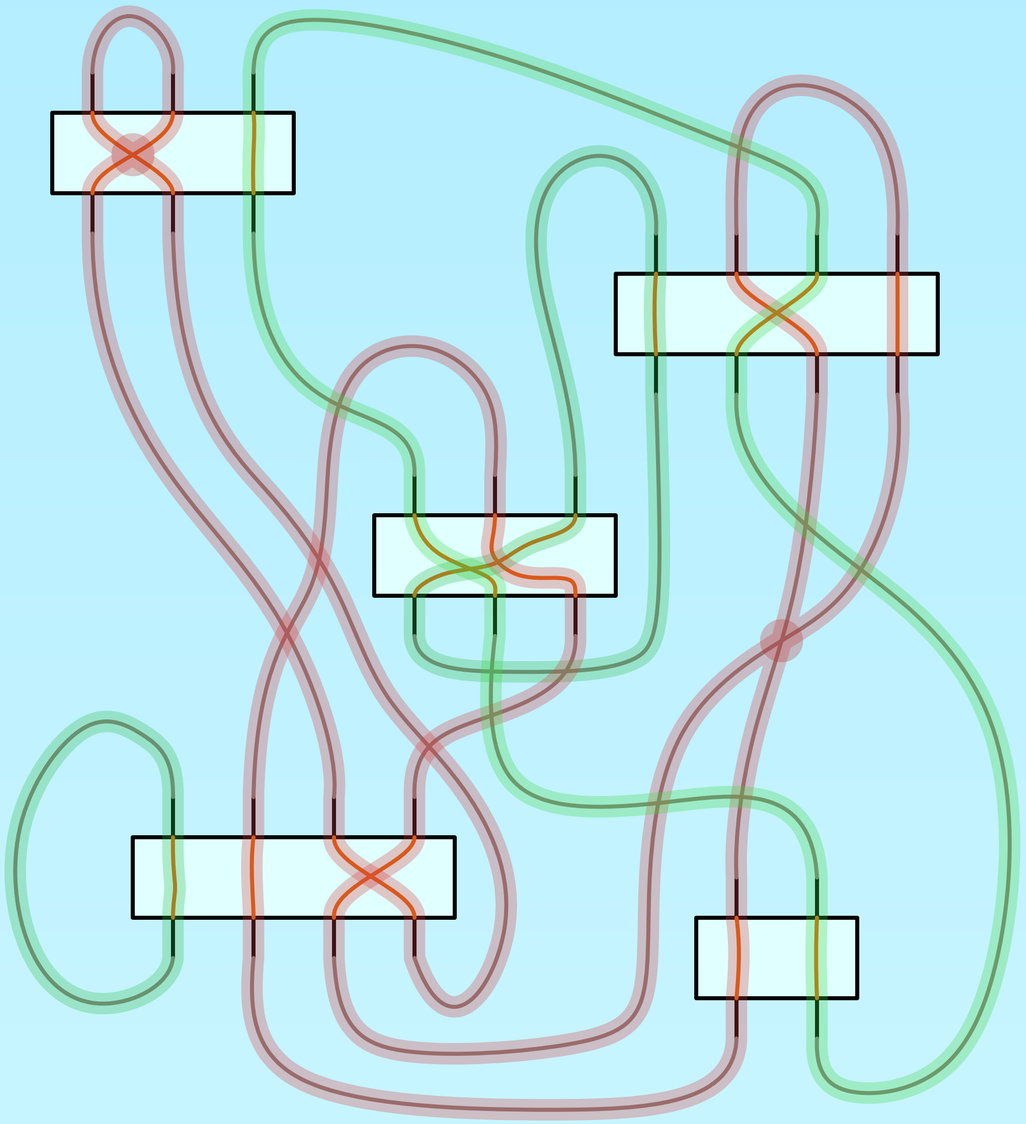

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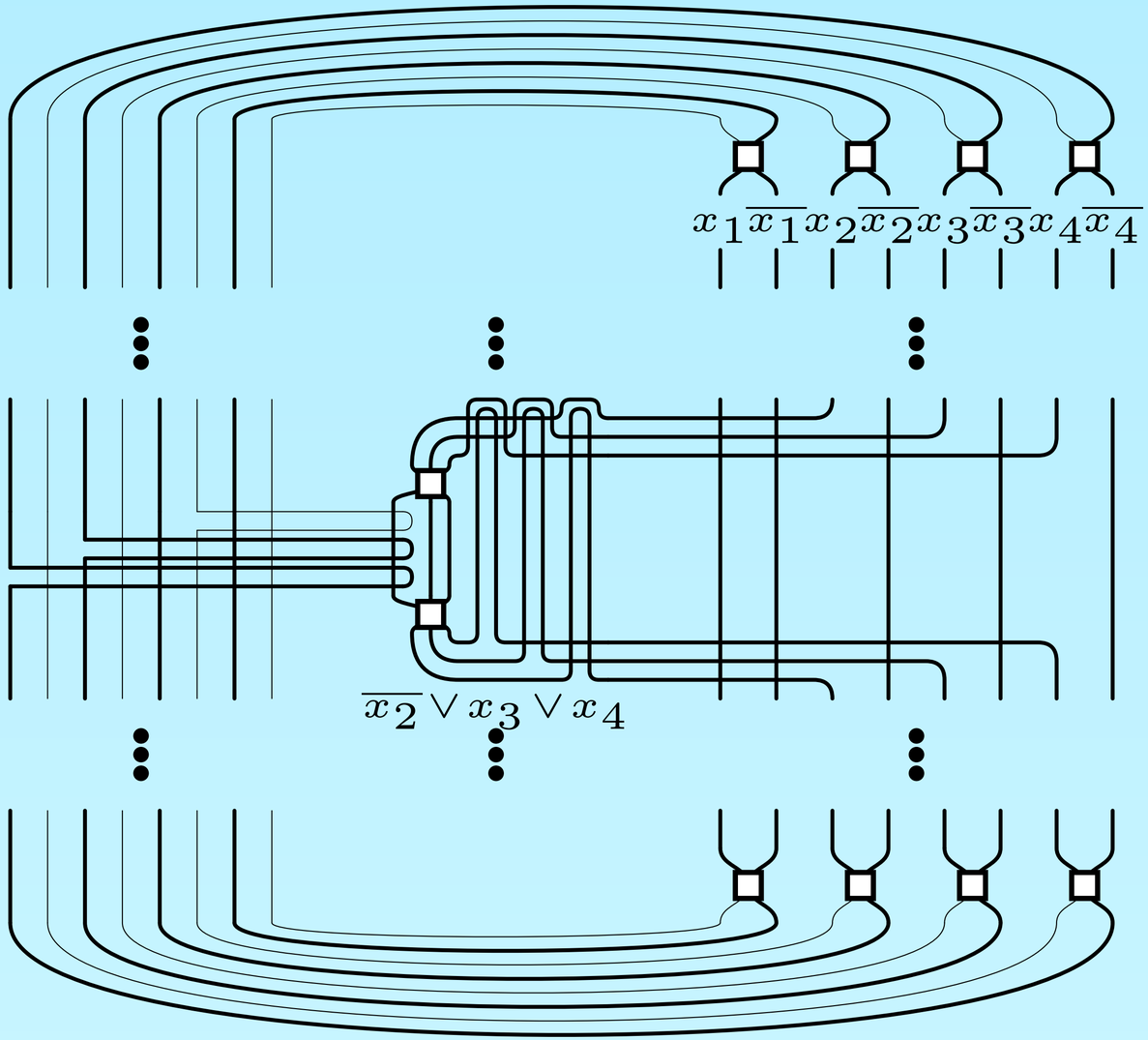

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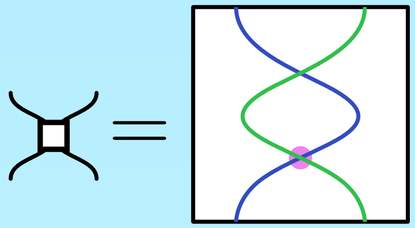

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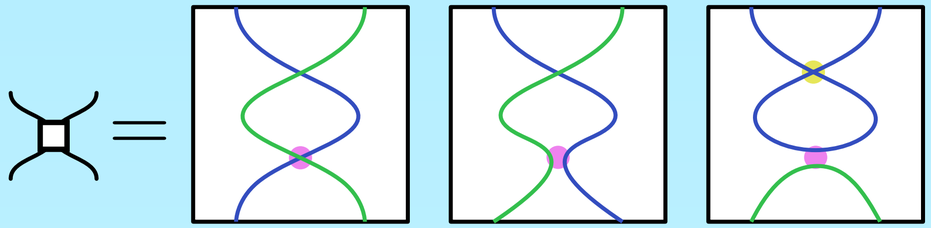

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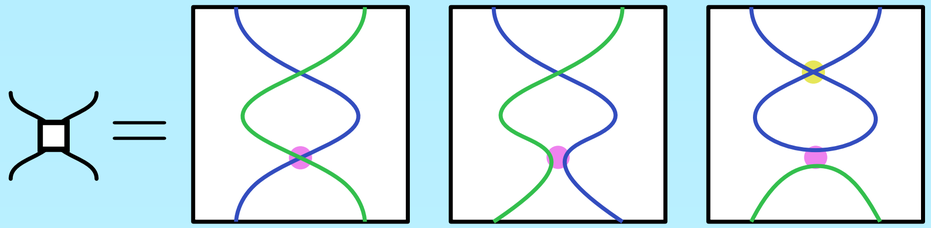

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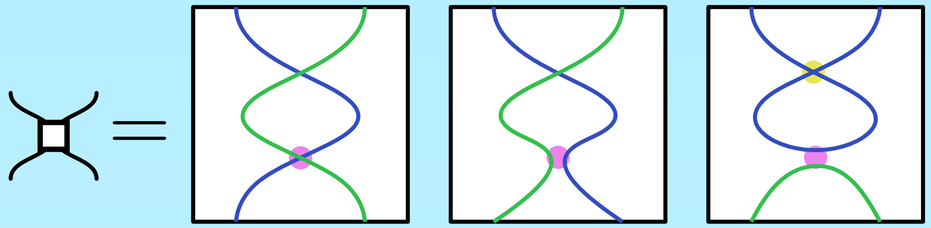

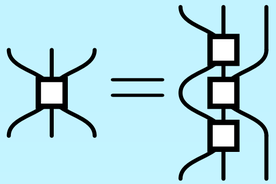

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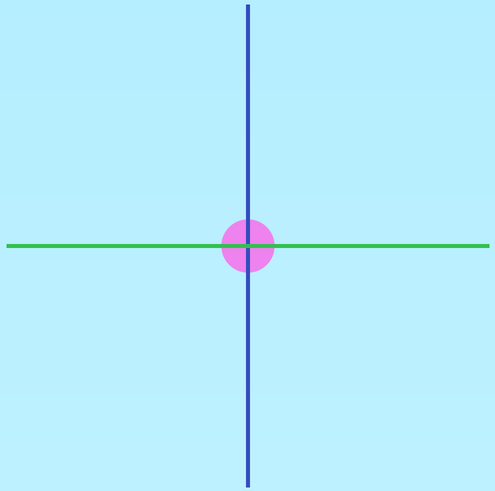

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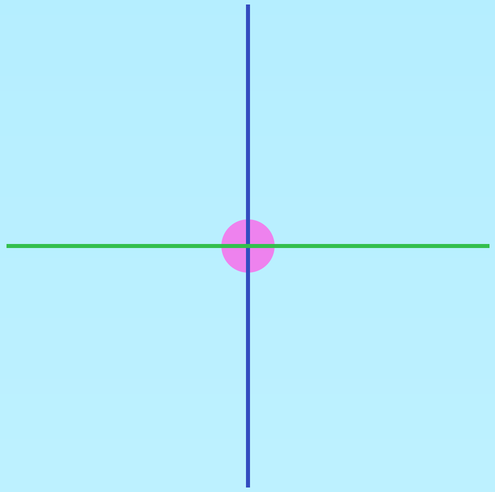

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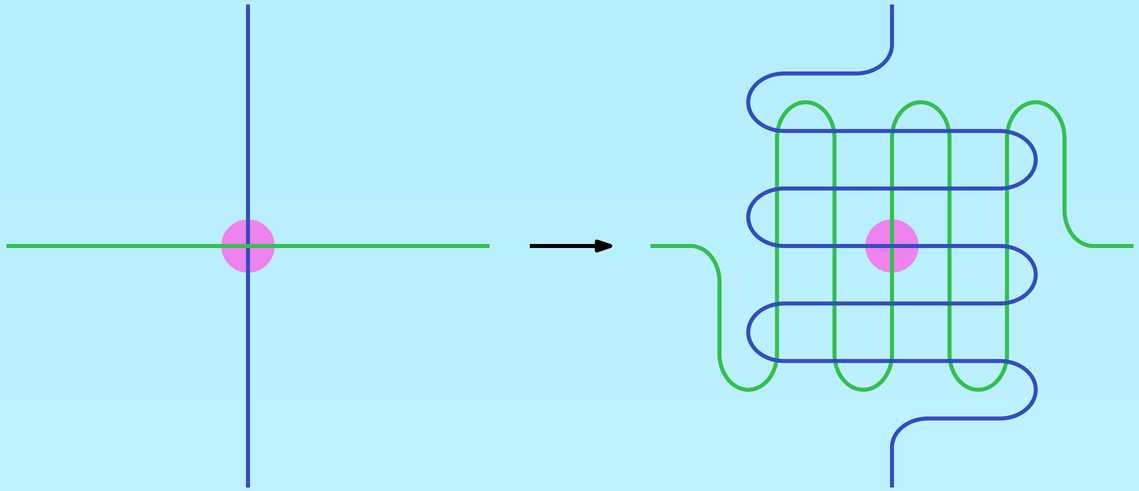

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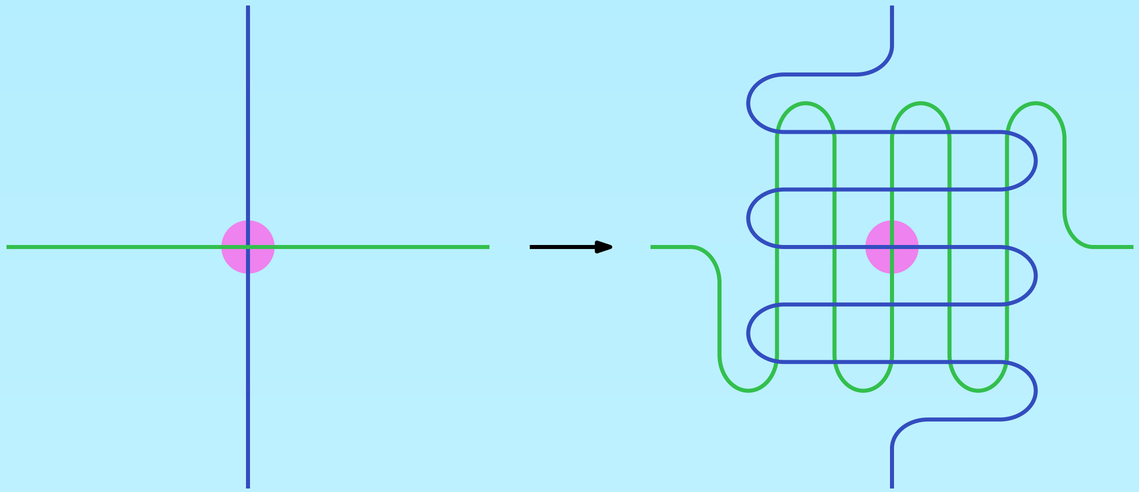

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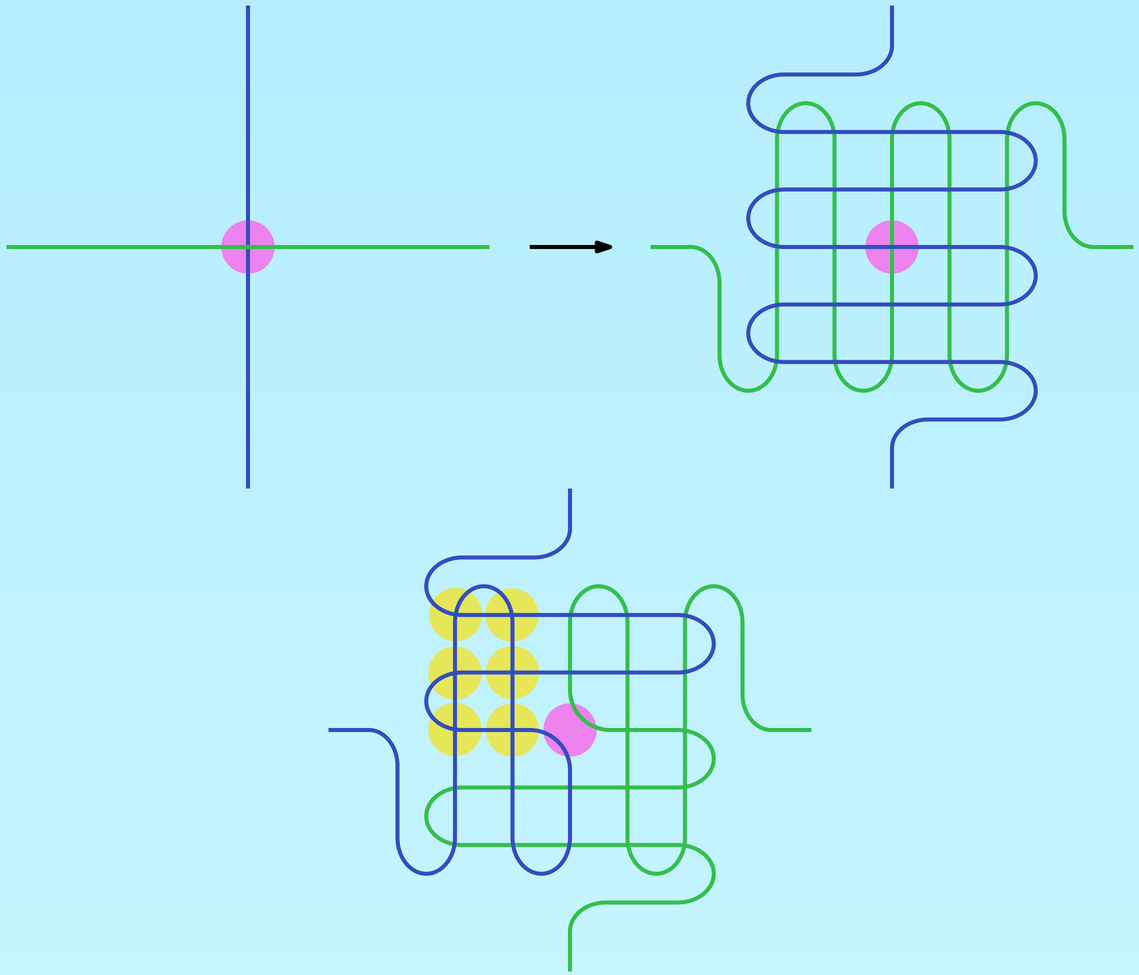

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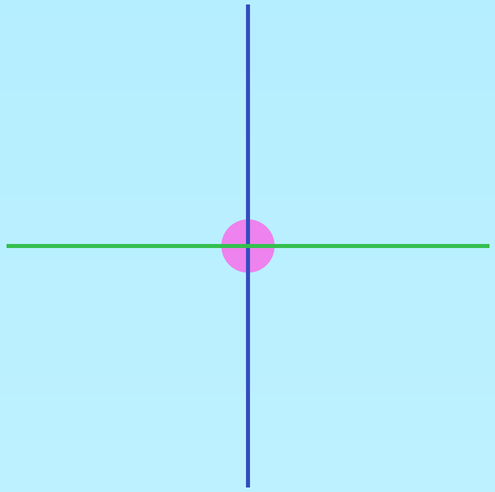

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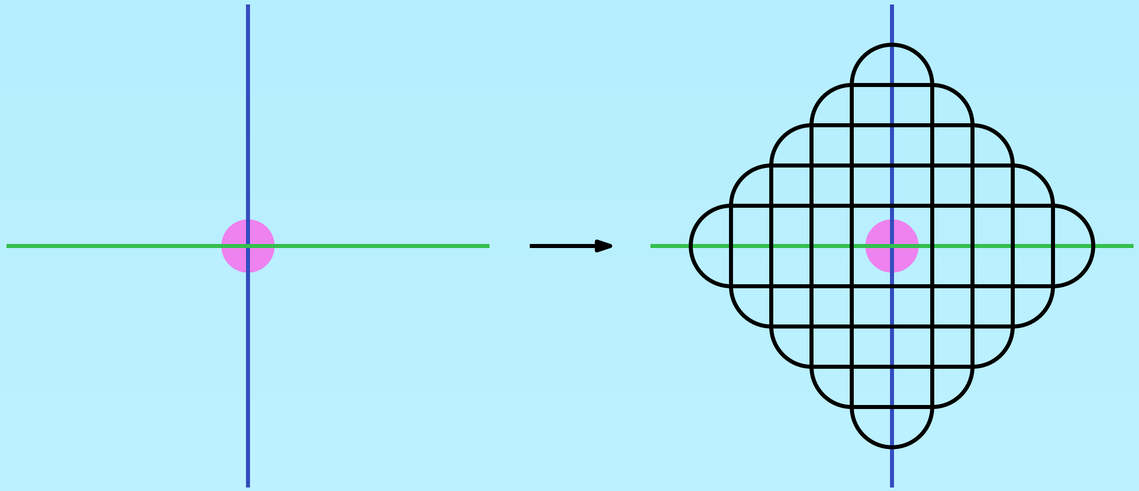

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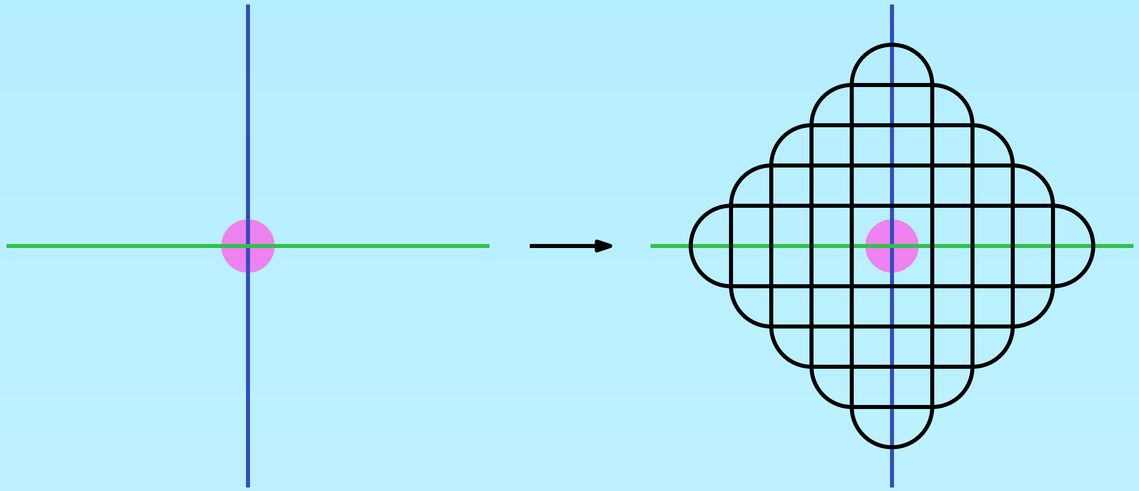

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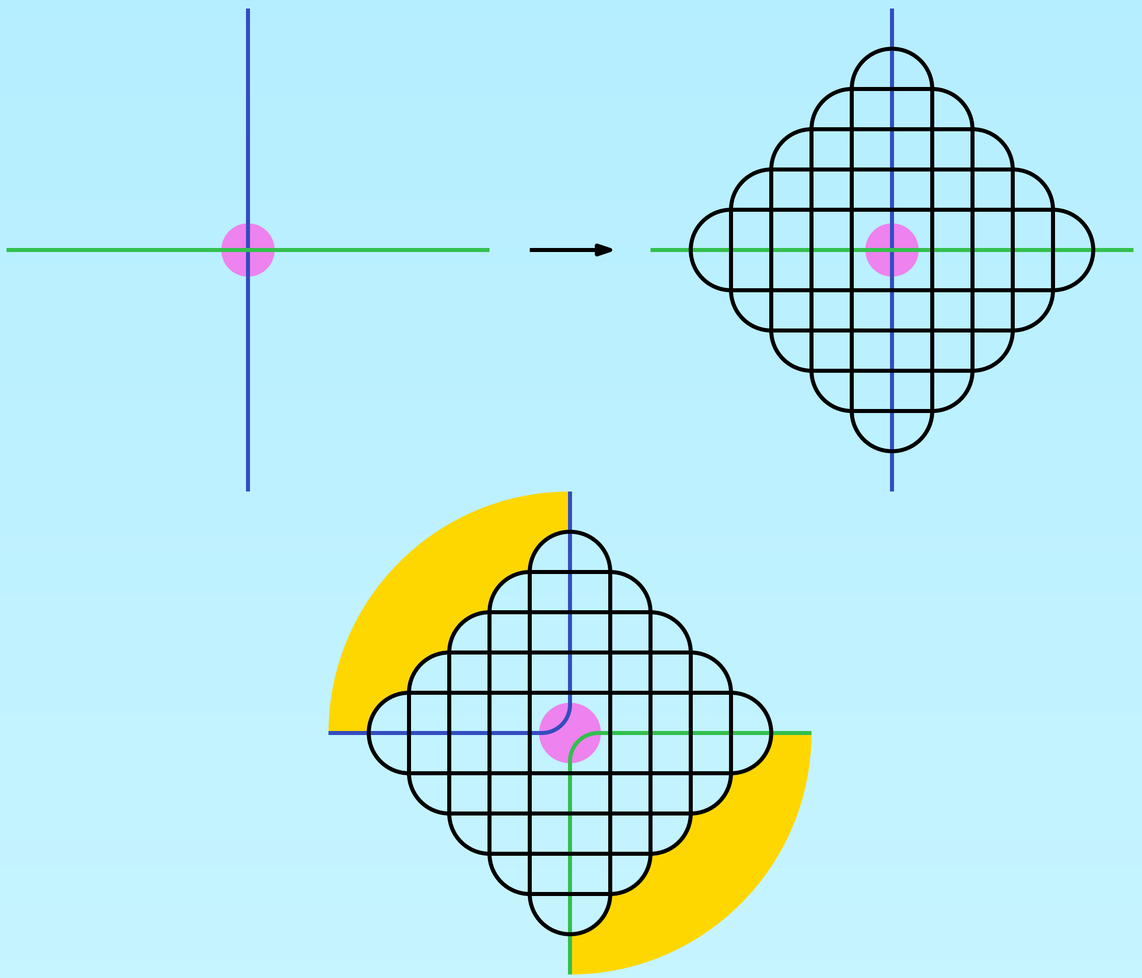

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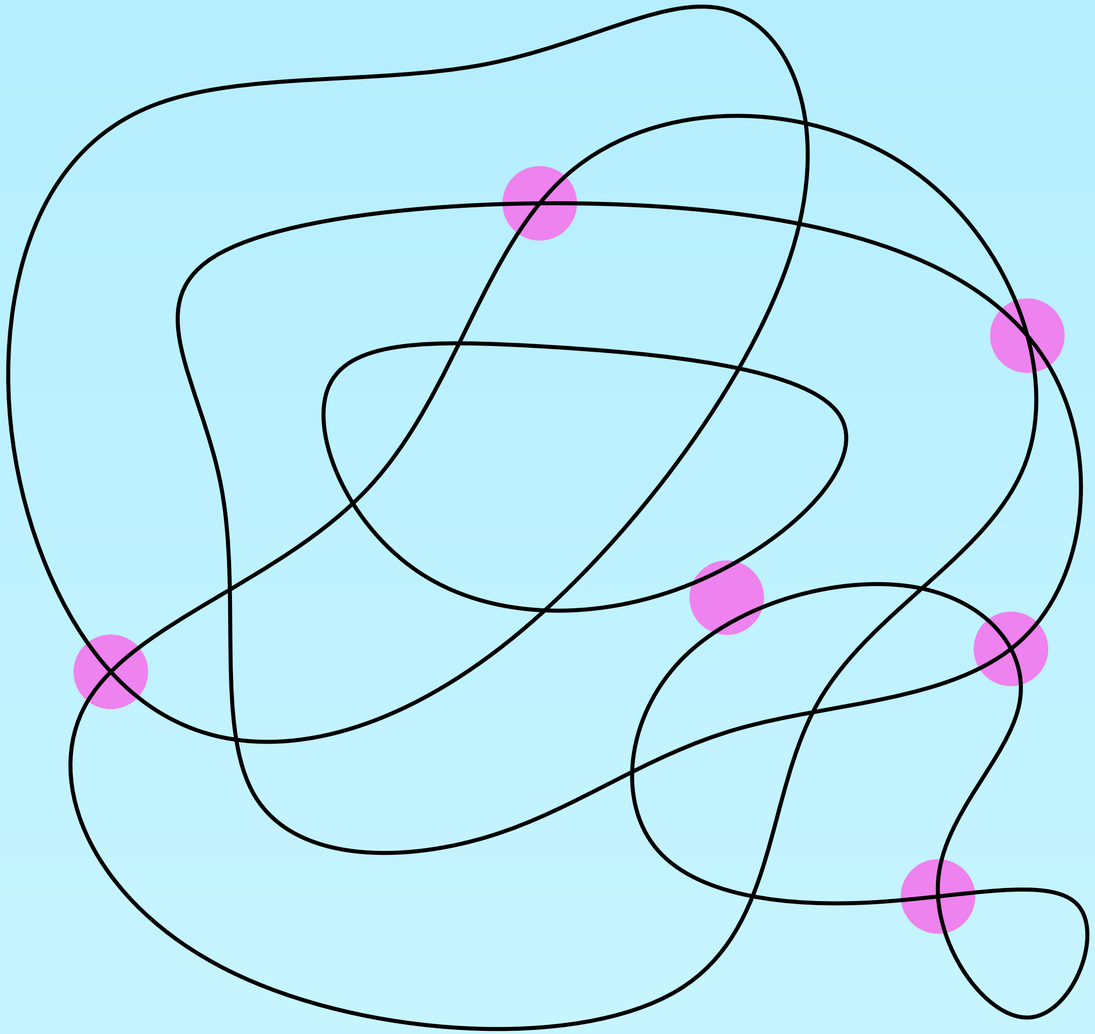

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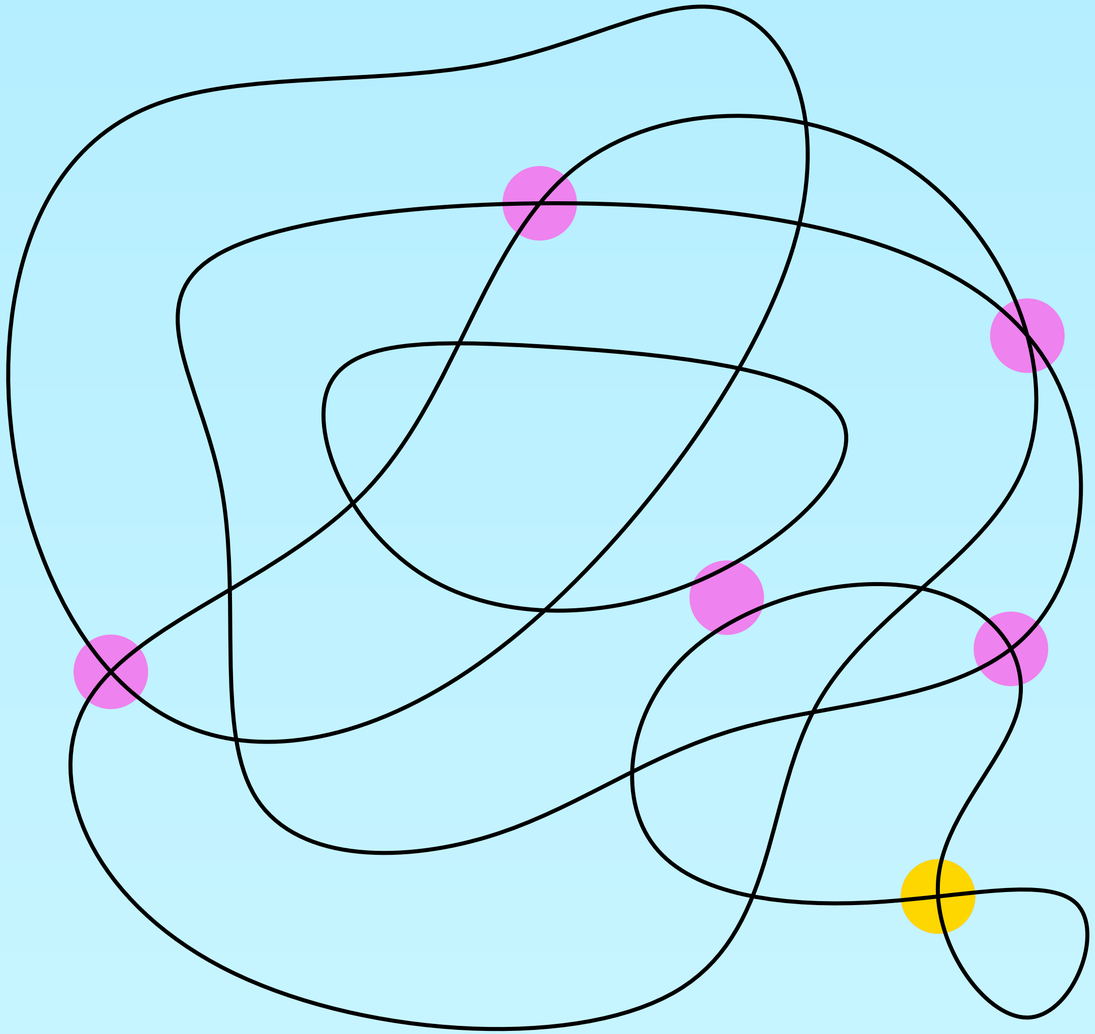

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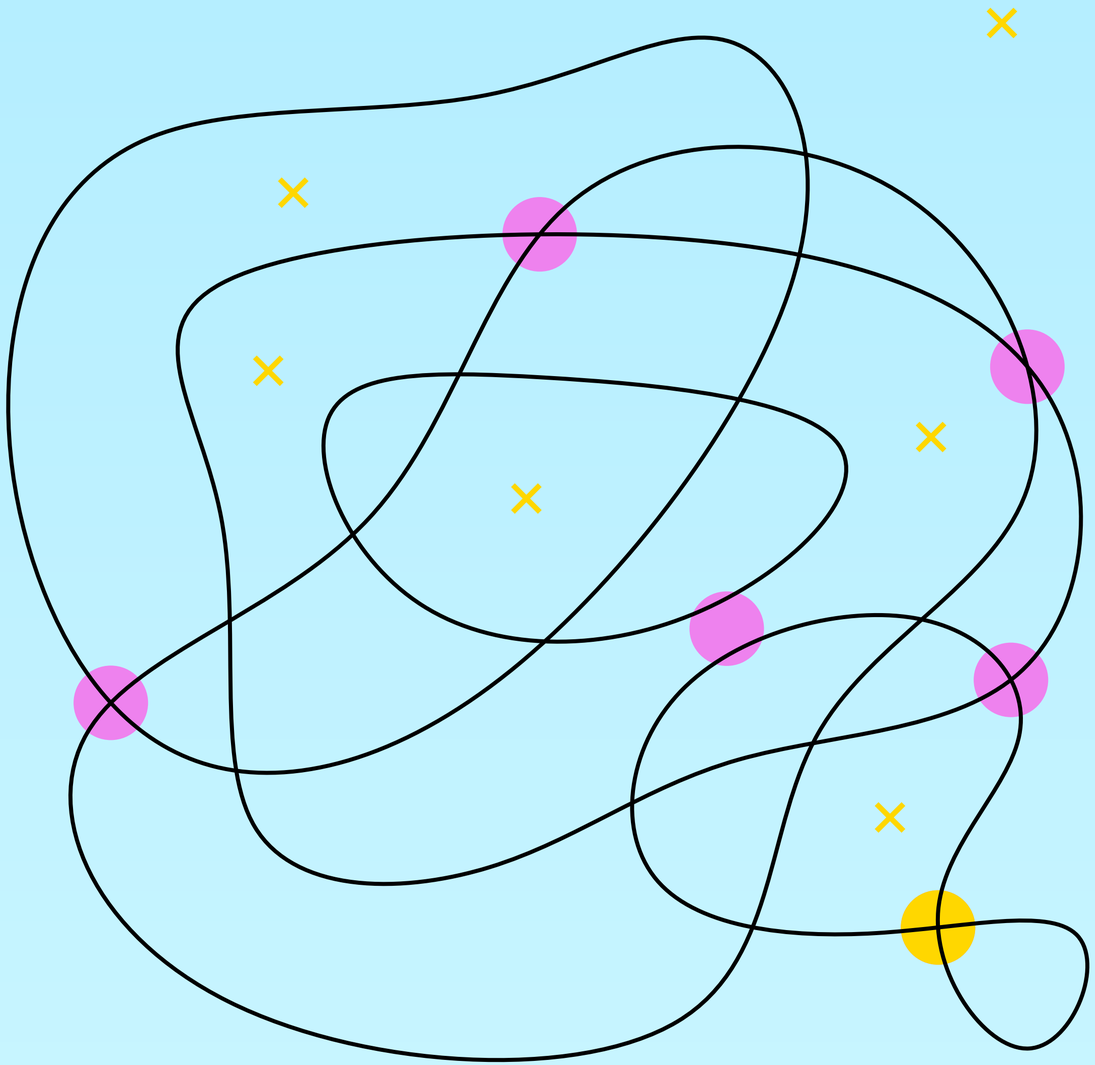

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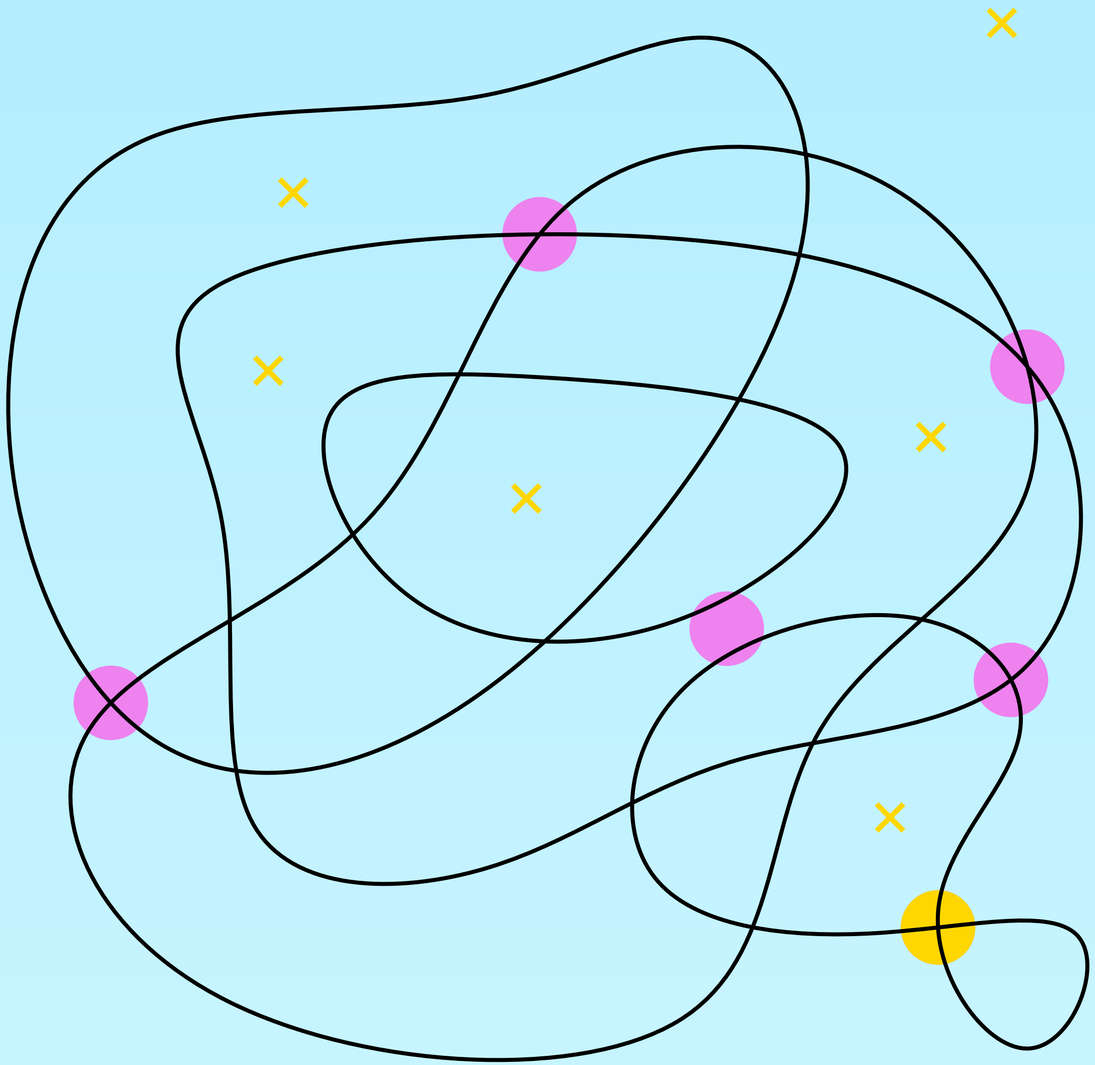

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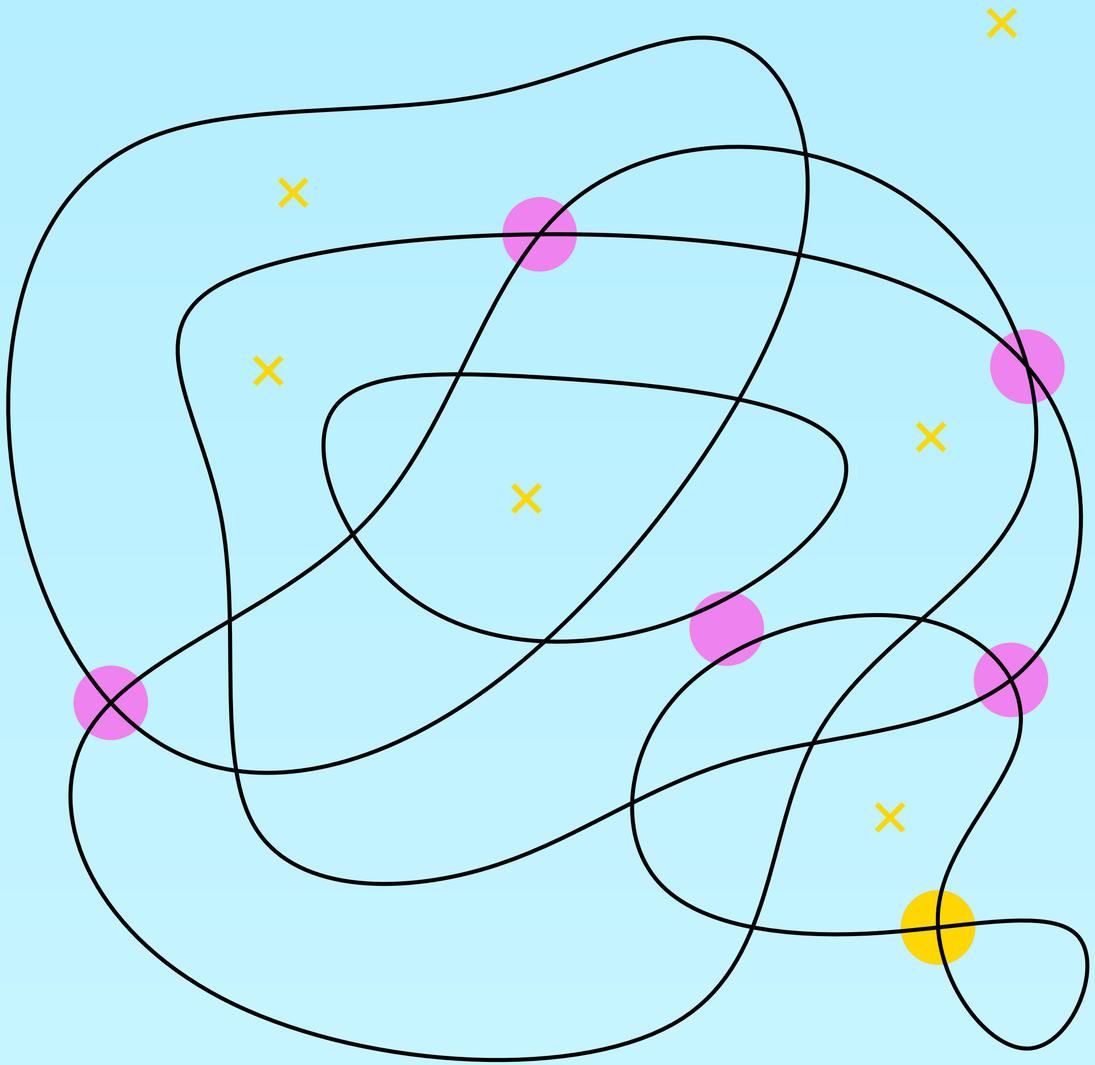

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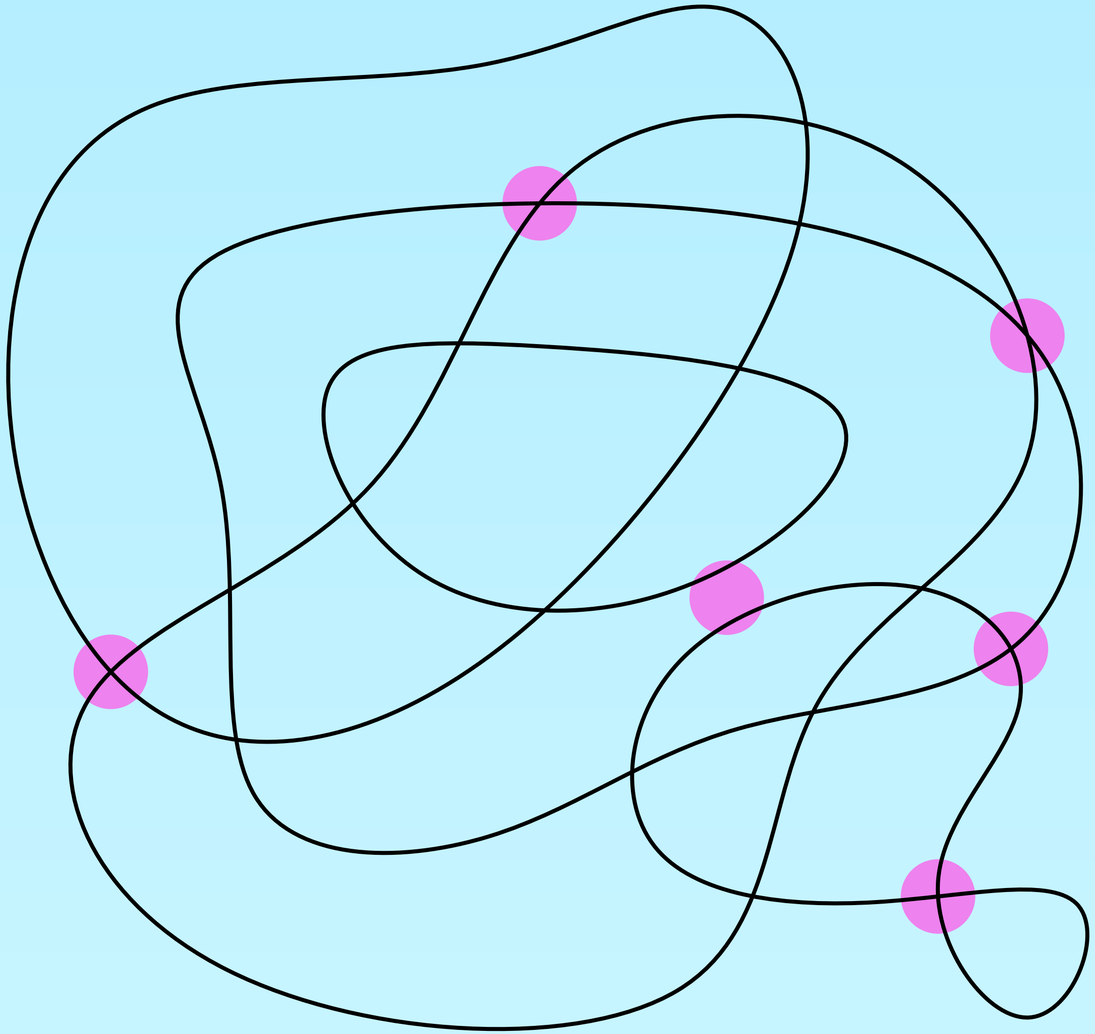

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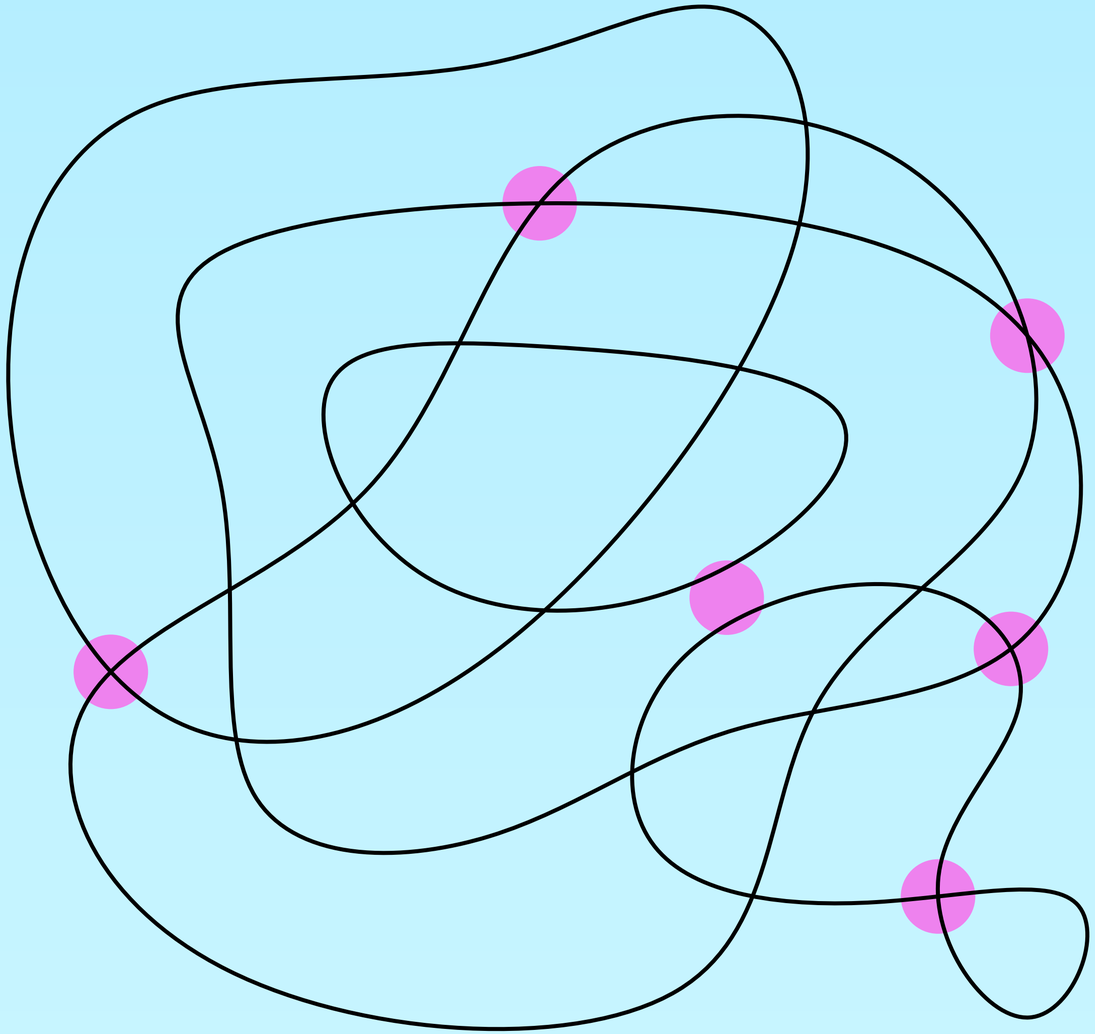

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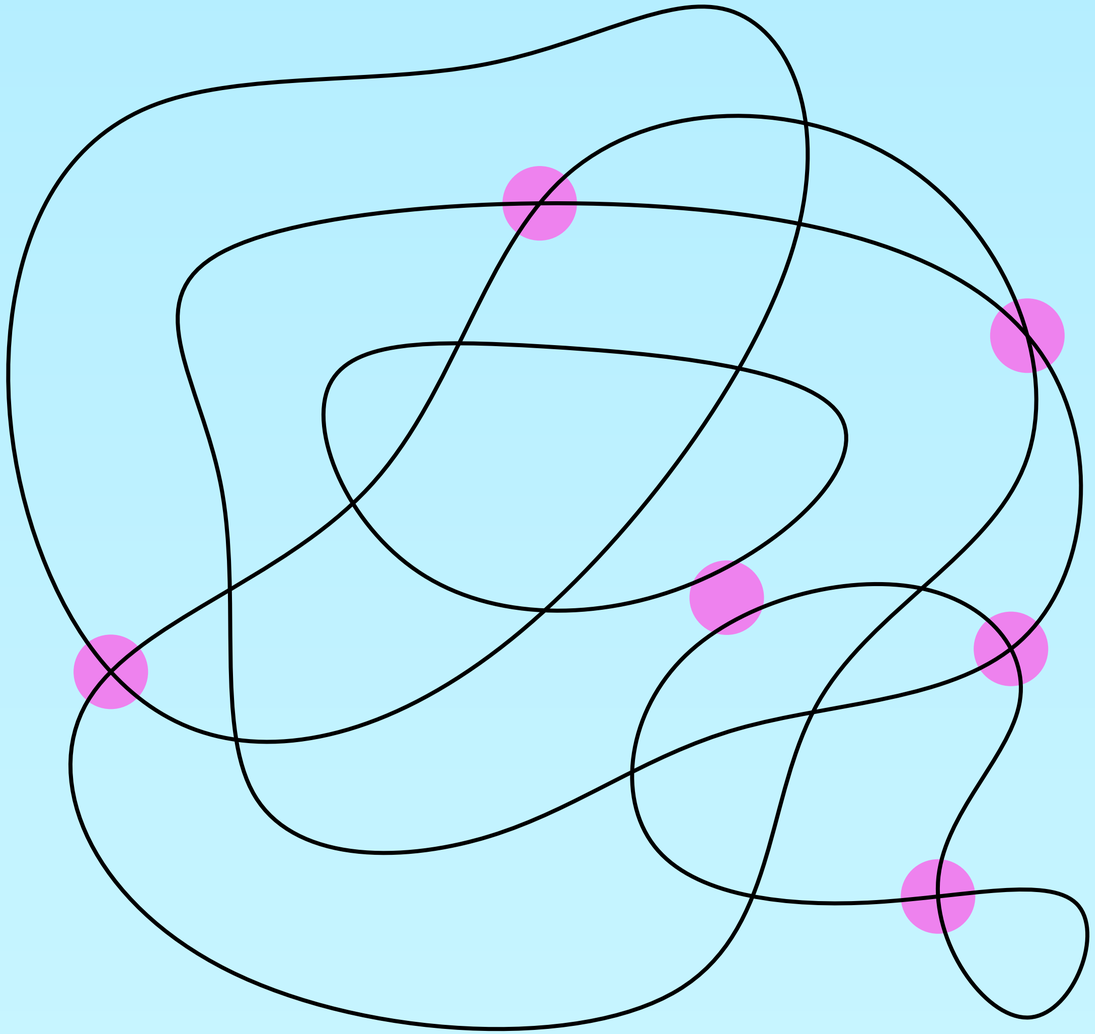

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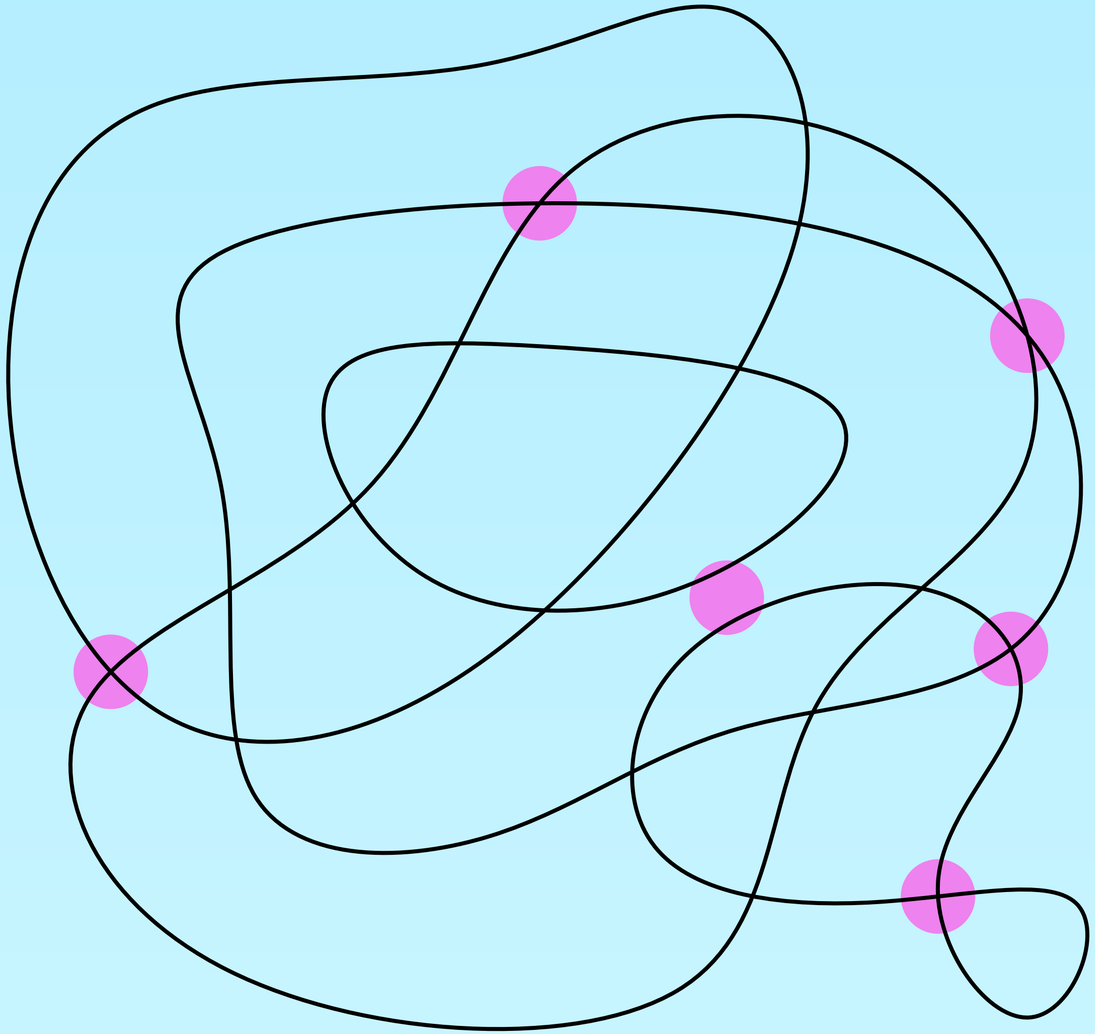

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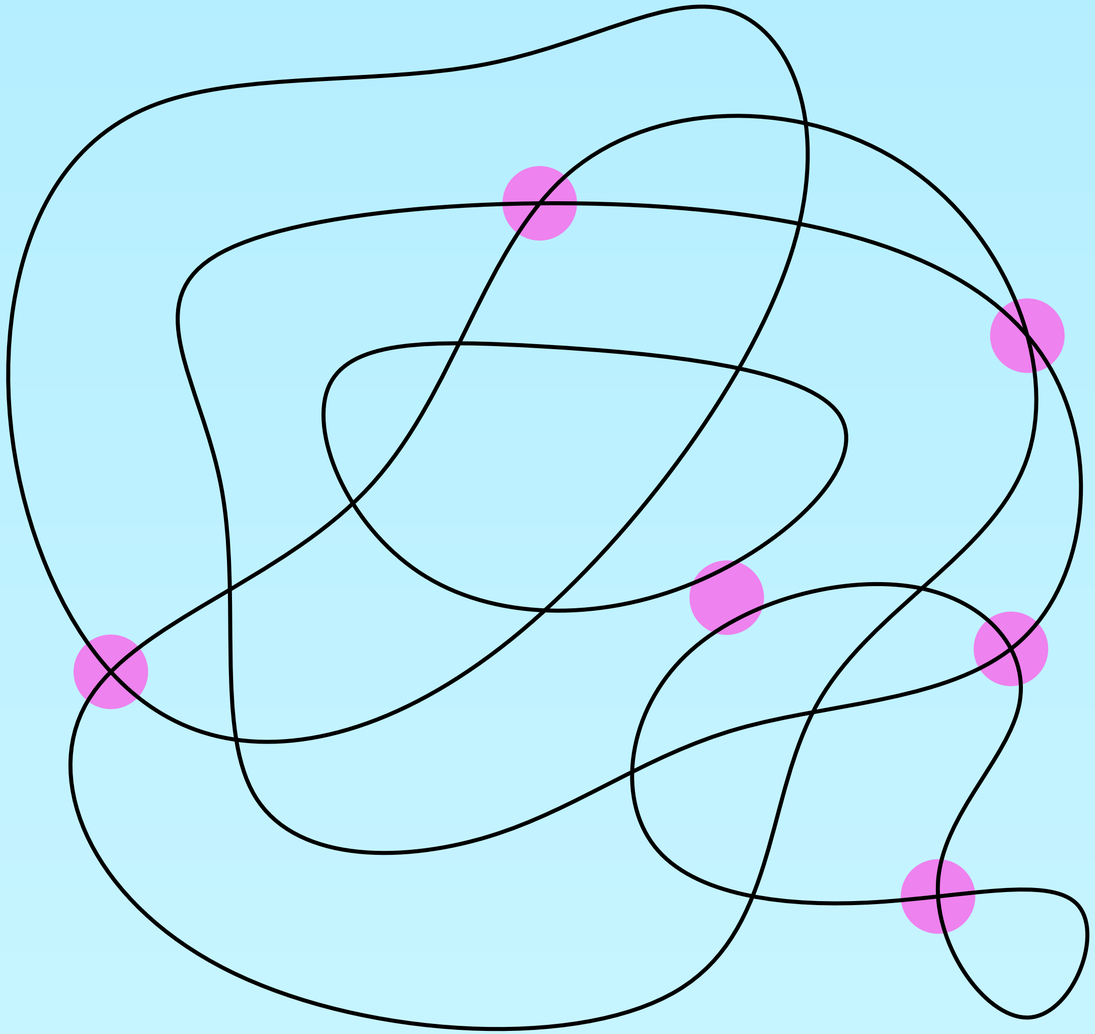

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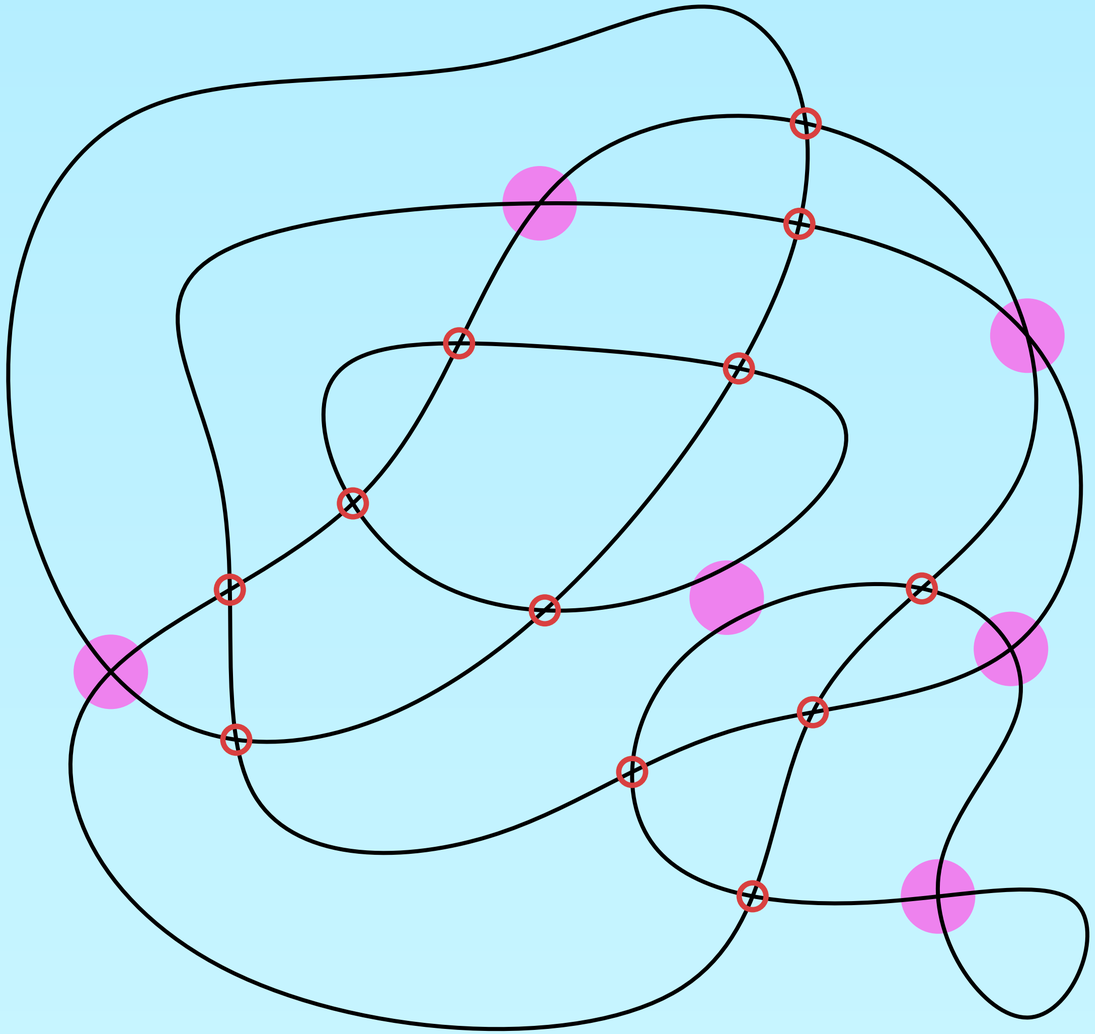

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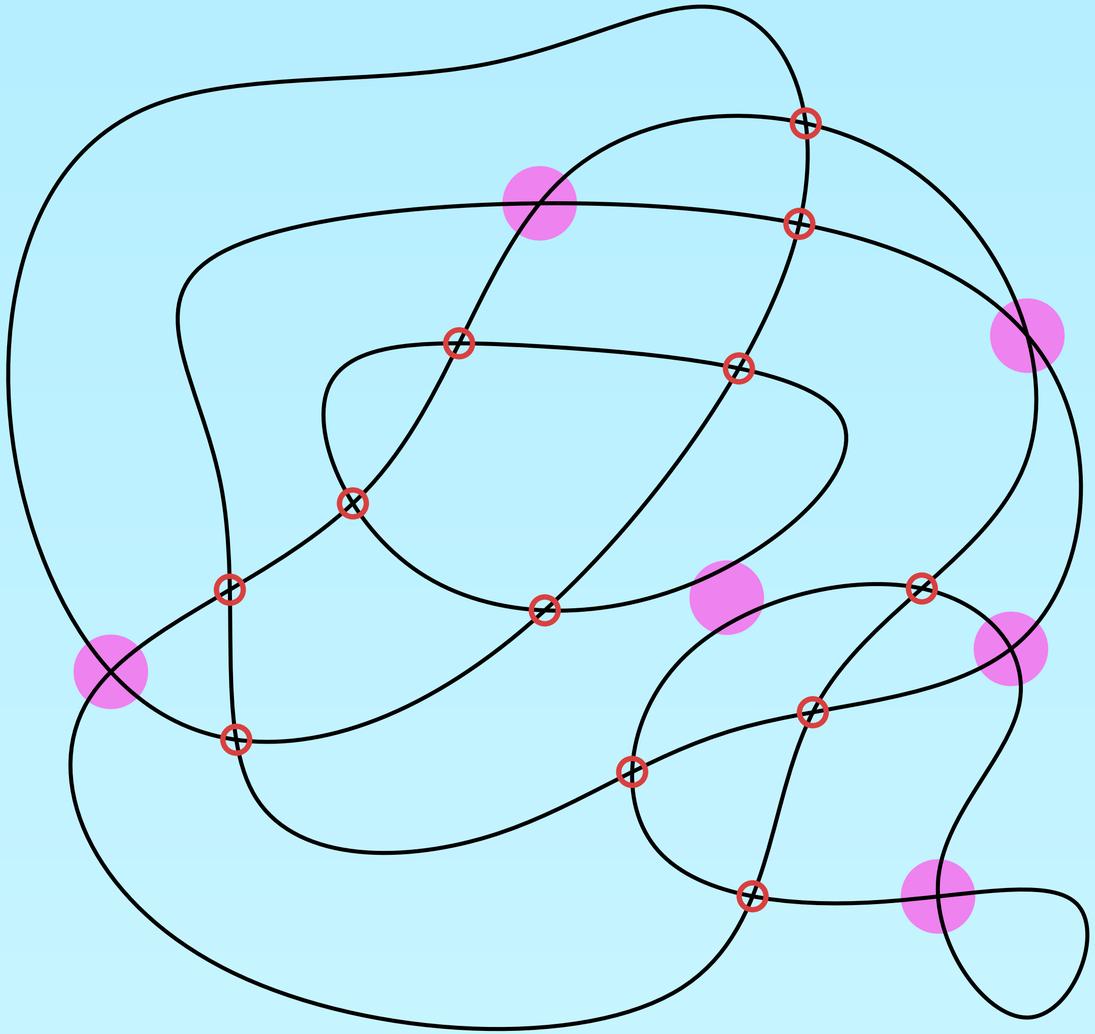

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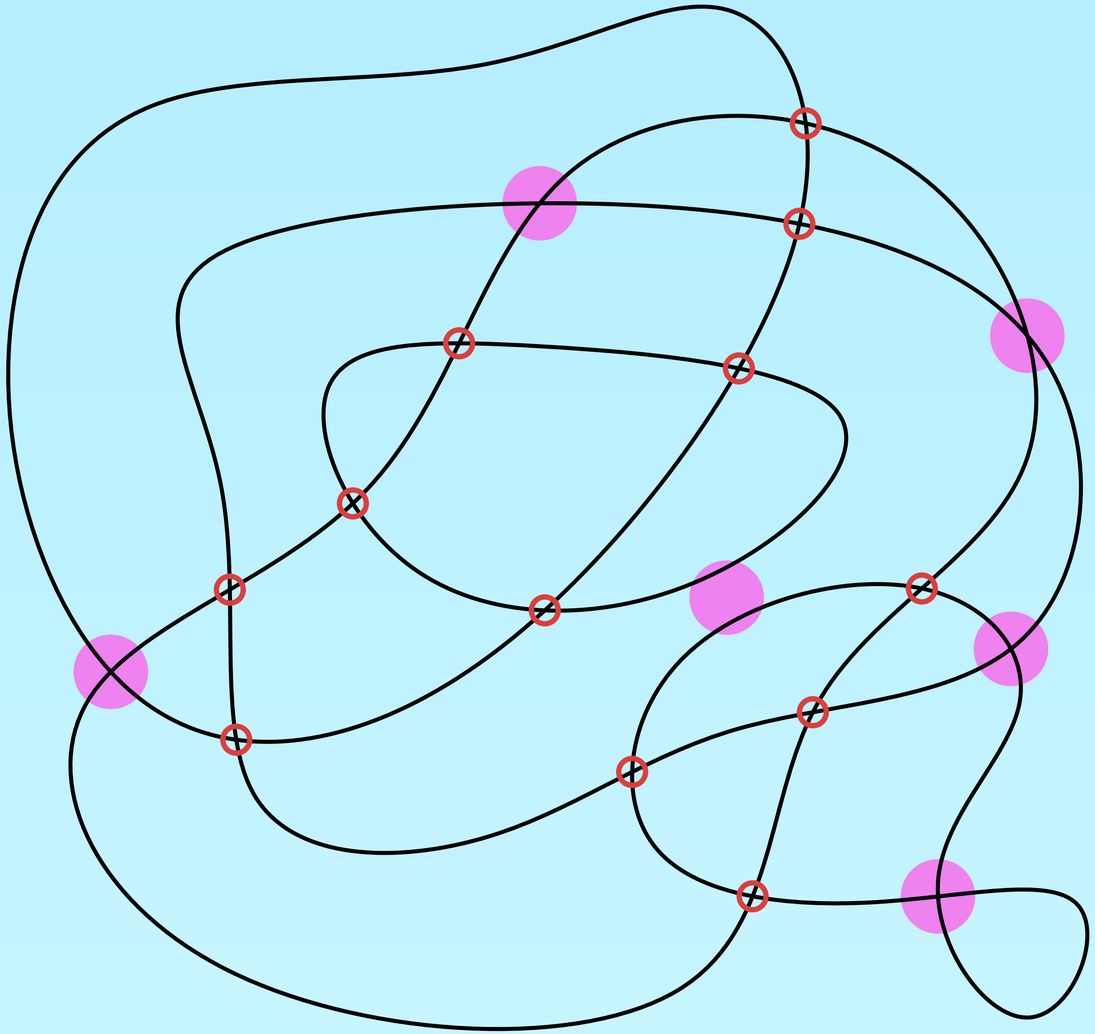

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Remember, when there are 0 non-switchable switches, the problem is trivial.

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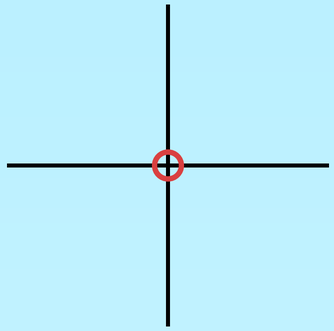

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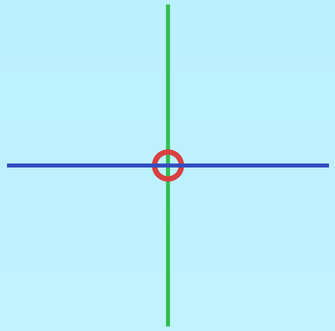

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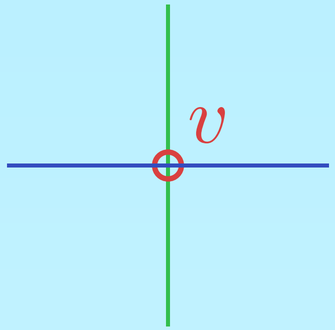

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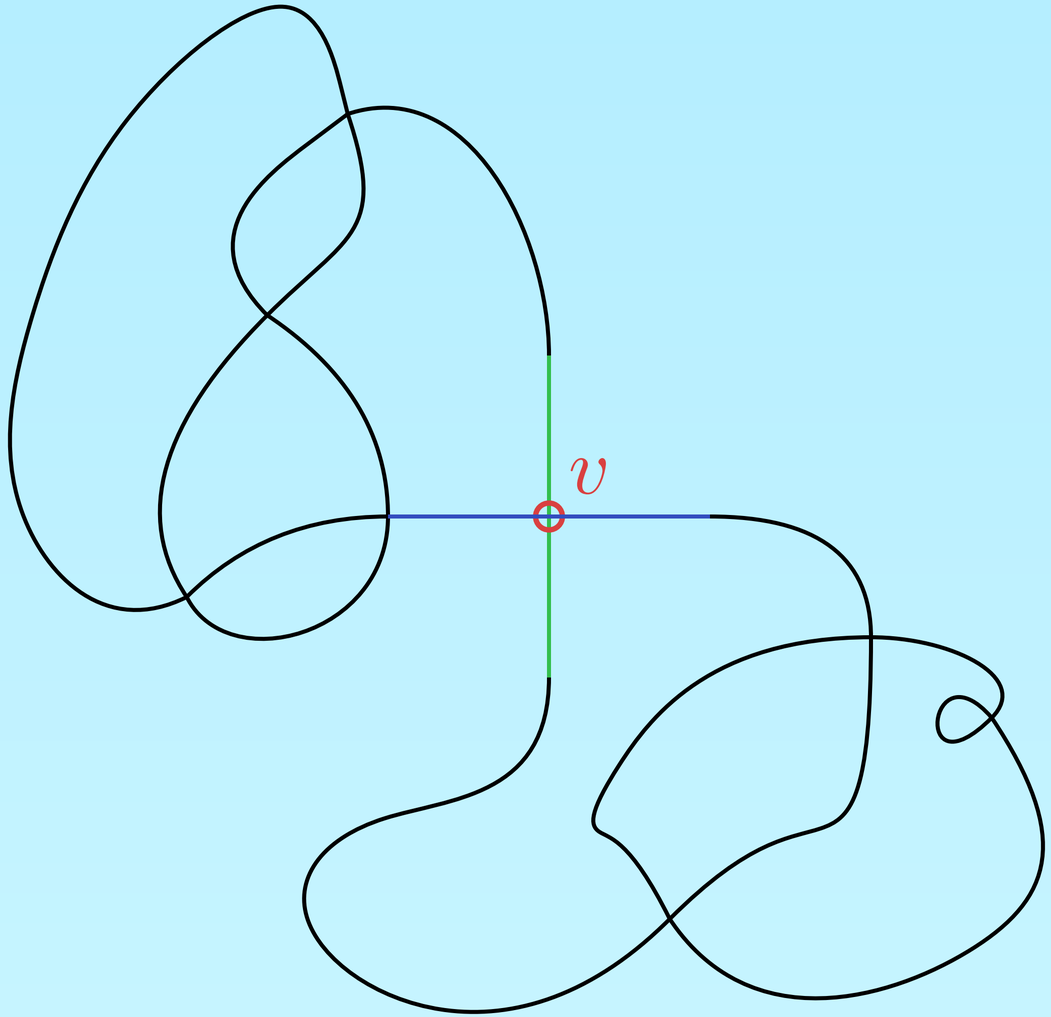

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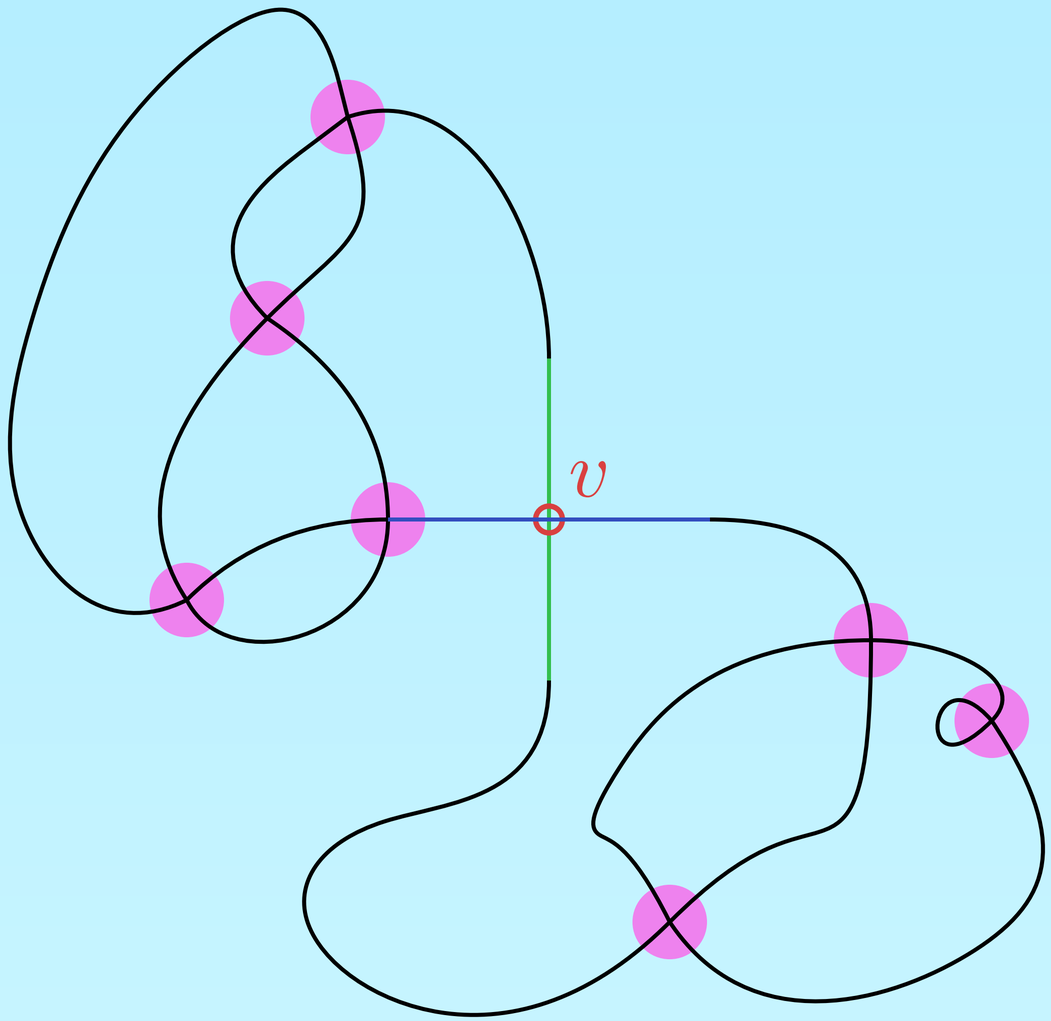

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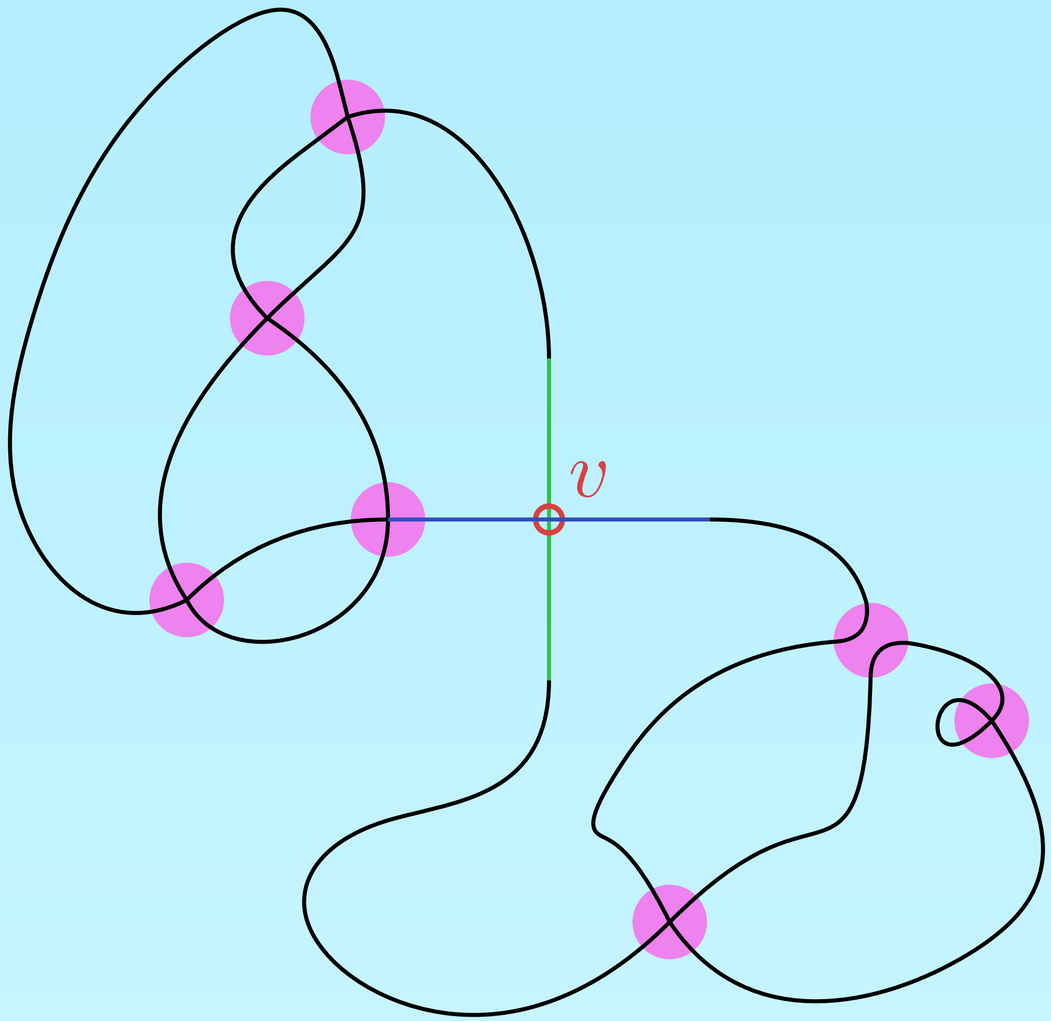

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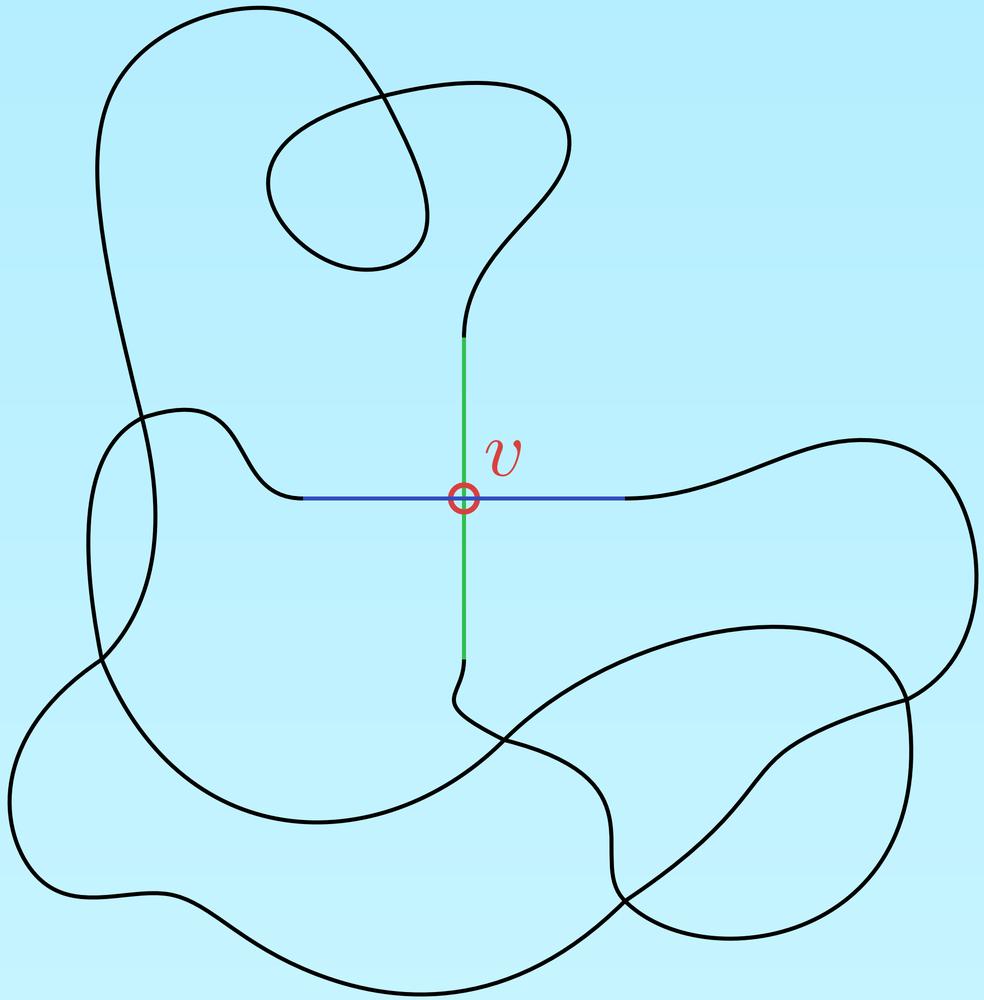

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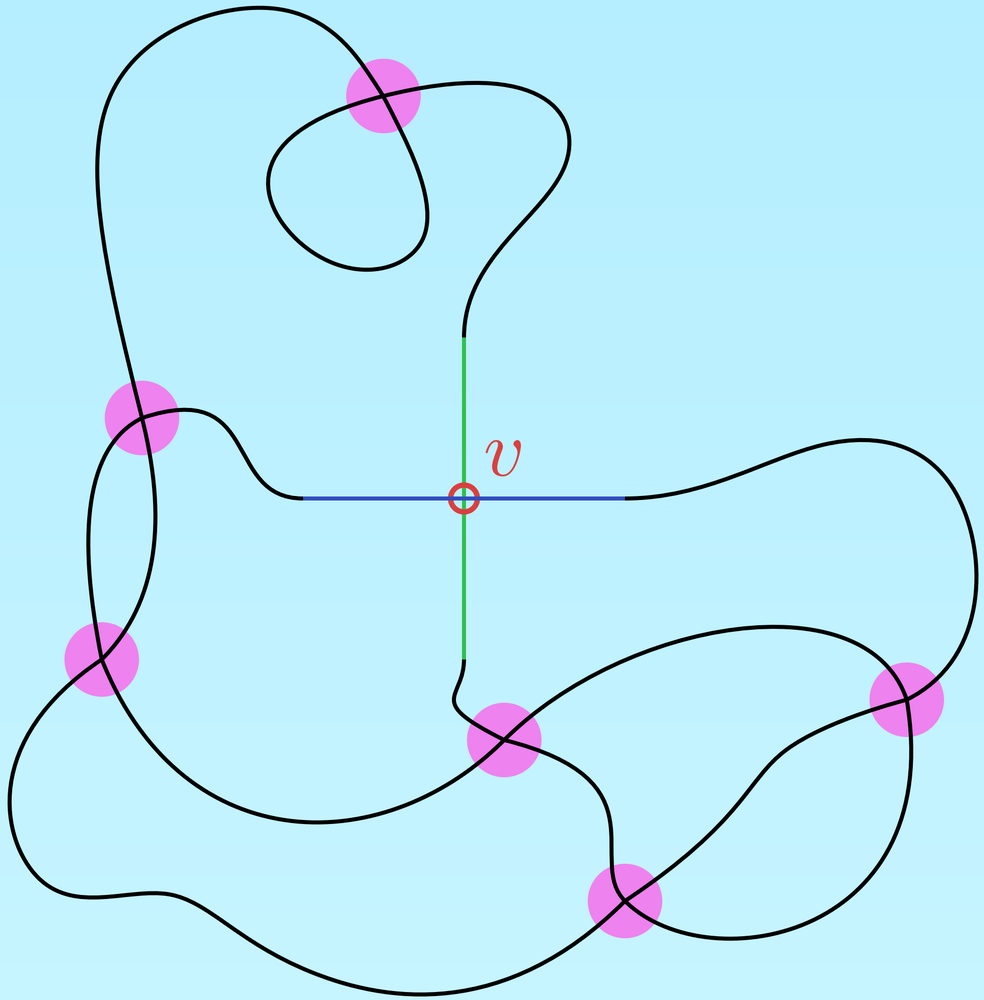

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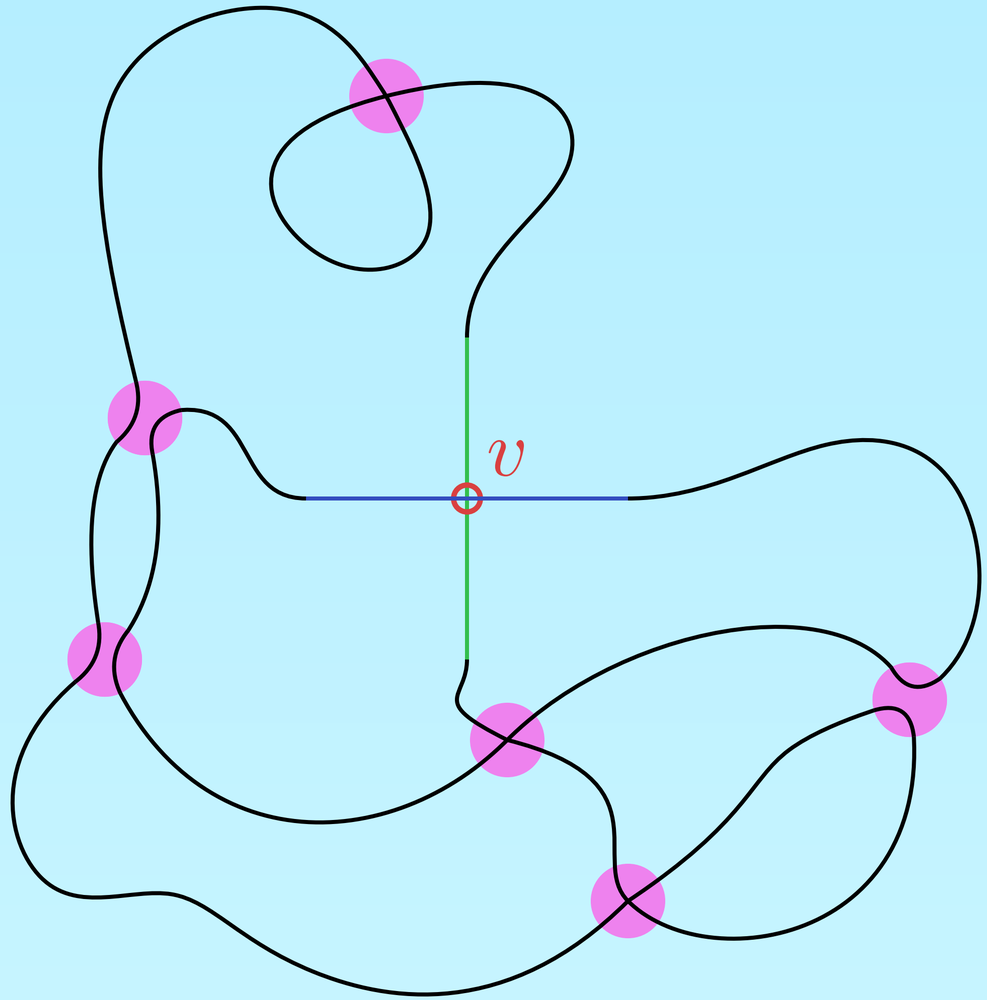

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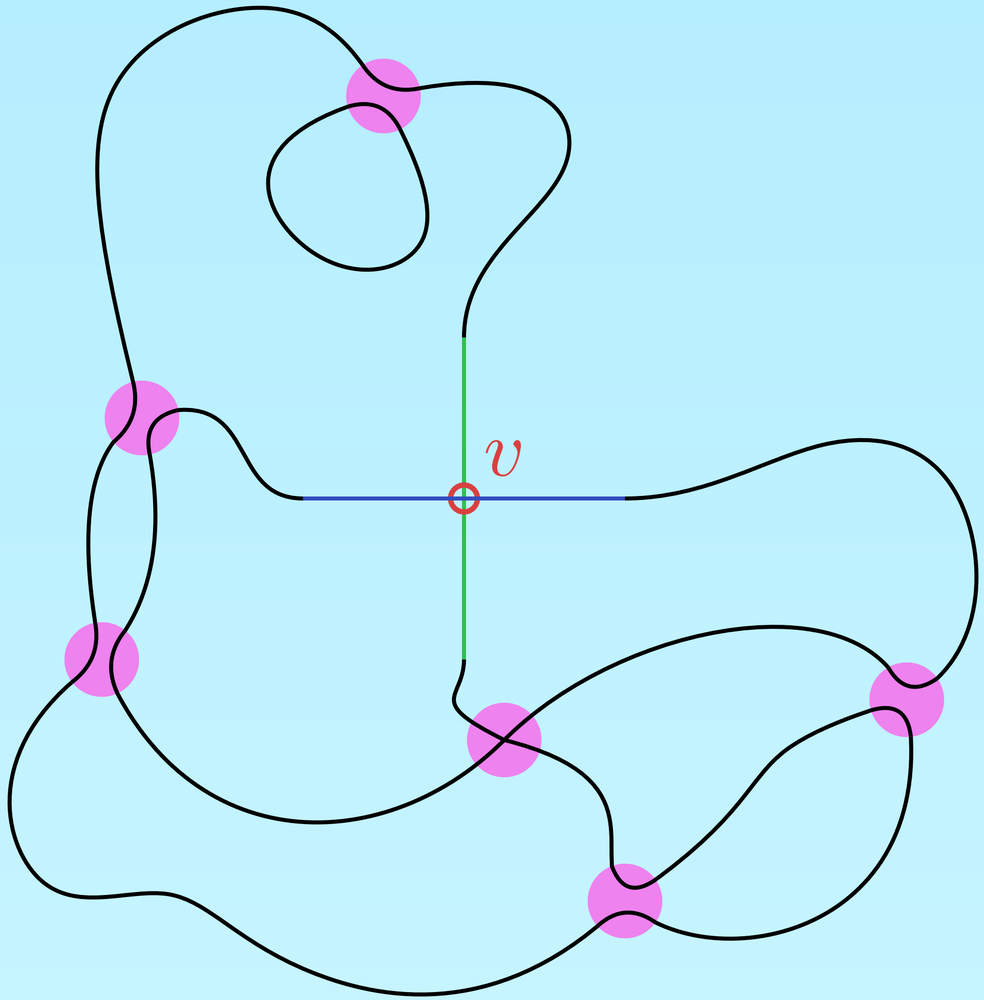

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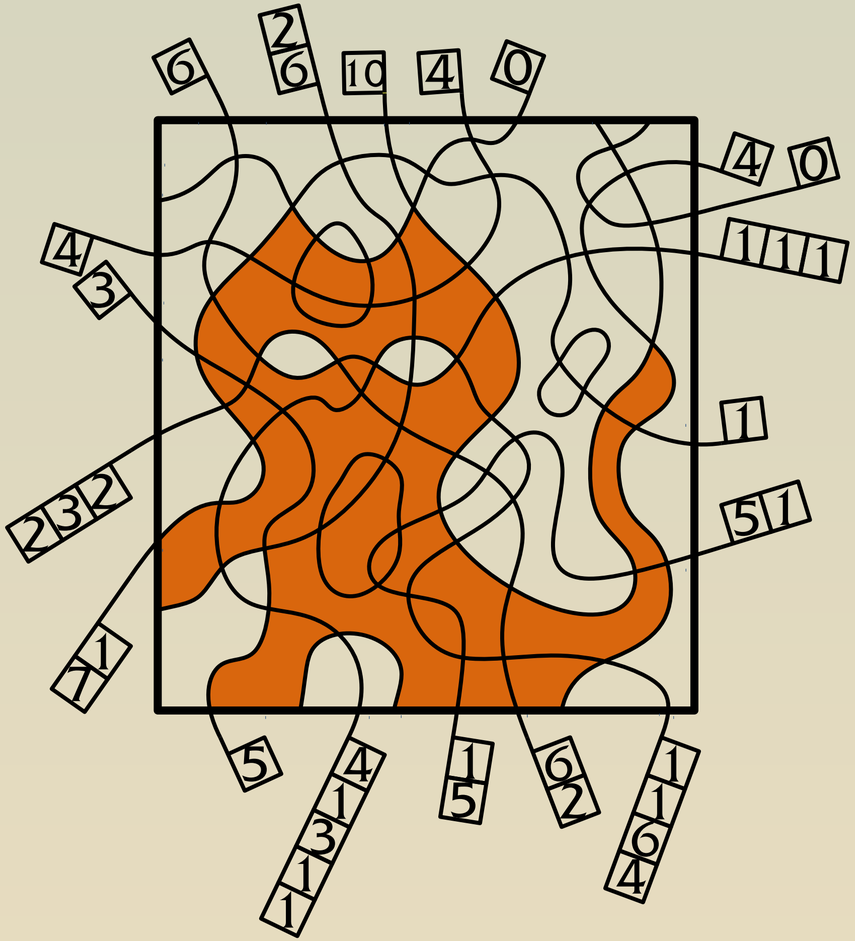
Thank You!